Comment on
"Sigma-Point Kalman Filter Data Assimilation Methods for Strongly Nonlinear Systems"

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Ambadan and Tang (2009; hereafter "AT09") recently performed a study of several varieties of a "sigma-point" Kalman filter (SPKF) using two strongly nonlinear models, Lorenz (1963; hereafter L63) and Lorenz (1996; hereafter L96). In this comparison, a reference benchmark was the performance of a standard ensemble Kalman filter (EnKF) of Evensen (1994, 2003), presumably with perturbed observations following Houtekamer and Mitchell (1998) and Burgers et al. (1998). We have identified problems in the description of the EnKF as well as its application with the L63 and L96 models.

a. Problem in the description of the EnKF.

AT09 stated (page 262, column 1) as a drawback of the EnKF that it " … assumes a linear measurement operator; if the measurement function is nonlinear, it has to be linearized in the EnKF." This statement is incorrect; the EnKF is routinely applied with nonlinear measurement operators; the standard formulation for this is shown in Hamill (2006), eqs. 6.11, 6.14, and 6.15.

b. L63 experiments.

AT09's examination of the EnKF with small ensembles was potentially misleading. They chose to include a white-noise model of unknown model errors in their assimilating model. This representation of model error was particularly poorly suited for use with EnKFs; in fact, AT09 showed that a 19-member ensemble had a root-mean square error (RMSE) more than three times larger than a 1000-member ensemble. However, a 19-member ensemble in fact has an RMSE that is only 1.05 times that of the

1000-member ensemble when white noise is removed from the assimilating model. This is consistent with the successful application of EnKFs with 10 to 100 members, even in large numerical weather prediction models (e.g., Houtekamer et al. 2005, 2009, and Whitaker et al. 2008).

c. L96 experiments.

AT09's EnKF reference was badly degraded by not using covariance localization and/or other methods to stabilize the filter.

Much has been learned about the performance of the ensemble-based data assimilation methods since the preliminary studies of the 1990s, lessons that AT09 apparently did not incorporate into their L96 EnKF reference. Since the early implementations of the EnKF, several now standard modifications are commonly considered to be essential in spatially distributed systems; the first is some form of "localization" of covariances (Houtekamer and Mitchell 2001; Hamill et al. 2001). Another common technique for the stabilization of the EnKF is the enlargement of the prior through "covariance inflation" (Anderson and Anderson 1999) or through additive noise (Houtekamer et al. 2005, Hamill and Whitaker 2005). Without judicious application of such techniques, poor performance or even filter divergence may occur in ensemble filters.

To review briefly, covariance localization modifies the estimate of covariances provided directly by the ensemble. When assimilating a given observation, the ensemble estimates of the cross covariance between the state at the observation location and the state at surrounding grid points are multiplied by a number between 0 and 1; typically, grid points near the observation location have their covariances multiplied by a number near 1.0, and the further distant a grid point is from the observation, the nearer the multiplication factor to 0.0. This function is usually a smooth function of distance between the observation and grid point, and compactly supported. As explained in Hamill et al. (2001), covariance localization provides several beneficial effects: it greatly increases the effective rank of the background-error covariance matrix, it filters out noisy far-field covariances, and consequently it can prevent an over-fitting to the observations and a collapse of spread in the ensemble. Because of its strongly positive effect on EnKF performance, some form of localization is now common in almost all implementations of the EnKF for large-dimensional, spatially distributed systems. Further, there is now a substantial body of research on various alternative localization techniques, the benefits and the tradeoffs. Mitchell et al. (2002) and Lorenc (2003) have pointed out that despite the beneficial effects, covariance localization can introduce state imbalances; Anderson (2006) has discussed an adaptive localization technique that does not require tuning. Hamill (2006) provides a review of localization and pseudo-code for a filter that includes this. Hunt et al. (2007) demonstrate in their local ensemble transform Kalman filter that an effect similar to localization can be achieved through distance-dependent re-weighting of observation-error variances. Buehner and Charron (2007) discuss localization in spectral space. Zhou et al. (2008) discuss a multi-scale localization alternative. Bishop and Hodyss (2009ab) discuss how localization may be improved through power transformations. Kepert (2009) discusses balance issues as well as how localization may be improved through a transformation of the state vector.

Another common modification is to increase the variances in the prior as a guard against over-fitting and eventual filter divergence. These are helpful in perfect-model scenarios and nearly ubiquitous in real-world scenarios with model error. Anderson and Anderson (1999) proposed a method now known as "covariance inflation" whereby perturbations around the mean state are inflated by some constant somewhat greater than 1.0. Recently, Anderson (2007) has proposed an adaptive method for controlling the amount of inflation to apply, based on innovation statistics. Another general method is the addition of structured noise to each member of an ensemble, as demonstrated in Houtekamer et al. (2005) and Hamill and Whitaker (2005). Zhang et al. (2004) propose a method they called "relaxation to prior" whereby after the data assimilation step, the posterior perturbations are enlarged somewhat in the direction of the prior perturbations.

Do these details have a profound effect? In the case of the simulations presented in AT09, the answer is clearly "*yes*." In section 5c of their article, they compared their 200-member sigma-point Kalman filter (SPKF) against a 200-member EnKF using the L96 model with a state vector of 960 elements. They correlated the time series of ensemble-mean analyses and forecasts with the truth, obtaining a correlation of 0.59 for the SPKF and 0.10 for the EnKF (their correlations were calculated from the first element of the 960-dimensional state). However, in our simulations where both covariance localization and a mild 2% covariance inflation were used, profoundly higher scores were obtained. When localization was applied using the compactly supported near-Gaussian function of Gaspari and Cohn (1999; eq. 4.10) with a localization radius of 10 grid points (the multiplication factor is 0.0 for 10 grid points and beyond), and with a 2 percent covariance inflation, the correlation increased from 0.10 to 0.92. Figure 1 provides a time series for this configuration of the EnKF, corresponding with AT09's EnKF in Fig. 15c. Note also that it may not be necessary to determine an effective localization radius and a covariance inflation value by trial and error; techniques that automatically determine appropriate values as part of the assimilation process are in widespread use in atmospheric applications (Anderson 2006, 2009).

A general conclusion that may be drawn from these L96 results is that any filter with a covariance model whose effective rank is much smaller than the effective number of degrees of freedom in the model is unlikely to produce high-quality analyses. Covariance localization, despite the noted drawbacks, provides a computationally tractable way of increasing the effective rank of a EnKF's background-error covariance matrix, filtering noisy ensemble estimates, and consequently improving ensemble performance. Again, evidence of the performance of such filters in real, high-dimensional weather prediction models can be found, for example, in Houtekamer et al. (2005), Whitaker et al. (2008), and Houtekamer et al. (2009).

There is great interest in the development of ensemble filters and reduced-rank filters. We argue that for all future manuscripts, should the authors wish to compare against an ensemble data assimilation method as a benchmark, they must make a good-faith effort to compare against some *state-of-the-art* version of the filter. In 2009, this means a filter with some incorporation of the concepts of localization plus inflation and/or additive noise. Whitaker and Hamill (2002, 2006) provide an example of the sort of exploration of this parameter space that is warranted when choosing the configuration of a filter, and the subsequent range of filter performance.

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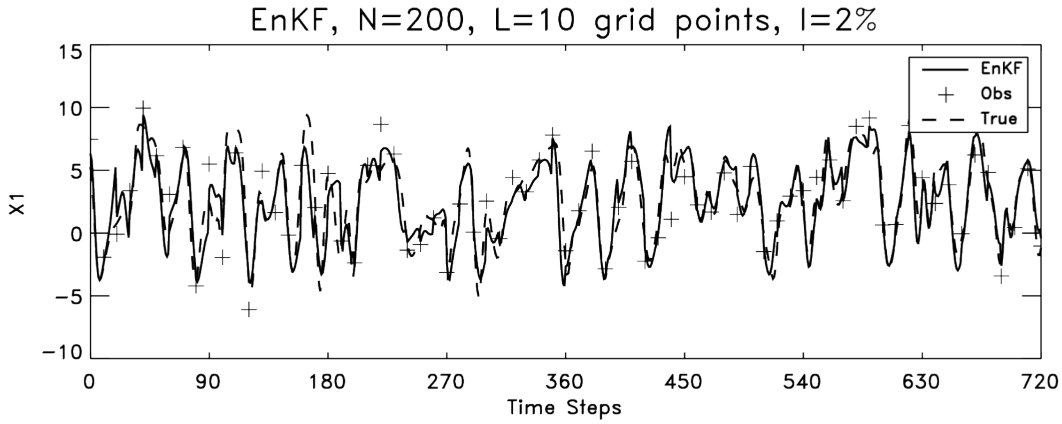

Figure 1: Time series for the first element of a 960-dimensional Lorenz ’96 system. Solid line denotes the trajectory of the EnKF ensemble-mean analysis and forecast; + signs denote the observations that were assimilated, and dashed line denotes the truth.