Chemical Reactions and Chemical Equilibria

- Chemical reactions will be considered from the perspective of a thermodynamic path.
- A reaction will be proceed until no further macroscopic change is observed, at which point equilibrium has been attained.
- Notation must be carefully considered.
- Consider the oxidation of ethanol to carbon dioxide and water:

 $C_2H_5OH(l) + 3O_2(g) \rightleftharpoons 2CO_2(g) + 3H_2O(l)$

- The equation is balanced
- Any factor could multiply all the stoichiometric coefficients and still leave a balanced equation.
- If the multiplying factor is a negative number, the roles of reactants and products will be reversed.
- All calculations must be consistent with the direction in which the equation is written.
- The double arrow \rightleftharpoons indicates the attainment of equilibrium.

- The phase of each species must be specified.
 - (g) denotes the gas phase
 - (l) denotes the liquid phase
 - (s) denotes the solid phase
 - (aq) denotes an aqueous solution.

$$NaCl(s) \rightleftharpoons Na^+(aq) + Cl^-(aq)$$

describes a saturated aqueous NaCl solution.

• In order to describe a reaction path in a thermodynamic sense, consider a chemical reaction in the general form:

$$\alpha A + \beta B \rightleftharpoons \gamma C + \delta D$$

where α, β, γ and δ are the stoichiometric coefficients.

- Consider the initial condition at a given T and P with $n_A^{\circ}, n_B^{\circ}, n_C^{\circ}$ and n_D° as the initial amounts of A, B, C, and D.
 - Let the reaction ensue.
 - Some time later find n_A, n_B, n_C and n_D as the amounts of A, B, C, and D present.

• Define ξ as the extent of reaction such that:

$$\frac{n_A^{\circ} - n_A}{\alpha} = \frac{n_B^{\circ} - n_B}{\beta} = \frac{n_C - n_C^{\circ}}{\gamma} = \frac{n_D - n_D^{\circ}}{\delta} = \xi$$

- Note that ξ has units of moles.
- ξ is also known as the degree of advancement or the reaction progress variable.
- The differential degree of advancement is:

$$d\xi = -\frac{dn_A}{\alpha} = -\frac{dn_B}{\beta} = \frac{dn_C}{\gamma} = \frac{dn_D}{\delta}$$

- Positive values of $d\xi$ correspond to the disappearance of reactants (dn_A and dn_B are negative) and the appearance of products (dn_C and dn_D are positive).
- The initial value of ξ is always zero and can either decrease or increase as the reaction proceeds.
- ξ is an extensive variable used to describe reaction path.

Energy and Entropy Changes in Chemical Reactions

• Consider first entropy and the following reaction with reactants in stoichiometric proportions:

$$\frac{1}{2}H_2(g) + \frac{1}{2}F_2(g) \rightarrow HF(g)$$

carried out at constant T and P.

- "→" means that the reaction goes stoichiometrically completely to products.
- The initial entropy is that of the isolated reactants:

$$S(\text{isolated reactants}) =$$

$$\left(\frac{1}{2} \operatorname{mol}\right) \overline{S}(H_2(g)) + \left(\frac{1}{2} \operatorname{mol}\right) \overline{S}(F_2(g))$$

• Now allow the reactants to mix, but not react:

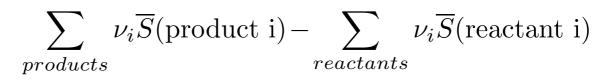
 $S(\text{mixed reactants}) = S(\text{isolated reactants}) + \Delta S_{mix}$

• Now allow reaction to go to completion:

$$S(\text{isolated product}) = (1 \text{mol}) \overline{S}(HF(g))$$

• The molar entropy change associated with the reactions $\Delta_r \overline{S}$ may be written:

$$\Delta_r \overline{S} =$$



where the ν_i are the dimensionless stoichiometric coefficients.

- In this case complete conversion of reactants to products corresponds to $\xi=1$ mol.
- Note that the entropy of mixing does not contribute to $\Delta_r \overline{S}$.
 - The role of mixing will be considered separately later.
- Now consider the enthalpy changes $(\Delta_r H)$.
- A particularly convenient thermodynamic path to consider is one in which the bonds in the reactants are broken, then the bonds in the product molecules are formed.
 - This would permit the calculation of $\Delta_r \overline{H}$ to be determined from bond energies.

- But bond energies can be difficult to determine, especially in polyatomics.
- Real reactions do not follow this type of path.
- All alternate but useful path is one in which the compounds of interest are formed from compounds for which the bond energies are known or are easily determined.
 - One example of this is the formation of compounds from elements in their common stable standard state.
- Consider:

$$2CO(g) + O_2(g) \rightarrow 2CO_2(g)$$

• The carbon containing compounds can be formed from elemental carbon and oxygen.

$$2C(\text{graphite}) + O_2(g) \rightarrow 2CO(g)$$

 $C(\text{graphite}) + O_2(g) \rightarrow CO_2(g)$

The energies association with these reactions may be determined by calorimetry and are known as the standard enthalpies of formation.

- In this case the standard enthalpy of formation coincides with the standard enthalpy of combustion.
- These and other named enthapies and reference states are discussed in your text.

Chemical Reaction Equilibria

• Consider the schematic chemical reaction:

$$\alpha A + \beta B \rightarrow \gamma C + \delta D$$

where with initial state n_A° , n_B° , n_C° , and n_D° moles of compound isolated from one another.

• The initial Gibbs free energy is:

$$\label{eq:G} \begin{split} G(\text{initial}) &= n_A^\circ \mu(A) + n_B^\circ \mu(B) + n_C^\circ \mu(C) + n_D^\circ \mu(D) \\ \text{where} \end{split}$$

$$\mu(A) = \mu(\text{pure A}) = \mu^{\circ}(T) + RT \ln f(A)$$

• Now allow the species to mix but not yet react. The free energy become:

$$G(\text{poised to react}) = G(\text{initial}) + \Delta G_{mix}$$
$$= n_A^{\circ} \mu_A + n_B^{\circ} \mu_B + n_C^{\circ} \mu_C + n_D^{\circ} \mu_D$$

where

$$\mu_A = \mu_A(T) + RT \ln(f_A/f_A^\circ)$$

- Note that:
 - f_A is the fugacity of A in the mixture and depends on composition.
 - f(A) is the fugacity of pure A.
 - f_A° is the fugacity of A in the reference state defining $\mu_A^{\circ}(T)$.
 - Instead of fugacity, partial pressure, activities, activity coefficients, mole fractions, etc. may be used.
- Once the reaction starts to progress, the Gibbs free energy must be expressed as a function as the extent of reaction, ξ
- For any composition:

$$G = n_A \mu_A + n_B \mu_B + n_C \mu_C + n_D \mu_D$$

where:

- $n_A = n_A^\circ \alpha \xi$
- $n_B = n_B^\circ \beta \xi$
- $n_C = n_C^\circ + \gamma \xi$
- $n_D = n_D^\circ + \delta \xi$

• Thus:

$$G(\xi) = n_A^{\circ} \mu_A + n_B^{\circ} \mu_B + n_C^{\circ} \mu_C + n_D^{\circ} \mu_D + \xi (\gamma \mu_C + \delta \mu_D - \alpha \mu_A - \beta \mu_B)$$

• This may be interpreted as:

 $G(\xi) = G(\text{poised to react})$

+G(due to amount ξ of reaction).

where ξ may be positive or negative.

- ξ can increase from zero as reactants are consumed.
- ξ can decrease from zero as products are consumed.
- The value of ξ (and its sign) are determined by the spontaneous direction of change of the reacting mixture such that G is minimized.

• At ξ_{eq} ,

$$\left(\frac{\partial G}{\partial \xi}\right)_{T,P} = 0$$

• At equilibrium:

$$dG = 0$$
$$VdP - SdT + \sum \mu_i dn_i = 0$$

• But T and P are constant. Therefore: $\sum \mu_i dn_i = 0$

• Recall:

$$d\xi = -\frac{dn_A}{\alpha} = -\frac{dn_B}{\beta} = \frac{dn_C}{\gamma} = \frac{dn_D}{\delta}$$

or:

$$dn_A = -\alpha d\xi$$
$$dn_B = -\beta d\xi$$
$$dn_C = \gamma d\xi$$
$$dn_D = \delta d\xi$$

Therefore:

$$\left(\sum_{products} \nu_i \mu_i - \sum_{reactants} \nu_i \mu_i\right) d\xi = 0$$

Because ξ is an independent variable,

$$\left(\sum_{products} \nu_i \mu_i - \sum_{reactants} \nu_i \mu_i\right) = \left(\frac{\partial G}{\partial \xi}\right)_{T,P} = 0$$

• For the example under consideration:

$$\gamma\mu_C + \delta\mu_D - \alpha\mu_A - \beta\mu_B = 0$$

• Since for each component:

$$\mu_i = \mu_i^\circ + RT \ln\left(\frac{f_i}{f_i^\circ}\right)$$

which gives:

$$\gamma \mu_C^{\circ} + \delta \mu_D^{\circ} - \alpha \mu_A^{\circ} - \beta \mu_B^{\circ}$$
$$= -RT \ln \left[\frac{\left(\frac{f_C^{eq}}{f_C^{\circ}}\right)^{\gamma} \left(\frac{f_D^{eq}}{f_D^{\circ}}\right)^{\delta}}{\left(\frac{f_A^{eq}}{f_A^{\circ}}\right)^{\alpha} \left(\frac{f_B^{eq}}{f_B^{\circ}}\right)^{\beta}} \right]$$

• The standard Gibbs free energy of reaction is defined with respect to:

$$\alpha A + \beta B \rightarrow \gamma C + \delta D$$

 \mathbf{as}

$$\Delta_r \overline{G}^\circ = \gamma \mu_C^\circ + \delta \mu_D^\circ - \alpha \mu_A^\circ - \beta \mu_B^\circ$$

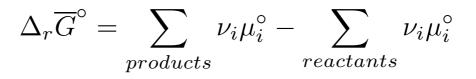
• Thus:

$$-\frac{\Delta_r \overline{G}^{\circ}}{RT} = \ln \left[\frac{\left(\frac{f_C^{eq}}{f_C^{\circ}}\right)^{\gamma} \left(\frac{f_D^{eq}}{f_D^{\circ}}\right)^{\delta}}{\left(\frac{f_A^{eq}}{f_A^{\circ}}\right)^{\alpha} \left(\frac{f_B^{eq}}{f_B^{\circ}}\right)^{\beta}} \right]$$

• Define the thermodynamic reaction equilibrium coefficient as:

$$K_{eq}(T) = \left[\frac{\left(\frac{f_C^{eq}}{f_C^{\circ}}\right)^{\gamma} \left(\frac{f_D^{eq}}{f_D^{\circ}}\right)^{\delta}}{\left(\frac{f_A^{eq}}{f_A^{\circ}}\right)^{\alpha} \left(\frac{f_B^{eq}}{f_B^{\circ}}\right)^{\beta}}\right]$$

• In general form this becomes:



and

$$K_{eq} = \frac{\prod_{products} \left(\frac{f_i^{eq}}{f_i^{\circ}}\right)^{\nu_i}}{\prod_{reactants} \left(\frac{f_i^{eq}}{f_i^{\circ}}\right)^{\nu_i}} = \exp\left(-\frac{\Delta_r \overline{G}^{\circ}}{RT}\right)$$