Chapter 23

Electric Fields. Solutions of Home Work Problems

23.1 Problem 23.10 *(In the text book)*

Two small beads having positive charges $3q$ and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point $x = d$. As shown in Figure (23.10), a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?

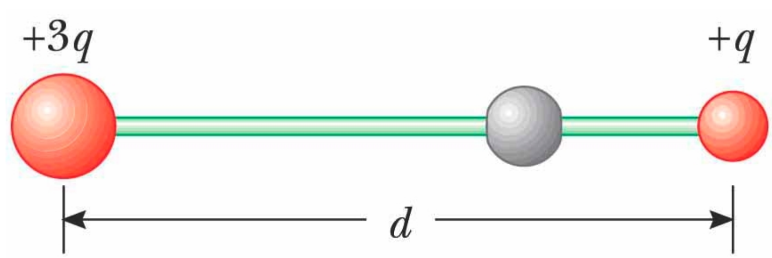

Figure 23.10:

Solution

Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by:

$$\boldsymbol{F} = \frac{k_e 3qQ}{x^2}\hat{i} + \frac{k_e qQ}{(d-x)^2} \times -\hat{i}$$

The net force will be zero if:

$$\frac{3}{x^2} = \frac{1}{(d-x)^2} \quad \text{or} \quad d-x = \frac{x}{\sqrt{3}} \quad \text{or} \quad d = x + \frac{x}{\sqrt{3}} = x \times \frac{1+\sqrt{3}}{\sqrt{3}} \quad \text{and} \quad x = d\frac{\sqrt{3}}{1+\sqrt{3}}$$

The equilibrium position of the third bead will be at:

$$x = \frac{\sqrt{x}}{1+\sqrt{3}}d = 0.634d$$

The equilibrium will be stable if the charge is positive and unstable if the charge is negative. When the third charge is positive the forces acting on it are repulsive, so displacing it slightly toward one of the other charges increases on repulsive force and decreases the other and as a result the third charge will go back to the equilibrium position. If, however, the third charge is negative, then the forces acting on it are attractive and cancel at the position of equilibrium. If the charge is slightly displaced toward one of the other charges, then one attractive force becomes larger than the other and the charge will continue to move in the direction of the original displacement and will not return back to the position of equilibrium.

23.2 Problem 23.17 *(In the text book)*

Two point charges are located on the x axis. The first is a charge $+Q$ at $x = -a$. The second is an unknown charge located at $x = +3a$. The net electric field these charges produce at the origin has a magnitude of $2k_eQ/a^2$. What are the two possible values of the unknown charge?

Solution

The electric fields produced by tow charges at a point between them, are in general opposite in directions. Since we know that the charge Q is positive then it field will be pointing in the direction of the positive x (to the right) or in the direction of $+\hat{\boldsymbol{i}}$ where $\hat{\boldsymbol{i}}$ is the unit vector along the x-axis, i.e.

$$\frac{2k_eQ}{a^2}(\hat{\boldsymbol{i}}) = \frac{k_eQ}{a^2}(\hat{\boldsymbol{i}}) + \frac{k_eq}{(3a)^2}(-\hat{\boldsymbol{i}}) \tag{23.1}$$

The charge q can be $+ve$ or $-ve$. If the total field is positive, then q must be negative and we get:

$$\frac{2k_eQ}{a^2}(\hat{\boldsymbol{i}}) = \frac{k_eQ}{a^2}(\hat{\boldsymbol{i}}) + \frac{k_eq}{(3a)^2}(-\hat{\boldsymbol{i}}) \quad \text{or} \quad \frac{q}{9} = Q - 2Q \quad \text{and} \quad q = -9Q$$

If on the other hand, the total field is negative, then q must be positive and we get:

$$\frac{2k_eQ}{a^2}(-\hat{\boldsymbol{i}}) = \frac{k_eQ}{a^2}(\hat{\boldsymbol{i}}) + \frac{k_eq}{(3a)^2}(-\hat{\boldsymbol{i}}) \quad \text{or} \quad -\frac{q}{9} = -2Q - Q \quad \text{and} \quad q = +27Q$$

23.3 Problem 23.34 *(In the text book)*

(a) Consider a uniformly charged thin-walled right circular cylindrical shell having total charge Q, radius R, and height h. Determine the electric field at a point a distance d from the right side of the cylinder as shown in Figure (23.34). (Suggestion: Use the result of Example 23.8 and treat the cylinder as a collection of ring charges.)

(b) What If? Consider now a solid cylinder with the same dimensions and carrying the same charge, uniformly distributed through its volume. Use the result of Example 23.9 to find the field it creates at the same point.

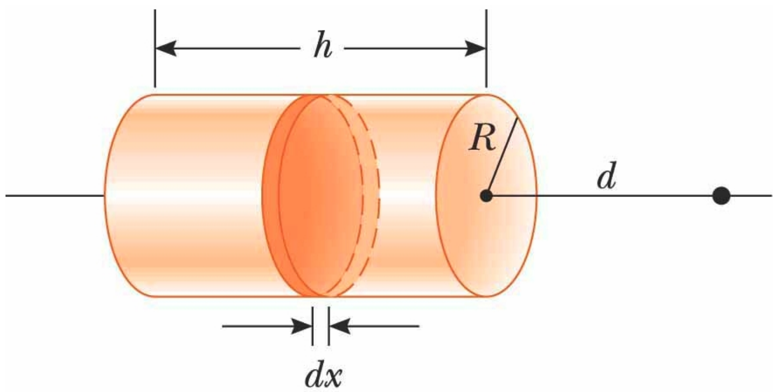

Figure 23.34:

Solution

(a) Let $x = 0$ be at the point where we are to find the field. The small ring with thickness dx withing the object and located at a distance x from the origin, carries a charge of dq:

$$dq = \frac{Q}{h}dx$$

The ring produces an electric field at the origin given by:

$$d\boldsymbol{E} = \frac{k_e x}{(x^2 + R^2)^{3/2}}\, dq(\hat{\boldsymbol{i}}) = \frac{k_e x}{(x^2 + R^2)^{3/2}} \left(\frac{Q}{h}\right) dx(\hat{\boldsymbol{i}})$$

The total field is:

$$\begin{aligned}
\boldsymbol{E} &= \int_{x=d}^{x=d+h} \frac{k_e Q x dx}{h\left(x^2+R^2\right)^{3/2}} \hat{\boldsymbol{i}} \\
&= \frac{k_e Q \hat{\boldsymbol{i}}}{2h} \int_{x=d}^{x=d+h} \left(x^2+R^2\right)^{-3/2} 2x dx
\end{aligned}$$

Let $x^2 + y^2 = z$, so $2xdx = dz$ and the above equation becomes:

$$\begin{aligned}
\boldsymbol{E} &= \frac{k_e Q \hat{\boldsymbol{i}}}{2h} \int_{x=d}^{x=d+h} z^{-3/2} dz \\
&= \frac{k_e Q \hat{\boldsymbol{i}}}{2h} \left[\frac{z^{-1/2}}{-1/2}\right]_{x=d}^{x=d+h} \\
&= \frac{k_e Q \hat{\boldsymbol{i}}}{h} \left[\frac{-1}{\sqrt{x^2+R^2}}\right]_{x=d}^{x=d+h} \\
&= \frac{k_e Q \hat{\boldsymbol{i}}}{h} \left[\frac{1}{\sqrt{d^2+R^2}} - \frac{1}{\sqrt{(d+h)^2+R^2}}\right]
\end{aligned}$$

(b) In case of solid cylinder we take our element of charge to be a solid disk with thickness dx instead of the ring in part (a). Assuming that the charge per unite volume on the whole cylinder is $\rho = Q/\pi R^2 h$.The charge on the disk is then given by:

$$dq = \rho \pi R^2 dx = \frac{Q}{\pi R^2 h} \pi R^2 dx = \frac{Q}{h}\, dx$$

and the charge per unit area σ on the disk is:

$$\sigma = \frac{dq}{\pi R^2} = \frac{Q}{\pi R^2 h}\, dx$$

The electric field produced by this disk at a distance x along the axis is:

$$d\boldsymbol{E} = 2\pi k_e \sigma \left[1 - \frac{x}{\sqrt{x^2+R^2}}\right] \hat{\boldsymbol{i}} = \frac{2k_e Q \hat{\boldsymbol{i}}}{R^2 h} \left[1 - \frac{x}{\sqrt{x^2+R^2}}\right] dx$$

and the total field is:

$$\boldsymbol{E} = \frac{2k_e Q \hat{\boldsymbol{i}}}{R^2 h} \int_{x=d}^{x=d+h} \left[1 - \frac{x}{\sqrt{x^2+R^2}}\right] dx$$

Using the same substitution we used in part (a), we get:

$$
\begin{aligned}
\boldsymbol{E} &= \frac{2k_eQ\hat{\boldsymbol{i}}}{R^2h}\int_{x=d}^{x=d+h}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]\,dx \\
&= \frac{2k_eQ\hat{\boldsymbol{i}}}{R^2h}\left[\int_{x=d}^{x=d+h}dx-\frac{1}{2}\int_{x=d}^{x=d+h}\frac{2xdx}{\sqrt{x^2+R^2}}\right] \\
&= \frac{2k_eQ\hat{\boldsymbol{i}}}{R^2h}\left[x-\frac{1}{2}\frac{\sqrt{x^2+R^2}}{1/2}\right]_{x=d}^{x=d+h} \\
&= \frac{2k_eQ\hat{\boldsymbol{i}}}{R^2h}\left[x-\sqrt{x^2+R^2}\right]_{x=d}^{x=d+h} \\
&= \frac{2k_eQ\hat{\boldsymbol{i}}}{R^2h}\left[d+h-d-\sqrt{(d+h)^2+R^2}+\sqrt{d^2+R^2}\right] \\
&= \frac{2k_eQ\hat{\boldsymbol{i}}}{R^2h}\left[h+\sqrt{d^2+R^2}-\sqrt{(d+h)^2+R^2}\right]
\end{aligned}
$$

23.4 Problem 23.49 *(In the text book)*

Protons are projected with an initial speed $v_i = 9.55 \times 10^3\ m/s$ into a region where a uniform electric field $E = -720\hat{\boldsymbol{j}}\ N/C$ is present, as shown in Figure (23.49). The protons are to hit a target that lies at a horizontal distance of 1.27 mm from the point where the protons cross the plane and enter the electric field in Figure (23.49). Find

(a) the two projection angles θ that will result in a hit and

(b) the total time of flight (the time interval during which the proton is above the plane in Figure (23.49)) for each trajectory.

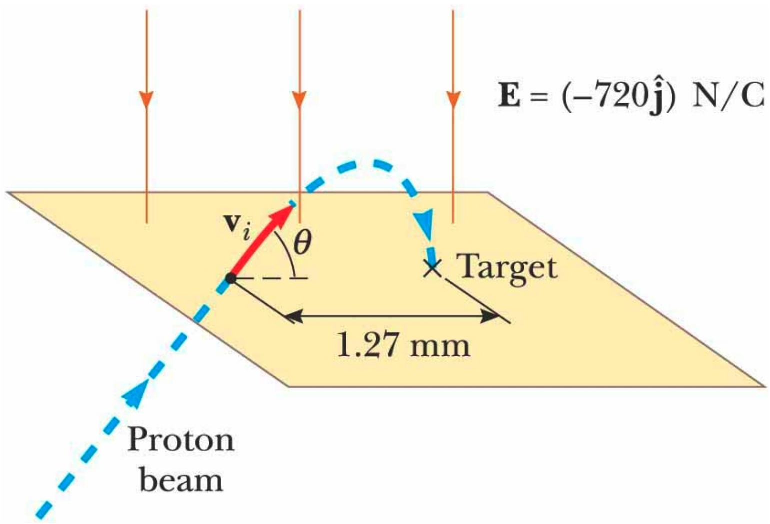

Figure 23.49:

Solution

This problem is similar to projectile problems. The proton acceleration produced by electric force acting acting on the positively charged proton plays the same role in this problem as the acceleration due to gravity in projectile problems. The initial velocity of the proton is $v_i = 9.55 \times 10^{3\ m/s}$.

(a) The proton moves along the $x-$axis with initial speed of $v_{xi} = v_i \cos\theta$ and along the $y-$axis with initial speed of $v_{yi} = v_i \sin\theta$. The proton does not accelerate in the x-direction. It does, however, accelerate downward in the $y-$direction. The proton then moves upward while accelerating downward. The vertical velocity will then diminish until the proton stops and then moves downward. During the time of moving upward to the maximum height and back down again, the horizontal motion would take the proton to the maximum range. The range can then be calculated from:

$$R = v_{xi}2t + \frac{1}{2}a_x 4t^2 \tag{23.2}$$

where t is the time it takes the proton to move to the maximum height, i.e. when $v_{yf} = 0$. This time can be calculated from the vertical motion as:

$$v_{yf} = v_{yi} - a_y t \qquad \text{or for } v_{yf} = 0, \qquad t = \frac{v_{yi}}{a_y}$$

Using Equation (23.2), noticing that $a_x = 0$, we get:

$$R = 2tv_{xi} = v_i\frac{2v_{yi}}{a_y} = v_i\cos\theta\frac{2v_i\sin\theta}{a_y} = v_i^2(2\cos\theta\sin\theta) = \frac{v_i^2\sin 2\theta}{a_y}$$

where we use $2\sin\theta\cos\theta = \sin 2\theta$. Under the effect of the electric filed, the proton accelerates downward with an acceleration a_y given by:

$$a_y = \frac{F_E}{m_p} = \frac{eE}{m_p} = \frac{1.60\times10^{-19}\times720}{1.67\times10^{-27}} = 6.90\times10^{10}\ m/s^2$$

The range of the proton then becomes:

$$R = \frac{v_i^2\sin 2\theta}{a_y} \qquad \text{or} \qquad \sin 2\theta = \frac{Ra_y}{v_i^2}$$

Since R is give to be $1.27\times10^{-3}\ m$, then theta becomes:

$$2\theta = \sin^{-1}\left(\frac{Ra_y}{v_i^2}\right) = \sin^{-1}\left(\frac{1.27\times10^{-3}\times6.90\times10^{10}}{(9.55\times10^3)^2}\right) = \sin^{-1}(0.961) \quad \text{or} \quad \theta = 36.9^\circ$$

Since $\sin 2\theta = \sin(180 - 2\theta)$, then $\theta' = \frac{1}{2}(180 - 2\times36.9) = 53.1^\circ$ will give the same range as 36.9°.

(b) The total time of flight $TOF = 2t$ is given by:

$$TOF = \frac{R}{v_{xi}} = \frac{R}{v_i\cos\theta} = \frac{1.27\times10^{-3}}{9.55\times10^3\cos(36.9^\circ)} = 1.66\times10^{-7}\ s$$

or for $\theta = 53.1^\circ$

$$TOF = \frac{1.27\times10^{-3}}{9.55\times10^3\cos(53.1^\circ)} = 2.21\times10^{-7}\ s$$

23.5 Problem 23.62 *(In the text book)*

Two small spheres, each of mass 2.00 g, are suspended by light strings 10.0 cm in length (Figure (23.62)). A uniform electric field is applied in the x direction. The spheres have charges equal to $-5.00 \times 10^{-8}\ C$ and $+5.00 \times 10^{-8}\ C$. Determine the electric field that enables the spheres to be in equilibrium at an angle $\theta = 10.0°$.

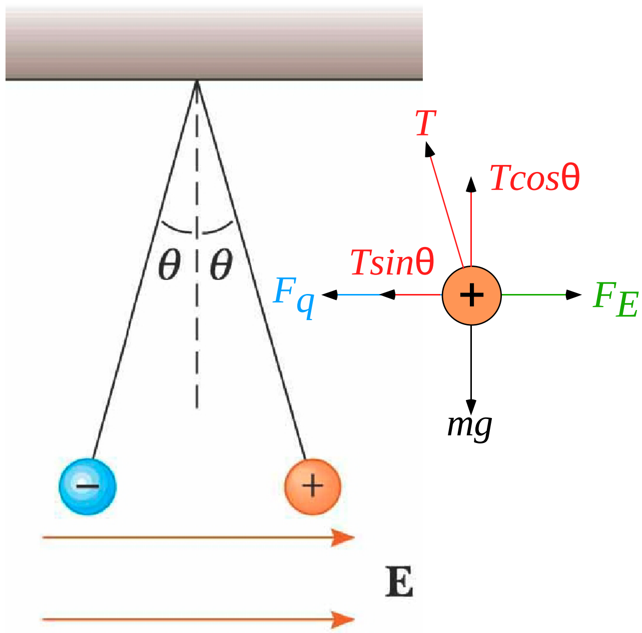

Figure 23.62:

Solution

At equilibrium the charges separated by a distance r given by:

$$r = 2L\sin\theta = 2 \times 0.100 \times \sin(10.0°) = 3.47 \times 10^{-2}\ m$$

Now consider the forces acting on the positive charge: F_q horizontally to the left, F_E horizontally to the right, T upward at angle θ with the vertical, $T\cos\theta$ vertically upward, $T\sin$ horizontally to the left, and the weight mg vertically downward, where F_q is the force due to the negative charge and F_E is the force due to the electric field. At equilibrium $\sum F_x = 0$ and $\sum F_y = 0$, we then get:

$$\sum F_y = 0 = T\cos\theta - mg \qquad \text{or} \qquad T = \frac{mg}{\cos\theta}$$

and

$$\sum F_x = 0 = F_E - T\sin\theta - F_q \qquad \text{or} \qquad F_E - F_q = T\sin\theta$$

Using the above two equations, we get:

$$F_E - F_q = T\sin\theta = \frac{mg}{\cos\theta}\sin\theta = mg\tan\theta = 2.00\times 10^{-3}\times 9.80\times\tan(10.0^\circ) = 3.46\times 10^{-3}\ N$$

F_q is given by:

$$F_q = \frac{k_e q_1 q_2}{r^2} = \frac{8.99\times 10^9\times 5.00\times 10^{-8}\times 5.00\times 10^{-8}}{(3.47\times 10^{-2})^2} = 1.87\times 10^{-2}\ N$$

Now:

$$\begin{aligned} F_E &= qE \\ &= 3.46\times 10^{-2} + F_q \\ E &= \frac{3.46\times 10^{-2} + F_q}{q} \\ &= \frac{3.46\times 10^{-3} + 1.87\times 10^{-2}}{5.00\times 10^{-8}} \\ &= 4.43\times 10^5\ N/C \end{aligned}$$