PGSS, Prince George, BC

Principles of Mathematics 11

Lesson Plans

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2/7/2008

Contents

1.1 Solving Systems of Linear Equations Graphically

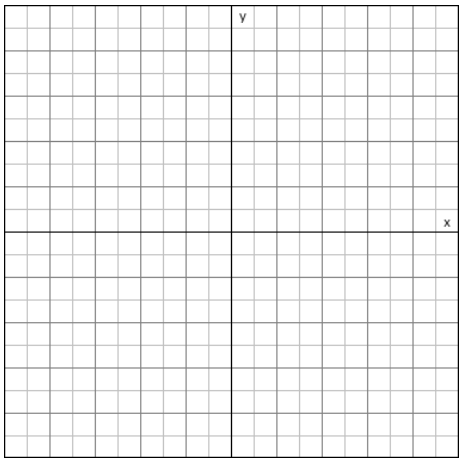
Learning Outcomes

- ✓ Solve systems of equations graphically using the slope and y-intercept form
- ✓ Solve systems of equations graphically using the intercepts
- ✓ Determine the number of solutions to a system of equations
- ✓ Analyze systems of equations to determine the number of solutions

Warm-Up

Questions for students to do:

1. Which point lies on the line with given equation?
   a. $x - 3y = 1$; $(5,2), (2,-1), (4,1), (-1,1)$
   b. $-4x + y = -2$; $(1,1), (2,1), (1,2), (3,-1)$
2. Write three ordered pairs that satisfy each equation (answers vary).
   a. $x + 3y = 3$
   b. $y - 2x = -2$
3. How is slope defined? Write it in words as a *ratio*.
4. Write each equation in the slope and y-intercept form. Use the slope and the y-intercept of each equation to graph it.
   a. $2x = 3y$
   b. $4y + x = 1$
   c. $2x = 5y - 8$



Activities

Show example of two lines intersecting on a graph.

Steps to solving linear equations graphically:

1. Change equations into slope and y-intercept form ($y = mx + b$)
2. Draw the lines (use the slope and intercept)
3. Identify the point at which the two lines intercept (if any)
4. To check that this point satisfies both equations, plug into both equations and verify.

Guided Practice (solving solutions)

(1) $3x + y = 2, x - y = 2$
(2) $4x - y = 8, x + 2y = -7$

Types of solutions to systems of linear equations:

Guided Practice (analyzing solutions):

(1) $2x + y = 3, \quad y - 8 = -2x$
(2) $2x + y - 4 = 0, \quad x + 2y - 6 = 0$

(2) The slope is -2, therefore lines are parallel or coincide. Slopes are different, so that there are no solutions.

(1) The slopes of the lines are -2 and $-\frac{1}{2}$. The lines are not parallel and do not coincide

Comment [M1]: Not going to use a graphing calculator yet

Comment [M2]: TODO

Graphs of Lines	Slopes of Lines	Intercepts	Number of Solutions
Intersecting	Different	Different unless the lines intersect on one axis or at the origin	One
Parallel	Same	Different	None
Coincident	Same	Same	Infinitely Many

Comment [M3]: Two equations that coincide are called equivalent equations

Assignment

One of

- Mathpower 11, p.3 #2-44 (even)
- Teacher's Resource Worksheet

And:

- Journal Response #1 (Due in one week)

1.3 Solving Systems of Linear Equations by Substitution

Learning Outcomes

- ✓ Solve systems of equations by substitution
- ✓ Find the exact solutions of a system of equations
- ✓ Solve investment problems and mixture problems involving systems of equations

Warm-Up

1. Write each equation in terms of the variable indicated.
   a. $x + 4y = 10$, x
   b. $2y - 3x = 1$, y
2. Solve for the variable.
   a. $2x - 4 = 3$,
   b. $-4x + 7 = -2$,
   c. $3x + 5x - 4 = 9, \quad x = \frac{13}{8}$

Comment [M4]: $x = 10 - 4y$
Comment [M5]: $y = \frac{3}{2}x + \frac{1}{2}$
Comment [M6]: $x = \frac{7}{2}$
Comment [M7]: $x = \frac{9}{4}$

Activities

Steps to solve a system of equations by substitution.
Example: $3x - y - 2 = 0, 5x + 2y = 3$

1. Solve one of the equations for one variable (y or x).
$$\mathbf{y = 3x - 2}$$
$$\mathbf{5x + 2y = 3}$$
2. Substitute the equation of this variable into the other equation.
$$5x + 2(3x - 2) = 3$$
$$5x + 6x - 4 = 3$$
$$11x = 7$$
$$x = \frac{7}{11}$$
3. Solve for the other variable by substituting the answer found in step 2.
$$y = 3x - 2$$
$$y = 3\left(\frac{7}{11}\right) - 2 = -\frac{1}{11}$$
4. Write the solution.
$$\left(\frac{7}{11}, -\frac{1}{11}\right)$$
5. Check the solution.

LHS	RHS
$5x + 2y$ $= 5\left(\frac{7}{11}\right) + \mathbf{2}\left(-\frac{\mathbf{1}}{\mathbf{11}}\right)$ $= \frac{35}{11} - \frac{\mathbf{2}}{11}$ $= \frac{\mathbf{33}}{\mathbf{11}}$ $= \mathbf{3}$	3

Comment [M8]: Solved for y

Guided Practice:

1. $2e + 3f = 5, \; e - 4f = -14$

Group Activity: Investing Money Worksheet

Assignment

One of:

- Mathpower 11, p.25 #2-24 (even), 27, 30, 35, 43

- Teacher’s Resource Worksheet

1.5 Solving Systems of Linear Equations by Elimination by Add/Sub

Learning Outcomes

- ✓ Solve a system of equation by elimination using addition
- ✓ Solve a system of equations by elimination using subtraction or addition

Homework Check

Comment [M9]: 25 min.

- Mathpower 11, p.25 #2-24 (even), 27, 30, 35, 43

Warm-Up

Comment [M10]: 20 min.

1. Find the lowest common multiple of each pair of numbers.
   a. $3, 5$
   b. $5, 6$
   c. $8, 12$
   d. $-10, -6$
   e. $7, -21$
2. Write the additive inverse (opposite) of each number.
   a. 7
   b. -20
   c. 102
   d. $-(-5)$
3. Group Activity: Investing Money Worksheet

Comment [M11]: 12 min.

Activities

Comment [M12]: 20 min.

Say: The substitution method works well when at least one variable in one or both equations has a coefficient of 1 or -1. With other coefficients, substitution may lead to complicated equations, and it may be better to use the elimination method.

Method of elimination uses this property of equality:

Since	$4 = 4$	If	$a = b$
and	$3 = 3$	and	$c = d$
then	$4 + 3 = 4 + 3$	then	$a + c = b + d$
and	$4 - 3 = 4 - 3$	and	$a - c = b - d$

Solving by addition.

The number of lace holes in a running shoe is represented by x, and the number of lace holes in a boot is represented by y. These two equations represent the relationship between the number of holes.

$$x + y = 20 \quad (1)$$

$$2x - y = 16 \quad (2)$$

How many lace holes does the running shoe have, how many lace holes does the boot have?

Have the students express the relationship in words. Then, have them write an equivalent equation to (1) and (2) by adding (1) and (2).

Show this! Write a line and a plus sign and indicate that they are to add the corresponding terms.

After this is complete, label this equation (3).

Questions:

- ❖ Why do you think you are asked to add (1) and (2) to add (3)?
- ❖ How many variables does (3) have?

- ❖ How could you solve (3)?
- ❖ How could you use the solution to (3) to find the value of the other variable in (1)?

Ask them to find the other variable using the variable just found.

- ❖ What is the solution to the given system?

$$(x, y) = (12,8)$$

- ❖ How can you check your solution?

Answer: plug the solution into the other equation.

LHS	RHS
$2x - y$ $= 2(12) - (8)$ $= 24 - 8$ 16	16

- ❖ Why do you think this method of solving a system of linear equations is called the method of elimination by addition?

Assignment

- p.

Solving by subtraction.

Present this system of equations to the class.

$$x - 2y = -5 \quad (4)$$

$$x + y = 1 \quad (5)$$

Ask:

- ❖ Can you use the method of elimination by adding to solve (4) and (5)?

  Not quite. We can't just add (See next step).

- ❖ What could you do to equation (4) and (5) in order to eliminate one of the variables?

  Elicit that we should subtract them. Write this on the screen.

- ❖ This method of solving a system of equations is called the method of elimination by subtraction. Explain.
- ❖ What is the solution to this system of equations?

$$(-1,2)$$

1.5 Solving Systems of Linear Equations by Elimination by Multiplication

Learning Outcomes

- ✓ Solve a system of equations by elimination using multiplication
- ✓ Solve a system of rational equations
- ✓ Solve problems involving systems of equations

Warm-Up

$$\frac{4}{15}+\frac{7}{3}=\frac{39}{15}=\frac{13}{5}$$

$$\frac{11}{24}-\frac{3}{16}=\frac{13}{48}$$

$$-\frac{4}{9}+\frac{3}{5}=-\frac{7}{45}$$

$$\frac{9}{25}+\frac{6}{15}=$$

Activities

Present this system of equation to the class.

$$2x-8y=-2 \qquad (1)$$

$$-3x+12y=3 \qquad (2)$$

Ask:

- ❖ Is it possible to eliminate one of the variables by adding or subtracting (1) and (2)?
- ❖ What would you have to do to (7) so that you could use the method of elimination by addition to solve the system?
- ❖ What would you have to do to (7) and (8) in order to use the method of elimination by subtraction to solve the system?

  This is a multi-step question. Recall the LCM we did in the warm-up; we will use this idea in this question.
  What is the LCM of 2 and -3?
  What is the LCM of -8 and 12?

  One half of the class will try to eliminate x, the other for y.

- ❖ This method of solving a system of equation is called the method of elimination by multiplication. Explain.
- ❖ What is the solution to this system of equations?

$$(3,1)$$

- ❖ How can you check your solution?

(Substitute back into one of the equations. Have them do this as well.)

Elimination Worksheet

Solving systems of rational equations

Example.

1. $\begin{cases} \frac{x-1}{4}+\frac{y+2}{5}=2 \\ \frac{x+3}{2}-\frac{y+1}{4}=3 \end{cases}$

2. $\begin{cases} 5(x-1)+4(y+2)=40 \\ 2(x+3)-(y+1)=12 \end{cases}$

$$\begin{cases} 5x+4y=37 \\ 2x-y=7 \end{cases}$$

Comment [M13]: $\times 4$

$$\begin{cases} 5x+4y=37 \\ 8x-4y=28 \end{cases}$$

$$13x=65$$

$$x=5$$

$$y=3$$

Assignment

One of:

- p.39 # 17-35 (odd), 39, 44
- Teacher Resource Worksheet

1.6 Solving Systems of Linear Equations in Three Variables

Learning Outcomes

- Solve systems of linear equations in three variables by elimination
- Solve problems involving systems of linear equations in three variables

Homework Correction

Quiz

Activity

Write:

Let's see if we can use the method of elimination by addition or subtraction on this system:

$$x + y + 3z = 12$$

$$2x + y + 3z = 14$$

$$x - y + 4z = 13$$

Students complete steps:

1. Multiply the first equation by two and subtract it from the second equation.
2. Subtract the first and third equations.
3. The equations in step 1 and step 2 form a system of equations in two variables. Solve that system.

$$-y - 3z = -10$$

$$-4y + 2z = 2$$

1. Multiply the first equation by -4.

$$4y + 12z = 40$$

$$-4y + 2z = 2$$

2. Add the two equations.

$$14z = 42$$

$$z = 3$$

3. Find y. (Hint: use the second pair of equation we found).

$$y = 1$$

4. Find x. (Hint: use the first system of equations; now *two* substitutions).

$$x = 2$$

Assignment

p.44 #2-10 (even), 11-34 (first and last)

1.1 Solving Systems of Equations using a Graphing Calculator

Learning Outcomes

- ✓ Solve systems of equations graphically using the intercepts
- ✓ Determine the number of solutions to a system of equations

Homework Correction

p.44 #2-10 (even), 11-34 (first and last)

Comment [M14]: 30 min
(Period 3) 12:26-12:56

Activity

- TI-84 Tutorial
- Revisit to the Teacher's Resource worksheet (1.1 Solving Systems of Equations by Graphing) – Students should have this with them
    - Note: when working on problems that involve systems and graphing calculators, the equations must be solved for y (into $y = mx + b$ form)
    - Have students graph each system on their graphic calculators and check that their solutions (when they completed the worksheet by hand) are correct
    - Hand-in worksheet with corrected work
- Group Work (based on pg.12, Mathpower 11), Pairs, Not to be handed in
    - Students will answer the following questions; each student on a separate piece of paper
        1. For many systems solved graphically using a graphing calculator, the point of intersection does not fall within the standard viewing window. Consider the following systems of equations.

$$a.\quad y = -2x - 16,\quad y = 4x + 59$$

$$b.\quad y = x - 24,\quad y = -2x + 120$$

$$c.\quad y = x - 2,\quad y = \frac{x}{2} - 10$$

        2. Describe how you found suitable values for the windows variables in each case.
        3. Compare your answers to part 2 with those of your classmates.

Comment [M15]: 20 min
(Period 3) 12:56-1:16

Comment [M16]: 12 min
(Period 3) 1:16-1:28

Comment [M17]: 16 min
(Period 3) 1:28-1:44

Assignment

(None) or, if many students not done pg.44 homework, give one more day

Notes

Alyssa's absence on Friday, Feb 8 counted

Amber B. (Period 3) cellphone in class, leaving class early

Make-up quizzes: (Period 3) Joshua, Graham, (Period 4) James B., Jason C., Angela M., Alyssa Wiseman

Chapter 1 Review

Learning Outcomes

- ✓ To review the skills and concepts of Chapter 1

Review Activities

Review Booklet

2.1 Reviewing Linear Inequalities in One Variable

Learning Outcomes

- ✓ Solve inequalities
- ✓ Graph Inequalities
- ✓ Solve problems involving inequalities

Warm-Up

Comment [M18]: 1:50-2:05

Graph each of the following sets of numbers on a number line. Here x is a rational number.

a. $x > 2$
b. $x \leq -1$
c. $-5 < x \leq 3$
d. $x \geq -2$

Activities

Comment [M19]: 2:05-2:35

Notes.

Lead-up to rules for manipulation of inequalities.

$$4 > 3$$

Multiply both sides by -1 to get

$$-4 > -3$$

Ask: is this correct? Is -4 larger than -3?

Student answers may vary but they should remember the rule for multiplying by a negative.

No, this must be incorrect.

Write on overhead:

Suppose that $x > y$. Subtract x from both sides, and subtract y from both sides. What do you get?

Answer: $-y > -x$. But, this also means that $-x < -y$. So in general if $x > y$ then $-x < -y$. This is also true for the inequalities $\leq$ and $\geq$.

What happens when we take the reciprocal of both sides?

Again, take $4 > 3$. If we take the reciprocal of both sides, then we would have $\frac{1}{4} > \frac{1}{3}$. But is $\frac{1}{4}$ larger than $\frac{1}{3}$?

Students answers may vary but should illicit "no".

Challenge for tonight: show that if you take the reciprocal of any inequality $x < y$, you must reverse the inequality sign.

Example of Solving and Graphing.

Solve $5(x - 3) + 2 > 12$, check and graph the solution. $(x > 5)$

Closed dot on the number line indicates that the point is included ($\leq$ or $\geq$ means closed dot).

Open dot (empty) on the number line indicates that the point is not included ($<$ or $>$ means open dot).

Example of solving an inequality involving fractions.

Solve and graph the solution. $(x \geq)$

$$\frac{2}{5}x - \frac{3}{6}x \geq -1$$

Mention LCD during the process!

Assignment

pg. 63, #5,8,13,15,16 #22,23,24 #31,32 #36 #50 core set: {23, 32, 50}