Chapter 4

Bipolar Junction Transistors. Home Work Solutions

4.1 Problem 4.33

A single measurement indicates the emitter voltage of the transistor in the circuit of Figure (4.1) to be 1.0 V. Under the assumption that $|V_{BE}| = 0.7\ V$, what are $V_{B,}, I_B, I_E, I_C, V_C, \beta$ and α?

Solution

The emitter current I_E is given by:

$$
\begin{aligned}
I_E &= \frac{V_{EE} - V_E}{R_E} \\
&= \frac{5 - 1}{5 \times 10^3} \\
&= 0.8\ mA
\end{aligned}
$$

The base voltage V_B is given by:

$$
\begin{aligned}
V_B &= V_E - V_{BE} \\
&= 1.0 - 0.7 \\
&= 0.3\ V
\end{aligned}
$$

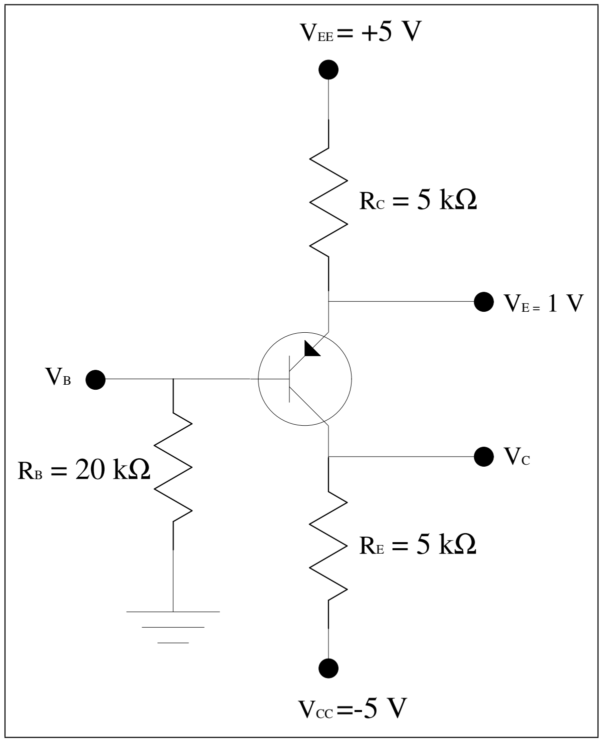

Figure 4.1:

The base current I_B is:

$$\begin{aligned} I_B &= \frac{V_B}{R_B} \\ &= \frac{0.3}{20 \times 10^3} \\ &= 15\ \mu A \end{aligned}$$

The collector current I_C is given by;

$$\begin{aligned} I_C &= I_E - I_B \\ &= 0.8 - 0.15 \\ &= 0.785\ mA \end{aligned}$$

The collector voltage V_C is given by:

$$\begin{aligned} V_C &= V_{CC} + I_C R_C \\ &= -5 + 0.785 \times 5 \\ &= -1.075\ V \end{aligned}$$

Now the transistor β is:

$$\begin{aligned} \beta &= \frac{I_C}{I_B} \\ &= \frac{0.785}{0.015} \\ &= 52.3 \end{aligned}$$

Finally, the transistor α is given by:

$$\begin{aligned} \alpha &= \frac{I_C}{I_E} \\ &= \frac{0.785}{0.8} \\ &= 0.98 \end{aligned}$$

4.2 Problem 4.46

For reasonably linear small signal operation of a BJT, v_{be} must be limited to no larger than 10 mV. To what percentage change of bias current does this correspond? For a design in which the required output voltage signal is 10 mA peak, what bias current required? What is the corresponding value of g_m?

Solution

The total (ac and dc) collector current i_C is:

$$\begin{aligned} i_C &= I_C e^{v_{be}/V_T} \\ &= I_C + i_c \\ \frac{i_c}{I_C} &= e^{v_{be}/V_T} - 1 \end{aligned}$$

Taking $v_{be} = 10$ mV and $V_T = 25$ mV, we get:

$$\begin{aligned} \frac{i_c}{I_C} &= e^{10/25} - 1 \\ &= 0.492 \\ &= 49.2\% \end{aligned}$$

For $i_c = 10$ mA while $v_{be} = 10$ mV, we get:

$$\begin{aligned} I_C &= \frac{10}{0.492} \\ &= 20.3\ mA \end{aligned}$$

Finally, for $I_C = 20.3$ mA, g_m is:

$$\begin{aligned} g_m &= \frac{I_C}{V_T} \\ &= \frac{20.3}{25} \\ &= 0.81\ A/V \end{aligned}$$

4.3 Problem 4.48

For the circuit of Figure (4.2), V_{BE} is adjusted so that $V_C = 2$ V. If $V_{CC} = 10$ V, $R_C = 2\ k\Omega$, and a signal $v_{be} = 0.004\ \sin \omega t$ volts is applied, find expressions for the total instantaneous quantities $i_C(t)$, $v_C(t)$, and $i_B(t)$. The transistor has $\beta = 100$. What is the voltage gain.

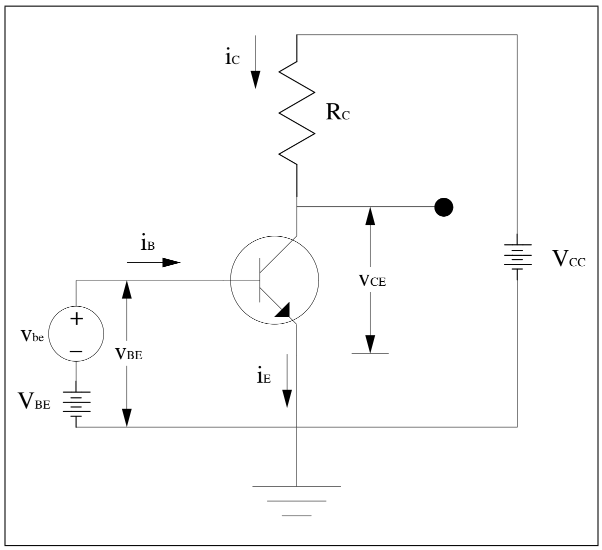

Figure 4.2:

Solution

Using the given values of V_C of 2 V, V_{CC} of 10 V, and R_C of 2 $k\Omega$, the collector current is:

$$\begin{aligned} I_C &= \frac{V_{CC} - V_C}{R_C} \\ &= \frac{10 - 2}{2 \times 10^3} \\ &= 4\ mA \end{aligned}$$

Using the the T-model equivalent circuit, the total collector current is:

$$
\begin{aligned}
i_C(t) &= I_C + g_m v_{be} \\
g_m &= \frac{I_C}{V_T} \\
i_C(t) &= I_C + \frac{I_C}{V_T} v_{be} \\
v_{be}(t) &= 0.004 \ \sin \omega t \\
i_C(t) &= I_C + \frac{I_C}{V_T} \times \sin \omega t \\
&= 4 + \frac{4}{2.5 \times 10^4} \times 0.004 \ \sin \omega t \\
&= 4 + 0.64 \ \sin \omega t \quad mA
\end{aligned}
$$

The total collector voltage $v_C(t)$is:

$$
\begin{aligned}
v_C(t) &= V_{CC} - R_C \, i_C(t) \\
&= 10 - 2 \times 10^3 \times (4 \times 10^{-3} + 0.64 \times 10^{-3} \ \sin \omega t) \\
&= 2 - 1.28 \ \sin \omega t \quad V
\end{aligned}
$$

The total base current $i_B(t)$ is:

$$
\begin{aligned}
i_B(t) &= \frac{i_C(t)}{\beta} \\
&= \frac{4 + 0.64 \ \sin \omega t}{100} \\
&= 0.04 + 0.0064 \ \sin \omega t \quad mA \\
&= 40 + 6.4 \ \sin \omega t \quad \mu A
\end{aligned}
$$

The peak value of the the ac-component of output voltage v_c is 1.28 V and the peak value of the input is 0.004 V, the voltage gain is then given by:

$$
\begin{aligned}
A_v &= -\frac{1.28}{0.004} \\
&= -320
\end{aligned}
$$

4.4 Problem 4.67

Sketch the $i_C - v_{CE}$ characteristics of an *npn* transistor having $\beta = 100$ and $V_A = 100$ V. Sketch characteristic curves for $i_B = 20$, 50, 80, and 100 μA. For the purpose of this sketch, assume that $i_C = \beta i_B$ at $v_{ce} = 0$. Also sketch the load line obtained for $V_{CC} = 10$ V and R_C $= 1k\Omega$. If the dc bias current into the base is 50 μA, write the equation for the corresponding $i_C - v_{CE}$ curve. Also, write the equation for the load line, and solve the two equations to obtain V_{CE} and I_C. If the input signal causes a sinusoidal signal of 30-μA peak amplitude to be superimposed on I_B, find the corresponding signal components of i_C and v_{CE}.

Solution

the equation for the characteristic $v_{CE} - i_C$ is given by:

$$i_C = m\, v_{CE} + C$$

where m is the slope of the line and C is the y-intercept. We are given the the y-intercept as $\beta\ i_B$ and the x-intercept as $V_A = 100$ V. The slope m can then be determined from the two intercepts as:

$$\begin{aligned} m &= \frac{C}{V_A} \\ &= \frac{\beta\, i_B}{100} \end{aligned}$$

So, the slopes m_1 of line 1, m_2 of line 2, m_3 of line 3, and m_4 of line 4 are 0.02, 0.05, 0.08, and 0.1 respectively. While the y-intercepts are 2, 5, 8, and 10 mA respectively. Of course all lines should meet at the x-intercept of -100 V. Using this information one then can plot the $v_{CE} - i_C$ characteristics for the four given base currents. The load line has a x-intercept of V_{CC} and a slope of $-1/R_C$, so in the graph it connects the points with coordinates of $(V_{CC}, 0)$ and $(0, V_{CC}/R_C)$ i.e. (10 V, 0) and (0, 10 mA). The four characteristic curves and the load line are shown in Figure (4.3).

The equation for the line with a base current $i_B = 50$ μA is:

$$\begin{aligned} i_C &= \beta\, i_B + \frac{\beta\, i_B}{VA} v_{CE} \\ &= 5 + 0.05\, v_{CE} \quad mA \end{aligned} \tag{4.1}$$

The load line equation is:

$$\begin{aligned} V_{CC} &= i_C R_C + v_{CE} \\ i_C &= \frac{V_{CC} - v_{CE}}{R_C} \\ &= 10 - v_{CE} \quad mA \end{aligned} \tag{4.2}$$

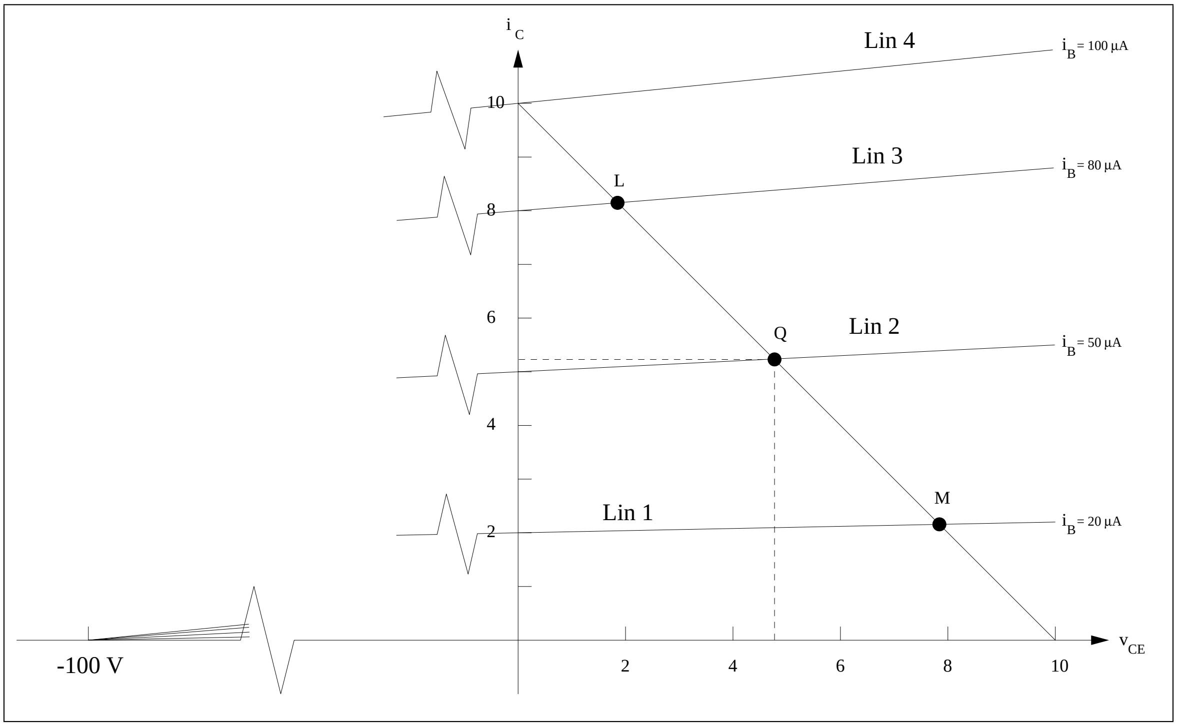

Figure 4.3:

Solving Equations (4.1) and (4.2) provides the intersection of the line for $i_B = 50\ \mu$A and the load line, i.e. the dc bias point Q with coordinates $(V_{CE},\ I_C)$. The solution is:

$$I_C = 5.24\ mA$$

and

$$V_{CE} = 4.76\ V$$

When a signal of 30 μA is superimposed on $I_B = 50\ \mu$A, the operating point moves along the load line between point L ($I_B = 80\ \mu$A) and point M ($I_B = 20\ \mu$A). To find the coordinates of point L we solve the load line equation Equation (4.2) and Line 3 equation given by:

$$i_C = 8 + 0.08\ v_{CE}$$

the solution gives:

$$\begin{aligned} i_C|_L &= 8.15 \quad mA \\ v_{CE}|_L &= 1.85 \quad V \end{aligned}$$

Similarly we can find the coordinates of point M to be:

$$\begin{aligned} i_C|_M &= 2.16 \quad mA \\ v_{CE}|_M &= 7.84 \quad V \end{aligned}$$

The ac output current signal is zero at point Q and will have a positive peak $\hat{i}_{c+}$ at point L and a negative peak $\hat{i}_{c-}$ at point M where:

$$\begin{aligned}\hat{i}_{c+} &= i_C|_L - I_C \\ &= 8.15 - 5.24 \\ &= 2.91\ mA \\ \hat{i}_{c-} &= I_C - i_C|_M \\ &= 5.24 - 2.16 \\ &= 3.08\ mA\end{aligned}$$

and the peak-peak current is 5.99 mA. Similarly one can calculate the voltage of the output signal as:

$$\begin{aligned}\hat{v}_{ce+} &= v_{CE}|_L - V_{CE} \\ &= 7.84 - 4.76 \\ &= 3.08\ V \\ \hat{v}_{ce-} &= V_{CE} - v_{CE}|_M \\ &= 4.76 - 1.85 \\ &= 2.91\ mA\end{aligned}$$

and the peak-peak voltage is 5.99 V. The output ac voltage (in volts) and the output ac current (in mA) are equal in magnitude because the slope of the load line is (-1/1$k\Omega$).

4.5 Problem 4.78

For the common emitter amplifier shown in Figure (4.4), let $V_{CC} = 9$ V, $R_1 = 27\ k\Omega$, $R_2 = 15\ k\Omega$, $R_E = 1.2k\Omega$, and $R_C = 2.2\ k\Omega$. The transistor $\beta = 100$ and $V_A = 100$ V. Calculate the dc bias current I_E. If the amplifier operates between a source for which $R_s = 10\ k\Omega$ and a load of 2 $k\Omega$, replace the transistor with its hybrid-π model, and find the values of R_i, the voltage gain v_o/v_s, and the current gain i_o/i_i.

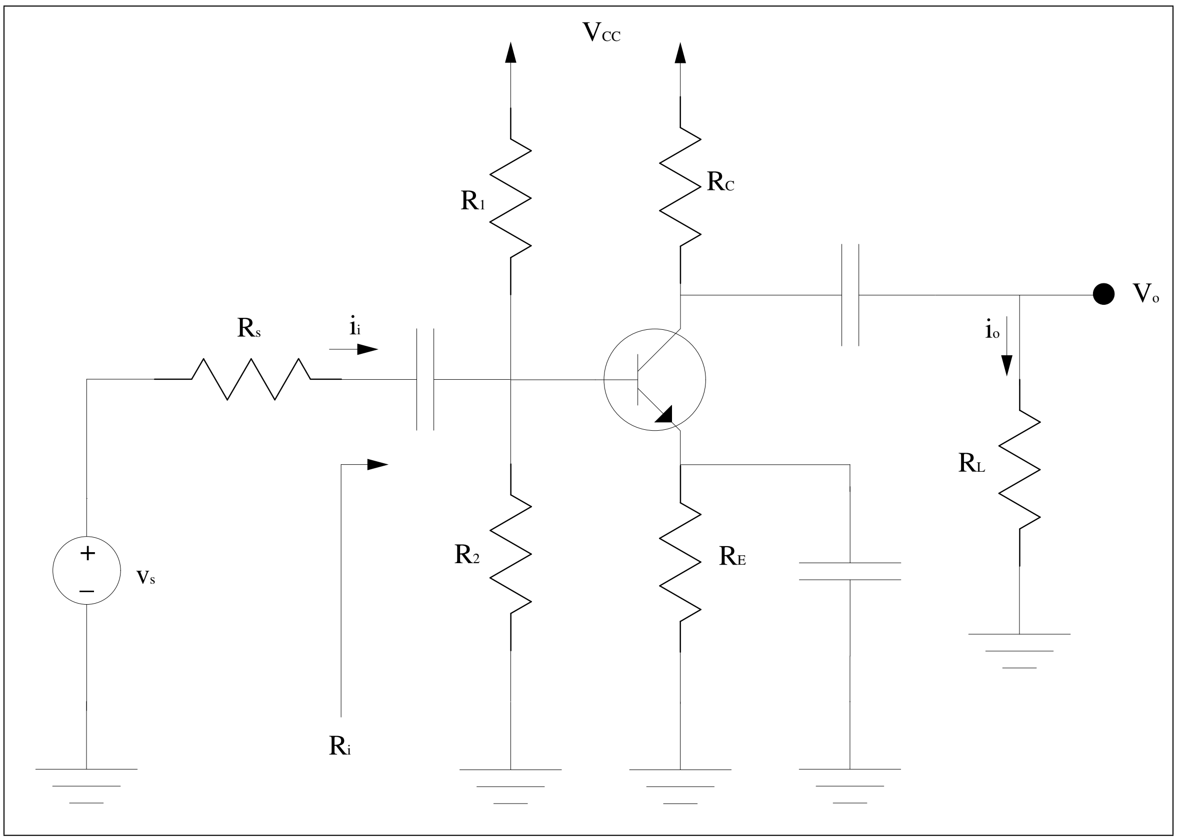

Figure 4.4: All capacitors are very large.

Solution

The hybrid-π model of the circuit is shown in Figure (4.5). the dc emitter current I_E is given by:

$$I_E = \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{\beta+1}}$$

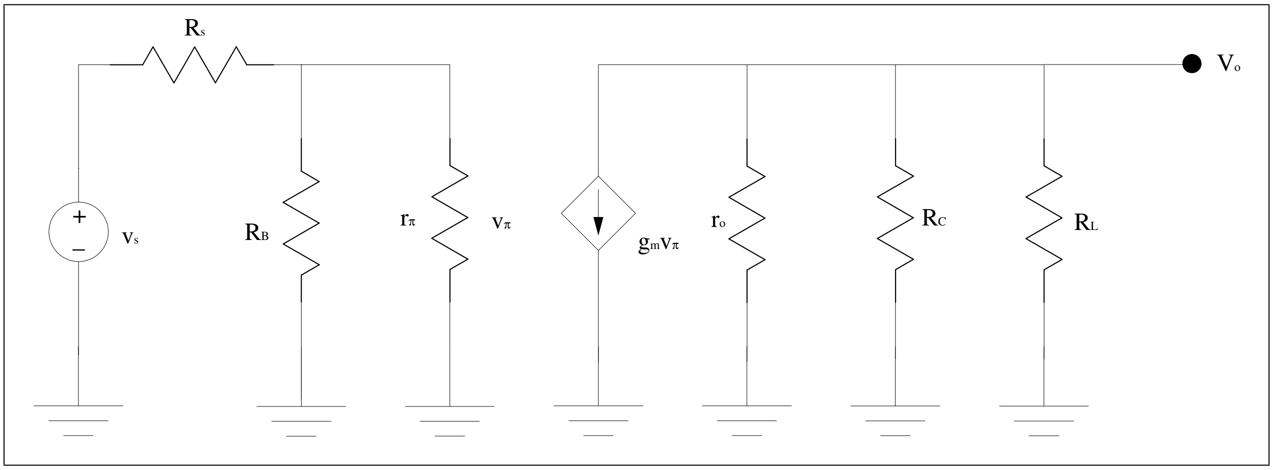

Figure 4.5: The hybrid-π-model of the circuit shown in Figure (4.4). $R_B = R_1//R_2$.

where:

$$\begin{aligned} V_{BB} &= V_{CC} \times \frac{R_2}{R_1 + R_2} \\ &= 9 \times \frac{15}{27 + 15} \\ &= 3.21\ V \\ &= 1.3\ k\Omega \\ R_B &= R_1//R_2 \\ &= \frac{R_1 R_2}{R_1 + R_2} \\ &= \frac{27 \times 15}{27 + 15} \\ &= 9.64\ k\Omega \end{aligned}$$

I_E is then:

$$\begin{aligned} I_E &= \frac{3.21 - 0.7}{1.2 + \frac{9.64}{101}} \\ &= 1.94\ mA \end{aligned}$$

We also have:

$$\begin{aligned} g_m &= \frac{I_C}{V_T} \\ &= \frac{\alpha I_E}{V_T} \end{aligned}$$

$$
\begin{aligned}
&= 1.3\ k\Omega \\
&= \frac{0.99 \times 1.94}{0.025} \\
&= 76.8\ mA/V \\
&= 1.3\ k\Omega \\
r_\pi &= \frac{\beta}{g_m} \\
&= \frac{100}{76.8} \\
&= 1.3\ k\Omega \\
r_o &= \frac{V_A}{I_C} \\
&= \frac{100}{0.99 \times 1.94} \\
&= 52.1\ k\Omega
\end{aligned}
$$

Figure (4.6) shows the equivalent circuit with the input and output resistances. Comparing Figure (4.5) and Figure (4.6), we get:

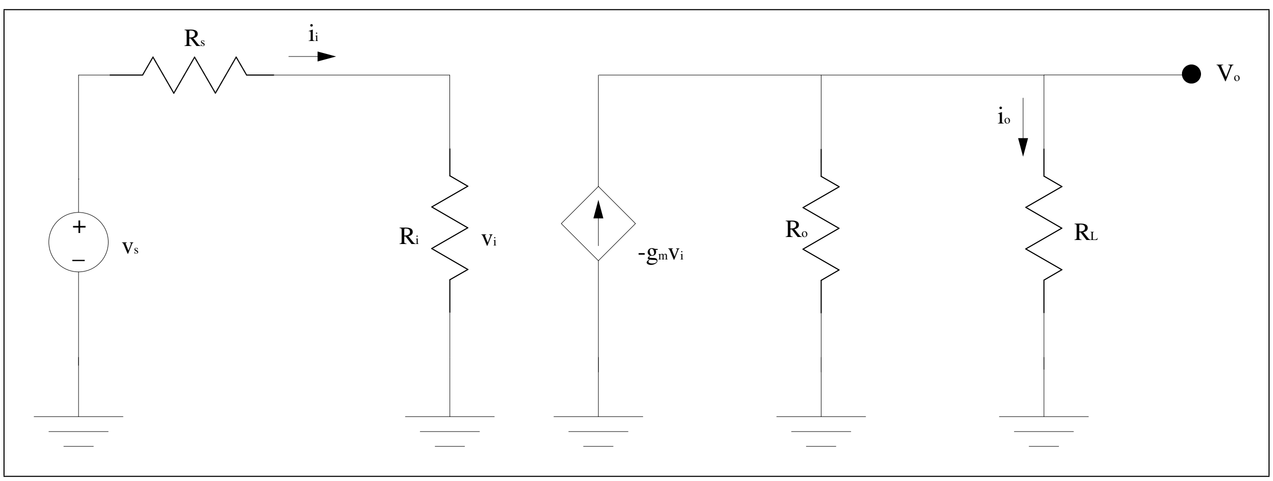

Figure 4.6:

$$
\begin{aligned}
R_i &= R//r_\pi \\
&= 9.64//1.3 \\
&= 1.15\ k\Omega \\
R_o &= R_C//r_o \\
&= 2.2//52.1 \\
&= 2.11\ k\Omega
\end{aligned}
$$

The voltage gain is given by:

$$
\begin{aligned}
A_v &= \frac{v_o}{v_s} \\
&= \frac{v_i}{v_s} \times \frac{v_o}{v_i} \\
v_i &= \frac{R_i}{R_s + R_i} \times v_s \\
v_o &= -g_m v_i \times (R_o//R_L) \\
A_v &= -\frac{R_i}{R_i + R_s} \times g_m \times (R_o//R_L) \\
&= -\frac{1.15}{10 + 1.15} \times 76.8 \times (2.11//2) \\
&= -8.13 \\
i_o &= \frac{v_o}{R_L} \\
i_i &= \frac{v_s}{R_s + R_i} \\
A_i &= \frac{i_o}{i_i} \\
&= \frac{v_o}{v_s} \times \frac{R_s + R_i}{R_L} \\
&= A_{v\times} \frac{R_s + R_i}{R_L} \\
&= -8.13 \times \frac{10 + 1.15}{2} \\
&= -45.3
\end{aligned}
$$

4.6 Problem 4.91

For the emitter follower circuit shown in Figure (4.7) the BJT used is specified to have β values in the range of 20 to 200 (a distressing situation for the circuit designer). For two extreme values of β ($\beta = 20$ and $\beta = 200$), find:

(a) I_E, V_E, and V_B,

(b) The input resistance R_i,

(c) The voltage gain v_o/v_s,

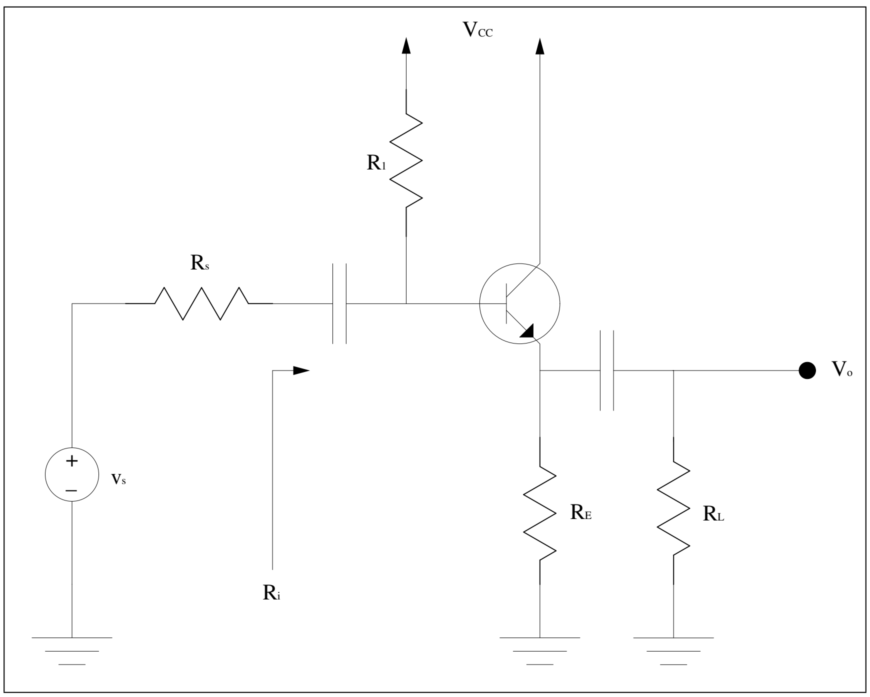

Figure 4.7: In the circuit $V_{CC} = 9$ V, $R_s = 10\ k\Omega$, $R_1 = 100\ k\Omega$, $R_E = R_L = 1\ k\Omega$. The capacitors are very large.

Solution

(a) The emitter current I_E, emitter voltage V_E and base voltage V_B are given by:

$$I_E = \frac{V_{CC} - V_{BE}}{R_E + \frac{R_1}{\beta+1}}$$

$$\begin{aligned}
V_E &= I_E R_E \\
V_B &= V_E + V_{BE} \\
I_{E20} &= \frac{9-0.7}{1+\frac{100}{21}} \\
&= 1.44\ mA \\
V_{E20} &= 1.44 \times 1 \\
&= 1.44\ V \\
V_{B20} &= 1.44 + 0.7 \\
I_{E200} &= \frac{9-0.7}{1+\frac{100}{201}} \\
&= 5.54\ mA \\
V_{E200} &= 5.54 \times 1 \\
&= 5.54\ V \\
V_{B200} &= 5.54 + 0.7 \\
&= 6.24\ V
\end{aligned}$$

(b) To find the input resistance R_i we use the T-model equivalent circuit shown in Figure (4.8). R_i is given by:

$$R_i = R_1 // R_i'$$

and R_i' is defined by:

$$R_i' = \frac{v_b}{i_b}$$

where v_b and i_b are the base voltage and current respectively. The base voltage is defined by:

$$\begin{aligned}
v_b &= i_e \times [r_e + (R_E//R_L)] \\
&= (1+\beta) \times i_b \times [r_e + (R_E//R_L)] \\
R_i' &= \frac{(1+\beta) \times i_b \times [r_e + (R_E//R_L)]}{i_b} \\
R_i &= R_1//(1+\beta) \times [r_e + (R_E//R_L)] \\
&= 100//(1+\beta) \times [r_e + (R_E//R_L)] \\
r_e &= \frac{V_T}{I_E} \\
&= \frac{0.025}{I_E} \\
R_i &= 100//(1+\beta) \times \left[\frac{0.025}{I_E} + (R_E//R_L)\right]
\end{aligned}$$

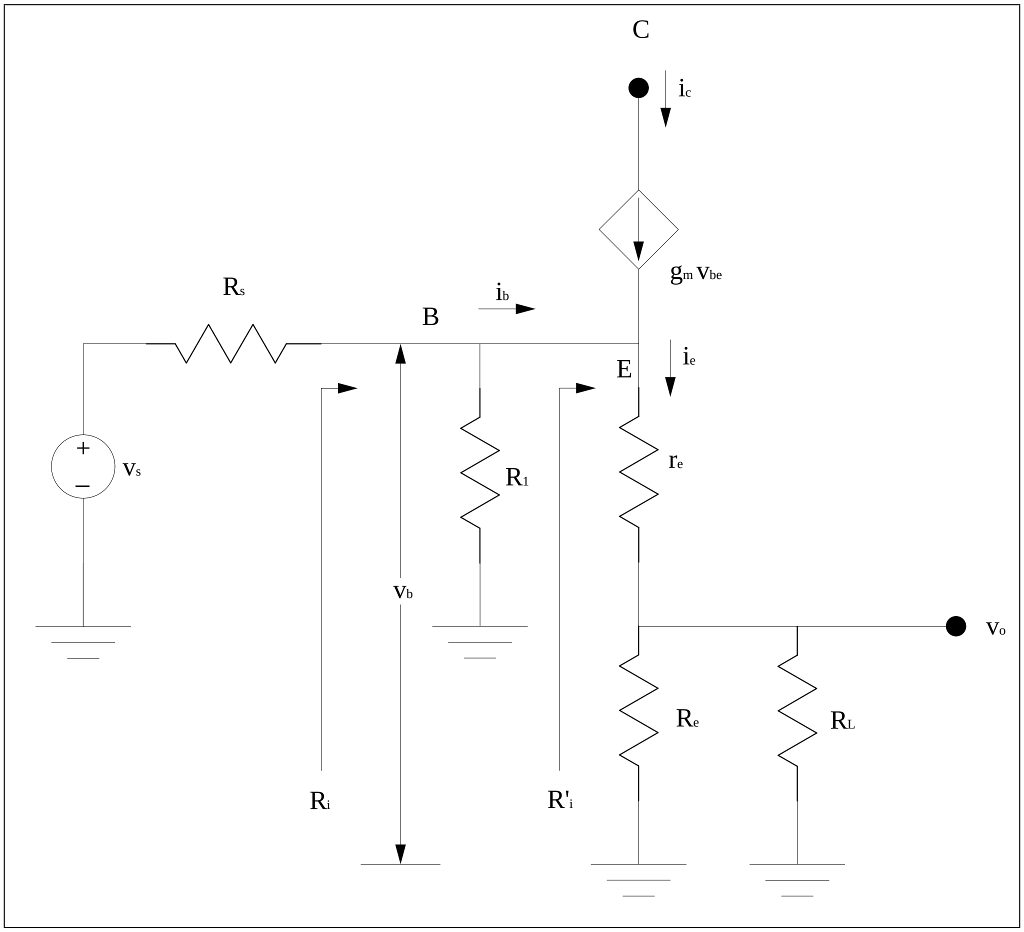

Figure 4.8:

$$
\begin{aligned}
R_{i20} &= 100//(1+20) \times \left[\frac{0.025}{I_{E20}} + (R_E//R_L)\right] \\
&= 100//21 \times \left[\frac{0.025}{1.44} + (1//1)\right] \\
&= 100//21 \times (0.0174 + 0.5) \\
&= 100//10.87 \\
&= 9.8\ k\Omega \\
R_{i200} &= 100//(1+200) \times \left[\frac{0.025}{I_{E200}} + (R_E//R_L)\right] \\
&= 100//21 \times \left[\frac{0.025}{5.54} + (1//1)\right] \\
&= 100//201 \times (0.0045 + 0.5) \\
&= 100//101.4 \\
&= 50.3\ k\Omega
\end{aligned}
$$

(c) The voltage gain v_o/v_s can be written as:

$$\frac{v_o}{v_s} = \frac{v_b}{v_s} \times \frac{v_o}{v_b}$$

Using Figure (4.8), we can see that at the input v_s, R_s, and R_i form a series voltage divider with v_b the voltage across R_i we then get:

$$\begin{aligned} v_b &= \frac{R_i}{R_i + R_s} \times v_s \\ \frac{v_b}{v_s} &= \frac{R_i}{R_i + R_s} \end{aligned}$$

while on the output we can see that v_b, r_e, and $R_E//R_L$ form another series voltage divider where v_o is the voltage across $R_E//R_L$, we then get:

$$\begin{aligned} v_o &= \frac{R_E//R_L}{r_e + (R_E//R_L)} \times v_b \\ \frac{v_o}{v_b} &= \frac{R_E//R_L}{r_e + (R_E//R_L)} \\ &= \frac{R_E//R_L}{\frac{V_T}{I_E} + (R_E//R_L)} \end{aligned}$$

The voltage gain is then:

$$\frac{v_o}{v_s} = \frac{R_i}{R_i + R_s} \times \frac{R_E//R_L}{\frac{V_T}{I_E} + (R_E//R_L)}$$

For $\beta = 20$, we get:

$$\begin{aligned} \frac{v_o}{v_s} &= \frac{9.8}{9.8 + 10} \times \frac{0.5}{\frac{0.025}{1.44} + 0.5} \\ &= 0.478 \end{aligned}$$

and for $\beta = 200$:

$$\begin{aligned} \frac{v_o}{v_s} &= \frac{50.3}{50.3 + 10} \times \frac{0.5}{\frac{0.025}{5.54} + 0.5} \\ &= 0.827 \end{aligned}$$