The calculation of climatically relevant singular vectors in the presence of weather noise as applied to the ENSO problem

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submitted to Journal of the Atmospheric Sciences in December 2002.

Abstract

We present an efficient technique for the extraction of climatically relevant singular vectors in the presence of weather noise. This technique is particularly relevant to the analysis of coupled general circulation models where the fastest growing modes are connected with weather and not climate. Climatic analysis however requires that the slow modes relevant to oceanic adjustment be extracted and so effective techniques are required to essentially filter the stochastic part of the system. The method developed here relies on the basic properties of the evolution of first moments in stochastic systems. We test the methodology for the climatically important ENSO problem and use two different coupled models. Firstly we test the method using a stochastically forced intermediate coupled model for which exact singular vectors are known. Here highly accurate estimates for the first few singular vectors are produced for the associated dynamical system without stochastic forcing. We then apply the methodology to a relatively complete coupled general circulation model which has been shown to have skill in the prediction of ENSO. The method is shown to converge rapidly with respect to the expansion basis chosen and also with respect to ensemble size. The first climatic singular vector calculated shows some resemblance to that previously extracted by other authors using observational datasets. The promising results reported here should hopefully encourage further investigation of the

methodology in a range of coupled models and for a range of physical problems where there exists a clear separation of time scales.

1 Introduction

Singular vectors are an important tool in constructing ensemble prediction schemes for weather prediction (see e.g. Palmer et al. 1993) because they allow a relatively efficient probing of the very high dimensional space of predictions consistent with small uncertainties in initial conditions (another important approach involving "bred" vectors has been also used, see e.g. Toth and Kalnay, 1997). The singular vectors have also proven particularly useful in analyzing the instability properties of the atmosphere (see e.g. Farrell and Ioannou, 1996). Climate prediction and the associated coupled model analysis are now very active and successful areas of investigation (see e.g. Latif et al., 1998 and Moore and Kleeman, 1996). As in numerical weather prediction, singular vectors have proven useful for predictability studies (e.g. Moore and Kleeman, 1998) as well as analysis (see e.g. Chen et al., 1997 and Moore and Kleeman, 2001).

Analysis to date of the coupled climate system and its predictability has taken place with intermediate complexity and hybrid models both of which have simplified steady state atmospheric models. An emerging challenge is to extend these investigations to coupled general circulation models (CGCMs) which are the most physically complete models and the future tool of choice for operational climate prediction (see e.g. Ji et al., 1994 and Stockdale et al., 1998). There is a fundamental difficulty in this program because the former models have atmospheric components which are in equilibrium with sea surface temperature (SST) while in the latter models this is not the case due to the presence of tropical weather “noise”. This has the consequence that ensemble divergence is much more rapid in CGCMs due to this stochastic forcing of the low frequency climate system. From the viewpoint of instability theory the atmospheric transients grow at a much faster rate than the climatic coupled instabilities. Clean separation of these modes by the choice of an appropriate “climate” norm is problematic since there is evidence that the stochastic forcing is effective in inducing variance in climatically relevant variables (see e.g. Kleeman and Moore, 1997, Blanke et al., 1997 and Eckert and Latif, 1997). This occurs because part of the noise is large scale and can thus project effectively onto the climate system which is most unstable at large scales (see Kleeman and Moore, 1997 for further details).

As a consequence of this, traditional calculation with adjoint versions of the CGCMs will prove difficult because the climate singular vectors will be swamped by the much faster growing atmospheric transients. In addition the separation between these scales is not necessarily clear cut. Similar problems are likely if more pragmatic “forward” calculations are used (see e.g. Chen et al. 1997, for the application of this technique to an intermediate model). Here one reconstructs the singular vectors (and other modes of interest) by applying the model to a suitable selection of initial condition perturbations. The problem here is that the stochastic forcing makes each integration of the forward model “non-unique” and so the estimate of singular vectors is likely to be non-robust (B. Kirtman, private communication).

Evidently a filtering device of some kind is required to remove the undesirable weather modes. We present in this contribution an efficient method for doing this and test it in two relevant coupled models of differing complexity. The rapid convergence and accuracy observed is very promising and deserving of further investigation in a range of coupled climate models in order to firmly establish the methods robustness.

2 A stochastically forced intermediate model as an analog for a CGCM

As is well known now the ENSO phenomenon consist of an oscillation of period around four years. This spectral peak is, however, fairly broad and the nature of this irregularity has been the subject of considerable investigation in recent years since it impacts directly of our ability to predict the phenomenon. There is now considerable evidence that a major source of the irregularity of ENSO is due to forcing of the low frequency climate manifold by atmospheric transients (see e.g. Kleeman and Power, 1994, Penland and Sardeshmukh, 1995, Kleeman and Moore, 1997, Blanke et al., 1997, Eckert and Latif, 1997 and Burgers, 1999). This forcing may be regarded as stochastic in the temporal domain but is required to have large scale coherency in order to project efficiently onto ENSO dynamics. Intermediate and hybrid coupled models which have such (prescribed) stochastic forcing added are able to account remarkably well for many aspects of observed interannual variability (see e.g Moore and Kleeman, 1999 and Blanke et al., 1997). Additionally they are able to account qualitatively and quantitatively for the observed ensemble error growth curves of CGCMs (see Kleeman and Moore, 1997 and Stockdale et al., 1998). Given this situation it seems reasonable to use such models to study the extraction of climatically relevant singular vectors. A further advantage of this approach is that the calculation of the singular vectors for the *unforced* intermediate (or hybrid) model is easy since adjoints exist for such models. Thus we are in a position to validate our methodology in an idealized setting. In this study we shall utilize the intermediate coupled model described in Kleeman et al. (1995) which has the dual advantages of an existing singular vector calculation (Moore and Kleeman, 1996) and a long successful record of operational ENSO prediction (Kleeman, 2001). A stochastically forced version of this model has been studied at length by Moore and Kleeman (1999) and we choose to use precisely this model in the present study.

3 Methodology and Results

A general dynamical system may be written compactly as

$$\overrightarrow{\psi}(t) = F(\overrightarrow{\psi}(t'))$$

where $\overrightarrow{\psi}$ is the state vector of the system and $t > t'$ is time. For "small" perturbations $\delta\overrightarrow{\psi}$ which do not grow "too much" during a particular time interval the linearized form of this equation holds approximately and may be written as:

$$\delta\overrightarrow{\psi}(t) = S(t, t-\Delta t)......S(t'+2\Delta t, t'+\Delta t)S(t'+\Delta t, t')\delta\overrightarrow{\psi}(t') \equiv R_n(t, t')\delta\overrightarrow{\psi}(t')$$

where Δt is the time-step of the associated numerical implementation of the system and for consistency $t - t' = n\Delta t$. The operators such as $S(t' +$

$\Delta t, t')$ may be referred to as the tangent linear operators and $S_n(t,t')$ as the propagator. Singular vectors for the time interval (t',t) are the eigenvectors of the operator $R_n^*(t,t')R_n(t,t')$.

In so-called forward methods for obtaining such vectors, one applies the propagator operator to two sets of initial conditions differing by the small perturbation $\delta\overrightarrow{\psi}(t')$ and at time t obtains $\delta\overrightarrow{\psi}(t)$. If this method is applied to a set of $\delta\psi$ spanning the systems state space then the appropriate matrix form of the desired operators can be constructed. In general for many models of interest (such as a CGCM) this state space is of enormous dimension so a reduced space that adequately resolves the fastest growing singular vectors is typically deployed (see Farrell and Ioannou, 2001 for a more complete discussion of these issues). Since application of the propagator to singular vectors that damp rapidly (typically most of the spectrum) results in very "small"[1] vectors such a truncation gives a reasonable matrix approximation for the action of the propagator on the most unstable part of the singular vector spectrum.

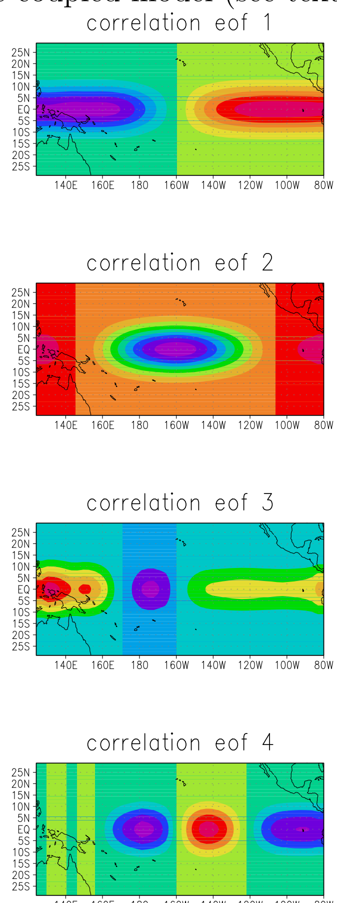
Obtaining a suitable reduced set of perturbations is an interesting question in its own right (Farrell and Ioannou, 2001) however for the case of ENSO coupled models Moore and Kleeman (2001) found a simple method which enabled the construction of reasonable reduced order models of the dynamical system studied. This was achieved by integrating the stochasti-

[1]More precisely the norm of such a resultant vector is simply the norm of the original singular vector multiplied by the corresponding eigenvalue or decay factor. This assumes that the singular vectors are calculated with reference to such a norm.

cally forced model for an extended interval and then calculating the EOF patterns of variability using the correlation matrix as opposed to the more traditional covariance matrix. An equivalent way of viewing this is that the usual EOFs of *normalized* variables were calculated. The normalization here is the (spatial) point value of the standard deviation of each variable under consideration. As has been emphasized by Farrell and Ioannou (2001), in non-normal dynamical systems there is often a great deal of difference in the spatial structure of regular EOFs and the singular vectors. This makes finding a suitable reduced state space for singular vectors using EOF expansion bases often quite problematic. In the context of a significant range of coupled ENSO dynamical models, this particular issue was explored at length in Moore and Kleeman (2001) and found to be a serious practical concern in recovering the correct singular vector structures. The use of correlation EOFs enabled this issue to be circumvented for some particularly strongly non-normal systems. The point of using correlation EOFs is that structures that may be small in amplitude but have strong large scale spatial and temporal coherency are emphasized. Singular vectors by the very nature of their excitation within dynamical systems are small vectors that grow into large vectors which span the typical large scale variability of the system. Use of the covariance matrix results in an adequate representation of large scale variability but not necessarily of the small vectors which "evolve" into it. On the other hand the correlation matrix approach simply emphasizes coherent structures which for many dynamical systems includes both singular vectors and EOFs both of which are present in extended integrations. In a linear sense the systematic methodology of Farrell and Ioannou is optimal in constructing reduced dynamical subspaces that recover both EOFs and singular vectors. It would be very instructive to inter-compare in detail the two methods for some simple systems (such as that below). The rather rapid convergence of our results both for simple and CGCM systems as well as the simplicity of the approach adopted here suggest that it may be an attractive avenue of exploration for reduced state space construction. This conclusion may however apply only to systems in which the irreducible dynamics are apparently particularly simple as appears the case for ENSO (see Moore et al., 2003 for evidence of this assertion).

In this contribution we restrict our attention to the SST part of the state space and calculate singular vectors for this variable only. The first four correlation EOFs of this variable may be seen in Figure 1. These patterns explain approximately 65%, 28%, 5% and 1.5% of the normalized variance respectively. This calculation uses a fifty year integration of our stochastically forced coupled model. Strongly apparent is the eastern Pacific pattern of ENSO variability. In addition, however, fairly high wavenumber western Pacific patterns may also be seen which is behavior characteristic of the dominant singular vectors with respect to a SST norm (Moore and Kleeman, 1997). For reconstruction of the first five singular vectors we find that retention of $5-10$ correlation EOFs is sufficient. In the results documented below six such vectors are retained.

Figure 1: The first four (normalized) correlation EOFs of sea surface temperature anomaly derived from an extended run of a stochastically forced intermediate coupled model (see text).



An important result in linear stochastic differential equations (Gardiner, 1985) is that for deterministic initial conditions, the mean of the evolving random state vector is simply the propagator applied to the initial conditions. Symbolically this may be written

$$\overline{\overrightarrow{\psi}(t)} = R(t,t')\overrightarrow{\psi}(t') \tag{1}$$

where we denote henceforth the propagator by R. Such a result suggests a rather simple strategy for reconstructing the *climatically relevant* propagator R in a CGCM: Apply the forward model to extremely similar initial conditions[2] and generate an ensemble of forward results. Use the mean of this ensemble and equation (1) to infer the structure of the climate propagator.

We apply then the stochastically forced model to initial condition pairs differing by the six dominant correlation EOFs. The particular initial condition chosen have zero anomalies in climatic variables and a background state appropriate for January. The perturbations are chosen to be the (spatially) L_2 normalized correlation EOFs multiplied by a factor of three which means typical eastern equatorial Pacific SST differences of order $0.5°C$. We repeat this forward integration using many different realizations of the stochastic forcing, construct the mean SST anomaly field and then infer the matrix structure of R on our six dimensional reduced state space. Algebraically this

[2]In principle initial conditions differing only at machine accuracy should be used however it was found empirically that differences roughly one or two orders of magnitude below the applied EOF perturbations (i.e. here typically around $0.01°C$) gave indistinguishable results.

process may be written as:

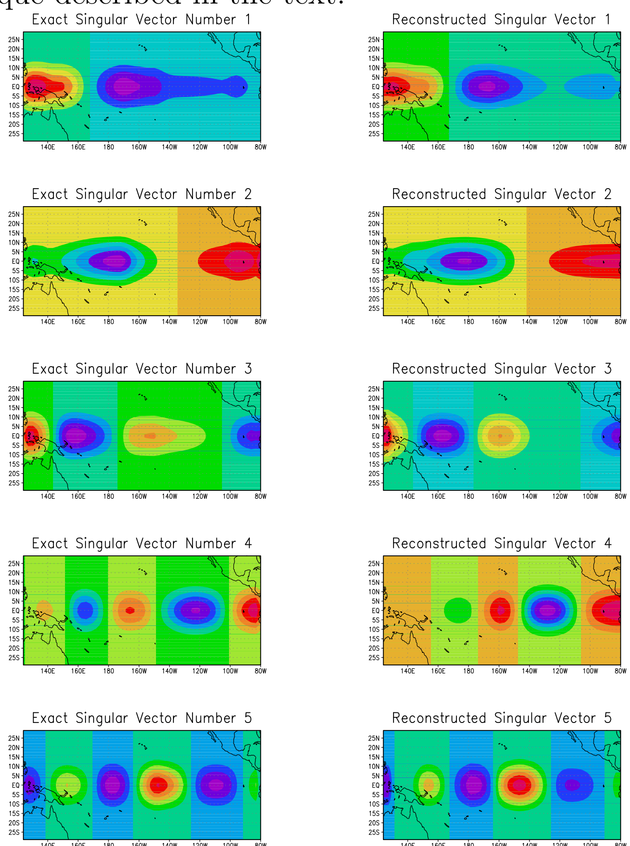
$$\Psi_{ij}^{+}(t) = F(t,t')\Psi_i^{+}(t')$$
$$\Psi_j(t) = F(t,t')\Psi(t')$$
$$\Psi_i^{+}(t') = \Psi(t') + 3 * e_i$$
$$\tfrac{1}{N}\textstyle\sum_{j=1}^{N}(\Psi_{ij}^{+}(t) - \Psi_j(t)) \equiv \sum_{k=1}^{m} R_{ik}(t,t')(\Psi_i^{+}(t') - \Psi(t'))$$

where the subscript $j = 1, ..., N$ denotes the particular stochastic forcing realization or ensemble member while the subscript $i = 1, ..., m$ represents the particular correlation EOF e_i added to the initial conditions. The operator $F(t,t')$ represents the action of integrating the stochastically forced model from time $t'$to time t.

A sample size of around $N = 50-100$ realizations of the stochastic forcing is found to be sufficient to obtain convergence of results. It was found that a sample of ten is too small to obtain robust results consistent with the results reported to the author by B. Kirtman (private communication).

Depicted in Figure 2 are the horizontal structures of the first five "climatic" singular vectors obtained by the above method and an ensemble of sixty realizations of the stochastic forcing. Also displayed for reference are the exact singular vectors calculated using the adjoint and tangent linear versions of the unforced intermediate coupled model and an SST norm. The agreement between the two very independent calculations for all five vectors is rather striking given that the growth factors (the corresponding eigenvalues) fall off rather rapidly (Moore and Kleeman, 1996) and the higher order

Figure 2: The first five singular vectors of the intermediate coupled model derived using two methods. The left hand panels are the exact vectors derived using the derived adjoint and tangent linear models of the coupled model. The right hand panels are the vectors derived from the reduced state space technique described in the text.



vectors have considerable zonal structure.

In terms of convergence of results with ensemble size, it was found that as the size is reduced the dominant vector starts to develop spurious structures of fairly small scale most noticeably in the eastern Pacific. This occurs for ensembles of less than 20-30 members.

4 Application of the methodology to a coupled general circulation model

Given the success of the methodology described above for a stochastically forced simple model we decided to test it with a relatively complete coupled general circulation model (CGCM). Evidently in this case we are unable to verify our results using the adjoint of the non-forced model since this does not exist; however we show that the method behaves robustly and produces disturbances that cause significant growth within the relevant climatic region of the eastern equatorial Pacific. We follow the following methodology:

(i) An ensemble of 50 predictions with lead times of 6 months are constructed by randomly perturbing a randomly chosen set of initial condition with 50 "very small" random patterns[3]. The ensemble mean is denoted by $\overline{\Psi}_0(t)$.

(ii) Each of the leading five normalized correlation-EOF modes e_i (i=1,2...5) are added (with a multiplication factor of 0.1 to ensure linearity) in turn to

[3] A white-noise pattern in space with a typical amplitude of around $0.01°C$ was used.

the initial condition described in (i), and new ensembles of 50 predictions produced. The corresponding ensemble means are denoted by $\overline{\Psi}_i(t)$.

(iii) One may now obtain a reduced state-space matrix version r_{ij} of the propagator R from the equations:

$$Re_i = \overline{\delta\Psi_i(t)} \equiv \overline{\Psi}_i(t) - \overline{\Psi}_0(t) = \sum_{j=1}^{5} r_{ij} e_j + Residual \tag{2}$$

The climatically relevant singular vectors for the CGCM are then the eigenvectors of the matrix $r^t r$. These five dimensional vectors may then be projected back to real SST space using the usual EOF basis vector expansion. The size and nature of the residual was examined for the calculation to be described below and was generally fairly small in amplitude and predominantly consisted of small scale features (see the comparison between Figures 6 and 7 below).

5 NSIPP coupled GCM

There are now a range of coupled general circulation models used in the operational statistical prediction of ENSO. One possible application of the methodology outlined in this paper is in improving current ensemble prediction schemes. For this reason and because it was thought prudent to select a model of reasonable prediction skill and veracity in the simulation of ENSO we decided to select the well tested NASA coupled model.

The NSIPP (NASA Seasonal-to-Interannual Prediction Project) coupled model is a fully coupled global ocean-atmosphere-land system developed at NASA Goddard Space Flight Center. It is comprised of the NSIPP atmospheric model (AGCM), the Poseidon ocean model (OGCM) and Mosaic land surface model (LSM). The NSIPP AGCM uses a finite-difference C-grid in the horizontal and a generalized sigma coordinate in the vertical. The model resolution used here is $2^o \times 2.5^o$ in the zonal and meridional directions as well as 20 layers in vertical. The top of the model atmosphere is at 10mb, where we assume $\sigma = 0$. The full model details may be found in Suarez (1996).

The OGCM is a Poseidon quasi-isopycnal ocean model (Schopf and Loughe, 1995). It is designed with a finite-difference reduce-gravity formulation which uses a generalized vertical coordinate to include a turbulent well-mixed surface layer with entrainment parameterized according to a Kraus-Turner bulk mixed layer model. The isopycnal region is treated in a quasi-isopycnal fashion, in which layers do not vanish at outcrops, but retain a thin minimum thickness at all grid points. The Pacanowski and Philander (1981) mixing scheme is used to parameterize the subsurface mixing and diffusion. The pressure field is designed with a reduced-gravity formulation. The version used here has horizontal resolution $1.25^o \times 0.5^o$ in the zonal and meridional directions and 20 layers in vertical.

The LSM computes area averaged energy and water fluxes from the land surface in response to meteorological forcing. The model allows explicit vegetation control over the surface energy and water balances. The land surface scheme is based on the simplified version of the Simple Biosphere, but extended to a more complicated framework accounting for sub-grid variability in surface characteristics through the "mosaic" approach. The model details can be found in Koster and Suarez (1996).

The atmospheric, ocean and land models are coupled by the Goddard Earth Modeling System, which produces routine experimental ENSO prediction of SST for the tropical Pacific region. The hindcasts during 1981-1998 show useful skill (i.e., NINO3 SST anomaly correlation scores of at least 0.6) for 6-7 month without ocean data assimilation, and the assimilation of altimeter data leads to better prediction skill (Rienecker, 2000).

The model was integrated freely for an extended period (50 years) in order to calculate the correlation EOFs, assess interannual variability as well as produce suitable initial conditions. The model exhibits considerable interannual variability with however a tendency for a bias toward biennial rather than the observed four year period variability.

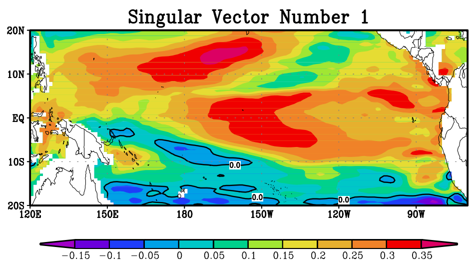
6 Results

6.1 The optimal perturbation and its growth rate

With the method described in the previous section, the singular vectors of the fully coupled model are obtained. We focus our discussion firstly on the fastest growing mode.

Figure 3 is the first singular vector mode, denoted by **S**. This pattern

Figure 3: The spatial pattern of the first singular vector {\textbf{S}} of SST derived from the NASA CGCM using the reduced state space technique.



shows a large-scale structure with major weighting located in the equatorial central and to a lesser extent in the eastern Pacific, and also rather surprisingly in the northern subtropics. The warm anomalies in the equatorial central and eastern Pacific are a common feature for the optimal perturbation in many intermediate ENSO models (e.g., Chen et al. 1997) but the anomalies in the Northwest Pacific region in Figure 3 are absent in simpler coupled models. Interestingly however there are similar anomalies in this region in the optimal perturbation derived from observed SST (Penland and Sardeshmukh, 1995). As with certain coupled intermediate models (e.g. Moore and Kleeman, 1996) it is notable that there is a strong weight to the singular vector in the dateline equatorial region as opposed to the eastern equatorial region where generally variance is greatest and the dominant (covariance) EOF has most loading.

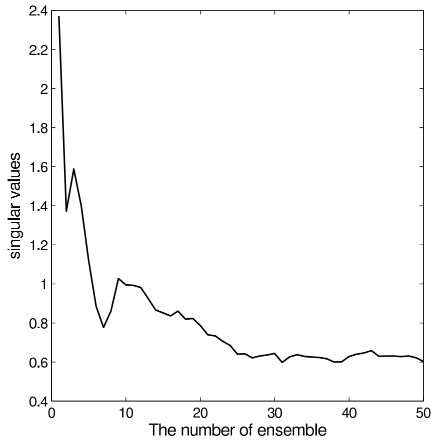
The growth rate corresponding to Figure 3 is 0.60, indicating that there are only decaying singular vectors in the fully coupled GCM model. This is rather different from the results obtained from either intermediate models or observation where there is at least one growing optimal perturbation. It is worth noting however that this growth is with respect to the truncated L_2 norm of the reduced state space. As we shall see below when a norm centered on the eastern equatorial Pacific is used significant growth is possible. Additionally even for the singular vector S significant growth occurs in the eastern equatorial region (see below).

Note also that the growth rate is likely sensitive to the reference trajectory to which the perturbation is applied. In this study, the perturbation starts in a particular December derived from a long coupled integration and and corresponds in that run to the onset of a La Niña event. Such a reference trajectory has been found to generate the smallest growth rate by many intermediate coupled models (Moore and Kleeman 1996; Chen et al. 1997), because the spring warming of the central Pacific is suppressed by a cold phase of ENSO cycle.

6.2 Verification of the methodology

Unlike in the intermediate model discussed above, there is no rigorous way of obtaining exact singular vectors for the CGCM. We can however, apply some indirect tests to verify that the singular vectors generated by the method are robust to the details of the technique and have the expected effect on the model when applied as perturbations. We firstly examine whether the com-

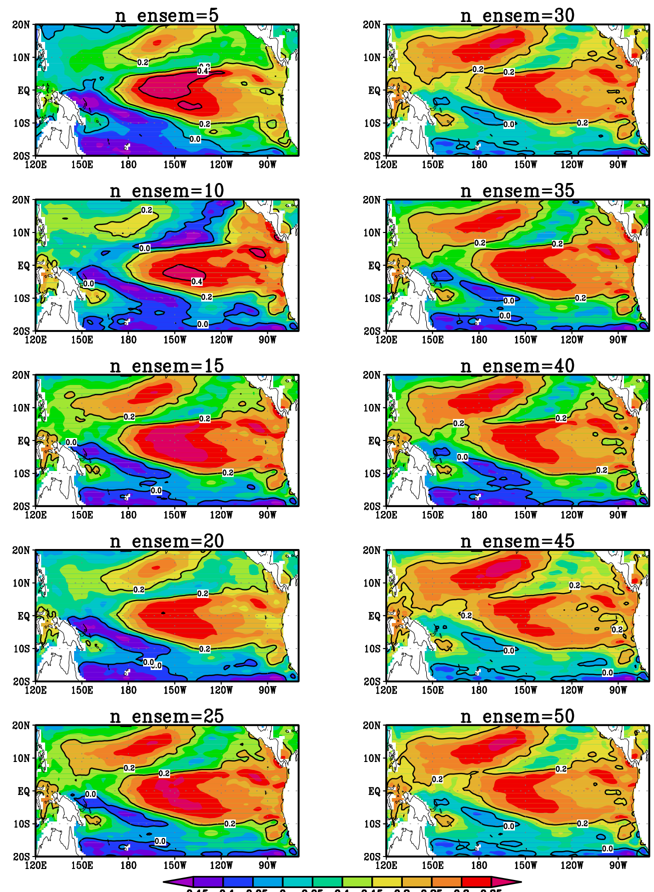
Figure 4: The convergence of the growth rate of the dominant singular vector derived from the NASA CGCM plotted as a function of the ensemble size used in its derivation (see text).



puted $\mathbf{S}$ converges with respect to increasing ensemble size. Plotted in Figure 4 is the first singular vector growth rate λ_1 as a function of the ensemble size and it is clear that convergence is achieved with $25-35$ members. The spatial pattern $\mathbf{S}$ shows similar convergence properties as seen in Figure 5.

In a second test of robustness we directly force the CGCM with the first singular vector to determine that the response of the model to the perturbation is similar to that calculated using matrices on the reduced state space. Thus we performed another ensemble experiment similar to the step (ii) in

Figure 5: The spatial pattern of the first SST singular vector ({\textbf{S}}) of the NASA CGCM plotted as a function of the ensemble size used in its derivation.



Section 1, but with the optimal perturbation $\mathbf{S}$ used instead of an EOF mode ($\mathbf{e}_i$) to perturb the model initial conditions. The ensemble size is chosen to be 30 in the light of the convergence properties discussed above.

Denote $\overline{\delta\Psi}(t) \equiv \overline{\Psi_{opt}(t)} - \overline{\Psi_0(t)}$, where the over-bars mean ensemble averages and the subscripts indicate how each ensemble is created. It is easily shown now that the growth rate of the first singular vector should satisfy the relation

$$\lambda_1^2 \sim \left(\lambda_1^d\right)^2 \equiv \overline{\delta\Psi}^T(t)\overline{\delta\Psi}(t)/\left(\mathbf{S}^T\mathbf{S}\right) \tag{3}$$

One may not expect this to be an exact relation given the truncation of the EOF basis and also the finite ensemble size. Nevertheless a direct calculation gives $\lambda_1^d = 0.66$ which compares well with our matrix eigenvalue λ_1.

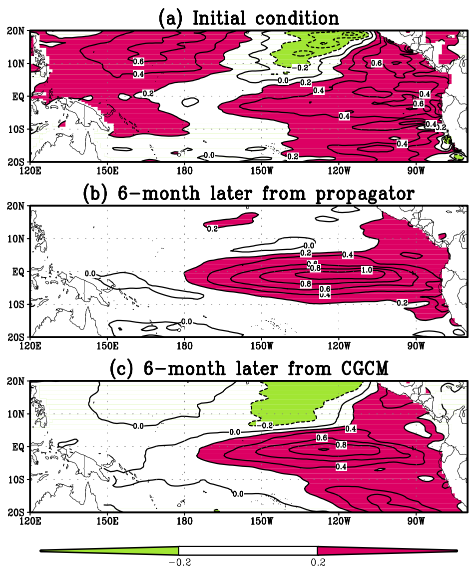
One may also check the singular value λ_1 directly by computing the amplification of the optimal perturbation in the reduced EOF space i.e.,

$$\lambda_1 \simeq \frac{\|\overline{\delta\Psi^{eof}(t)}\|}{\|\overline{\delta\Psi^{eof}(0)}\|} \tag{4}$$

where $\overline{\delta\Psi^{eof}(t)}$ is the projection of $\overline{\delta\Psi(t)}$ onto the reduced state-space. This gives the λ_1 estimate of 0.597, which is very close to that one obtains from the matrix calculation. This agreement between directly calculated values of the growth rate and those derived from our matrix on the reduced state space is reassuring.

The spatial structures of SST anomaly in the five dimensional reduced state space at initial time and after 6 months evolution are shown in Figure

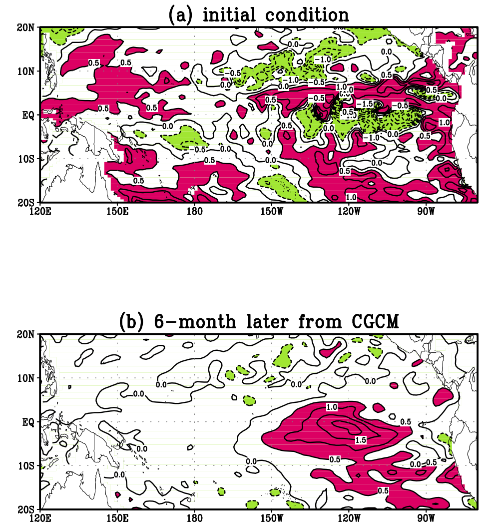
Figure 6: The time evolution of SST anomaly when a perturbation with the structure of the first singular vector is added to the specific initial conditions used for the singular vector calculation. a) The initial pattern in reduced state space. b) The time evolved pattern after six months as deduced from the reduced state space propagator. c) The same as b) but from a direct CGCM calculation and then projected onto the reduced space. In all cases the total SST anomaly projected onto the reduced state space is plotted.



6. Figure 6b is obtained using the matrix propagator and Figure 6c is the ensemble mean of the direct calculation described above. In Figure 6a the optimal perturbation is superimposed on the randomly chosen (December) initial condition SST anomalies. As can be seen, the final SSTA pattern obtained using the propagator is in good agreement with that obtained from the direct CGCM integrations, (there is a correlation of 0.84 between the two patterns).

It is quite striking that the final field is concentrated in the dynamically

Figure 7: Same as previous Figure but for the case in which the SST anomaly is not projected onto the reduced state space. In other words this represents the raw evolution of SST anomaly from the chosen initial conditions after the imposition of the first singular vector (see text).



important eastern equatorial Pacific and that in this region, unlike the domain as a whole, significant amplification occurs.

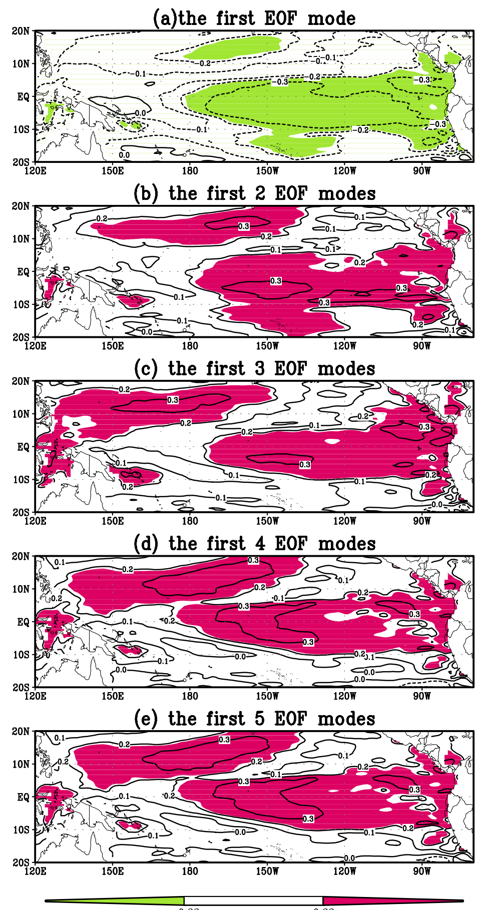
Figure 7 is the same as Figure 6ac but for the total state space rather than just the projection onto our reduced state space. Note that the initial conditions chosen already have significant SST anomalies associated with them that do not project onto our reduced state space hence the very different structure of the initial condition field in 7a versus 6a. The growth results are qualitatively the same as the previous Figure although smaller scale features are more evident as one may have anticipated since such features are absent from the reduced state space. Again we notice the strong amplification in the eastern equatorial Pacific.

6.3 Sensitivity of the dominant optimal perturbation to the reduced state space dimension

An important issue concerning the algorithm proposed here is computation cost given that an expensive CGCM is being integrated over many ensemble members and reduced state space basis vectors. As we have seen above an ensemble size of around 25-35 appears sufficient to ensure adequate convergence. Here we investigate how many basis vectors are required for convergence: In principle, the bigger the ensemble size and reduced space size, the more accurate the computed singular vectors.

Figure 8 shows the optimal perturbation obtained using different EOF modes for the reduced space. The ensemble size is 50 in all cases. As can be seen, the optimal perturbation tends toward a relatively stable structure when only around the first 3 EOF modes are retained. The correlation of the optimal perturbation from 3 dimensional reduced space and from standard 5 dimensional space is 0.91. The growth rate is also quite close to the five mode case.

Figure 8: The spatial structure of the first singular vector from the NASA CGCM plotted as a function of the dimension of the reduced state space used in its calculation (see text).



6.4 Norm dependency of results

An important feature of singular value calculations is their often significant dependence on the norm utilized. Given the large concentration of the evolved perturbation discussed above on the eastern equatorial Pacific one might expect that there may be some sensitivity of growth rate to the region used in the norm. This turns out to be the case here as well and if one uses the $(NINO3)^2$ norm utilized previously in the literature when considering intermediate coupled models then quite significant growth (a factor of around 2.7 for the first vector) occurs. The spatial structure of the vector also shows some sensitivity with respect to the Northwest Pacific region (not shown) although the overall large scale structure is qualitatively similar.

7 Summary and discussion

A fundamental problem in extracting climatically relevant singular vectors in a coupled general circulation model is the undoubted fact that the fastest growing modes will be heavily atmospheric in character. Evidently a filtering device of some kind is required in order to extract those perturbations responsible for large climatic as opposed to atmospheric deviations. Here an ensemble technique for extracting the slow "climatic" instability properties of the coupled ocean atmosphere system has been developed. Using a stochastically forced intermediate coupled model we are able to recover almost exactly the dominant "climatic" singular vector spectrum using a relatively inexpensive methodology. The results described here are simply a “proof of concept” and they need to be repeated with more complex hybrid coupled models (see Moore et al., 2003) and with a more complete state space than just SST. Nevertheless their striking accuracy is a promising sign.

Given the success with the intermediate model we then applied the methodology to a fully coupled GCM (the NSIPP model) which has reasonably realistic ENSO variability as well as skill in prediction. The results obtained show that the algorithm is an effective and robust method for the calculation of the *climatically relevant* singular vectors of CGCMs.

Sensitivity experiments show that the ensemble size of approximately 30 is sufficient to obtain converged climatic singular vectors for a fully coupled GCM model. Likewise a reduced state space of 3 EOF modes seems sufficient to resolve the fastest growing climate singular vector.

The spatial structure of the dominant singular vector differs from those obtained previously from intermediate models but shows quite some similarity to that obtained by Penland and Sardeshmukh (1995) from observational data.

8 Acknowledgments

RK would like to thank Max Suarez from NASA Goddard and Michael Tippett from the IRI for very useful discussions. RK and YT were supported by NASA grant NAG5-9871 and NSF grant ATM-0071342.

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