# Chapter 24

# Gauss's Law. Solutions of Selected Problems

## 24.1 Problem 24.7 *(In the text book)*

A pyramid with horizontal square base, 6.00 $m$ on each side, and a height of 4.00 $m$ is placed in a vertical electric field of 52.0 $N/C$. Calculate the total electric flux through the pyramids four slanted surfaces.

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### Solution

Since the pyramid's base is horizontal and the electric field is vertical, then the electric flux that goes through the slanted surfaces of the pyramid must also go also through the base (the lines get in through the slanted surface and get out through the base or in from the base and out through the slanted surfaces). Consider former case, the electric field and the base's normal are antiparallel, so the flux through the base is then given by:

$$\Phi_e = EA\cos\theta = EA\cos 180 = -52.0 \times 6 \times 6 = -1.87 \times 10^3 \; N \cdot m^2/C$$

So the flux through the slanted surfaces is $+1.87 \times 10^3 \; N \cdot m^2/C$

## 24.2 Problem 24.19 *(In the text book)*

An infinitely long line charge having a uniform charge per unit length $\lambda$ lies a distance $d$ from point $O$ as shown in Figure (24.19). Determine the total electric flux through the surface of a sphere of radius $R$ centered at $O$ resulting from this line charge. Consider both cases, where $R < d$ and $R > d$.

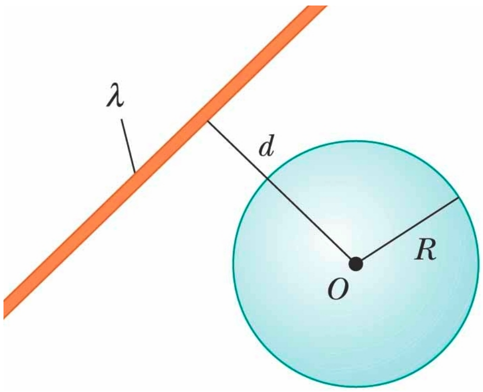


Figure 24.19:

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### Solution

If the radius of the spherical surface is $R \leq d$ then the sphere does not enclose any charge and the net flux through is:

$$\Phi_e = \frac{q_{in}}{\epsilon_\circ} = 0.$$

If, however, $R > d$ then there will be a part $\ell$ of the charged line that lies within the sphere, $\ell$ is given by (see Figure (24.20)):

$$\ell = 2\sqrt{R^2 - d^2}$$

and the the electric flux $\Phi_E$ through the spherical surface is:

$$\Phi_E = \frac{q_{in}}{\epsilon_\circ} = \frac{\lambda\ell}{\epsilon_\circ} = \frac{2\sqrt{R^2 - d^2}}{\epsilon_\circ}$$

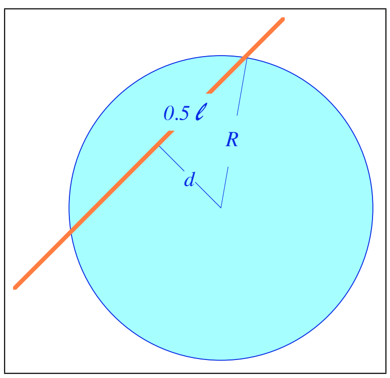


Figure 24.20:

## 24.3 Problem 24.35 *(In the text book)*

A uniformly charged, straight filament 7.00 $m$ in length has a total positive charge of 2.00 $C$. An uncharged cardboard cylinder 2.00 $cm$ in length and 10.0 $cm$ in radius surrounds the filament at its center, with the filament as the axis of the cylinder. Using reasonable approximations, find

(a) the electric field at the surface of the cylinder and

(b) the total electric flux through the cylinder.

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### Solution

The approximations that we need to use to solve these problem are:

1. The electric field within the length of the cylinder is uniform, this is normally the case if the cylinder's length is much smaller than the charged filament's length.

2. The cardboard cylinder does not disturb the electric field or the charge distribution along the filament.

(a) The electric field produced by the charged filament emanates radially outward. It can be show that at a point a distance $r$ away from the filament the electric field is:

$$E = 2k_e \frac{\lambda}{r}$$

where $\lambda$ is linear charge density of the filament. So,

$$E = 2k_e \frac{q}{\ell r} = 2 \times 8.99 \times 10^9 \times \frac{2.00 \times 10^{-6}}{7.00 \times 0.100} = 51.4 \times 10^3 \ N/C$$

(b) The electric flux is perpendicular to the surface and parallel to the the normal to the surface, so the flux is:

$$\Phi_E = EA\cos\theta = E\times(2\pi r\ell)\times\cos\theta = 51.4\times10^3\times(2\pi\times0.100\times0.020)\times\cos 0 = 646 \ N{\cdot}m^2/C$$

## 24.4 Problem 24.45 *(In the text book)*

Two identical conducting spheres each having a radius of 0.500 $cm$ are connected by a light 2.00-$m$-long conducting wire. A charge of 60.0 $C$ is placed on one of the conductors. Assume that the surface distribution of charge on each sphere is uniform. Determine the tension in the wire.

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# Solution

The charges on the charged conducting sphere are mobile. The charges will then flow through the conducting wire from the charged sphere to the uncharged conducting sphere. The charge flow will continue until the total charge is divided equally among the two identical spheres. Now, the two sphere are charged with similar charge $Q/2$, so they repel each other, like point charges at their centers, extending the wire and putting it under tension. The tension in the wire is then determined by the Coulomb force of repulsion between the spheres, i.e.

$$T = F_E = k_e \frac{q_1 q_2}{r^2} = k_e \frac{(Q/2) \times (Q/2)}{(L + 2R)^2} = k_e \frac{Q^2}{4(L + 2R)^2}$$

and

$$T = 8.99 \times 10^9 \frac{(60.0 \times 10^{-6})^2}{4 \times (2.00 + 0.01)^2} = 2.00\ N$$

## 24.5 Problem 24.68 *(In the text book)*

A point charge $Q$ is located on the axis of a disk of radius $R$ at a distance $b$ from the plane of the disk (Figure (24.68)). Show that if one fourth of the electric flux from the charge passes through the disk, then $R = \sqrt{3}b$.

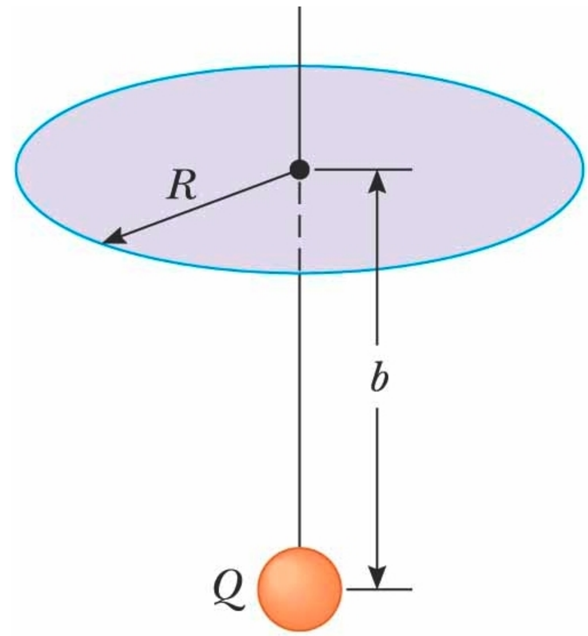


Figure 24.68:

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### Solution

The total flux produced by the charge is, according to Gauss's law, is $Q/\epsilon_\circ$. Only one quarter of this flux passes through the disk. The flux through the disk is given by:

$$\Phi_{disk} = \int \boldsymbol{E} \cdot d\boldsymbol{A}$$

where the integration cover the entire area of the disk. Evaluating this integral and set it equal to $Q/4\epsilon_\circ$ relates $b$ to $R$. As shown inFigure (24.69) we take $d\boldsymbol{A}$ to be the area of annular ring with radius $s$ and width $ds$. The electric field at the ring make an angle $\theta$ with the normal to to the ring, the flux through the ring is:

$$d\Phi_{ring} = \boldsymbol{E} \cdot d\boldsymbol{A} = E\ dA\ \cos\theta = E(2\pi s ds)\cos\theta$$

The magnitude of the electric field has the same value at all points on the ring, i.e.

$$E_{ring} = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{s^2+b^2}$$

in addition:

$$\cos\theta = \frac{b}{r} = \frac{b}{\sqrt{s^2+b^2}}$$

So, the flux through the ring becomes:

$$d\Phi_{ring} = \frac{Qb}{2\epsilon_\circ}\frac{s}{(s^2+b^2)^{3/2}}ds$$

To get the flux through the entire disk we integrate $d\Phi_{ring}$ from $s = 0$ to $s = R$:

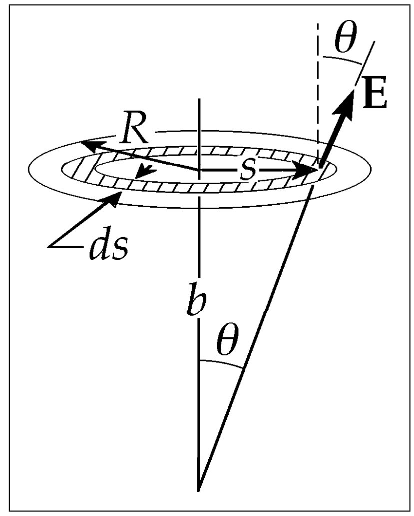


Figure 24.69:

$$\Phi_{E,disk} = \int_0^R d\Phi_{ring} = \frac{Qb}{2\epsilon_\circ}\int_0^R \frac{s}{(s^2+b^2)^{3/2}}ds = \frac{Qb}{4\epsilon_\circ}\int_0^R \frac{2sds}{(s^2+b^2)^{3/2}}$$

T carry out the integration, let us change variables to $x = s^2 + b^2$, so $dx = 2sds$. When $s = 0$, then $x = b^2$ and when $s = R$, then $x = R^2 + b^2$, we then get:

$$\begin{aligned}
\Phi_{E,disk} &= \frac{Qb}{4\epsilon_\circ}\int_0^R \frac{2sds}{(s^2+b^2)^{3/2}} \\
&= \frac{Qb}{4\epsilon_\circ}\int_{b^2}^{R^2+b^2} \frac{dx}{x^{3/2}} \\
&= \frac{Qb}{4\epsilon_\circ}\int_{b^2}^{R^2+b^2} x^{-3/2}dx \\
&= \frac{Qb}{4\epsilon_\circ}\left[\frac{x^{-1/2}}{-1/2}\right]_{b^2}^{R^2+b^2} = \frac{Qb}{2\epsilon_\circ}\left[-\frac{1}{\sqrt{x}}\right]_{b^2}^{R^2+b^2} \\
&= \frac{Qb}{2\epsilon_\circ}\left[\frac{-1}{\sqrt{R^2+b^2}} + \frac{1}{b}\right] \\
&= \frac{Q}{2\epsilon_\circ}\left[1 - \frac{b}{\sqrt{R^2+b^2}}\right]
\end{aligned}$$

Since the flux through the disk is already given by $Q/4\epsilon_\circ$, then:

$$\frac{Q}{4\epsilon_\circ} = \frac{Q}{2\epsilon_\circ}\left[1 - \frac{b}{\sqrt{R^2+b^2}}\right]$$

and we get:

$$4b^2 = R^2 + b^2 \quad \text{or} \quad R^2 = 3b^2 \quad \text{and} \quad R = \sqrt{3}b$$