Thinking about Time *

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Abstract

Time plays a fundamental role in forest management. The interactions between systems that develop over time with their environment and with management actions can be confusing. Useful formalisms for describing dynamic systems have been developed and are taken for granted in many disciplines, but generally they are not fully understood or consistently applied in forestry. I attempt a simple explanation of these ideas, emphasizing their conceptual basis.

Keywords: Forestry, dynamical systems, modelling, state space, scale.

Introduction

Time and models play a particularly important role in forestry. Planning is done over long time horizons. Compared to agriculture, for example, there is less value in experience, experimentation, and trial-and-error. Foresters are forced to make long-term predictions based on indirect observations and on an understanding of system behaviour.

I will not go into philosophical questions about the nature of time. The aim here is pragmatic, seeing how to best represent time-related concepts to facilitate understanding and prediction.

The way in which people think about time probably depends much on individual background and training. Judging from the bulk of the forestry literature, most foresters and forest researchers take what I will characterize

*Presented at FORCOM 2004, International Symposium on The Role of Forests for Coming Generations: Philosophy and Technology for Forest Resource Management. Utsunomiya, Japan, October 2004 as an input/output view. Perhaps the most natural, it derives from a long forestry tradition rooted in the 17th and 18th Centuries. Physicists and engineers, on the other hand, may find natural to resort automatically to a view based on rates of change, due largely to Isaac Newton. The, in many ways more efficient, "rates" view may appear obvious to many; in fact, it tends to seem obvious once understood. Nevertheless, although the ideas are basically simple, the insight necessary to make the switch may never come. My intention here is to trigger that process, with apologies to those already familiar with these matters.

The concepts are often intertwined with details pertaining to specific applications. In the 1960's, System Theory abstracted the basic principles from their origins in classical physics and in the theory of differential equations, producing and formalizing a "disembodied" general theory of dynamical systems, applicable to any system that evolves in time. Some expositions, in increasing order of rigour and mathematical difficulty, are found in Kalman et al. (1969, Ch. 1), Zadeh (1969), and Windeknecht (1971). Much of this is now part of Nonlinear Dynamics, although emphasis has shifted to chaos and other topics less relevant for our purposes. For some forestry applications see, for example, García (1990, 1994), Vanclay (1994), Franc et al. (2000).

I will attempt an intuitive, non-mathematical explanation of the basic principles, geared toward typical forestry situations.

Modelling time

Models

By models I do not mean necessarily mathematical models, but any representation of an aspect of reality. In particular, the mental models that are necessary for reasoning about a problem, models that may or may not be eventually verbalized and expressed in natural or mathematical language. Of course, there are philosophical issues about reality that we do not need to get into. Suffice to say that no model is "true"; models may have more to do with the structure of the human brain than with what exists "out there" (if anything!).

Deterministic or stochastic?

A common objection to deterministic models is that nature is variable. Every realization of a natural process is going to be different from every other one: models should be stochastic. However, if instead of the actual value of a future outcome we think of its expectation, most likely value, or the median, a deterministic model for such a location parameter is perfectly valid. In fact, that may be all what a decision-maker needs, wants, or is able to use.

More generally, in practice stochastic models are just "deterministic" models for a certain number (maybe even an infinite number) of probability distribution parameters. Whatever we might mean by *probability* (Barnett 1999, Jaynes 2003). To keep things simple I will consider deterministic models, which might describe, e. g., a most likely behaviour.

Input/output

Forestry frequently deals with relationships between functions of time. For instance, a yield table may predict volume over time for a given sequence of thinnings, specified by their timing and intensity. Or a process model might describe responses to a trend in temperature. In forest planning we might be interested in the trend of timber supply, measured by the price of timber, given past annual harvests. The time functions may be discrete (thinnings), continuous (temperature), or we might have the choice of modelling them either way (price, harvest). More generally, one may be interested in several responses (*outputs*), affected by a number of *input* variables.

Roughly, the general problem may be pictured as in Figure 1. The inputs and outputs can be lists of numbers (vectors), or even more general objects. Outputs depend on the whole history of past inputs.

This is a confusing state of affairs. A time-function is here a function of another time-function (what mathematicians call a *functional*). It is not at all clear how one could go about modelling or reasoning about this system. Many researchers have discovered correct or almost correct solutions along lines similar to those described below. Especially in growth and yield, however, others continue publishing models that are clumsy and/or logically flawed. Examples of this are most uses of variable-density yield tables for managed stands. I will show how to generate the functional through simpler rate functions.

Output functions

As an intermediate step, let us introduce a state description, information pertaining to the current situation at some point in time. For now, assume that this description includes everything that might be relevant to the char-

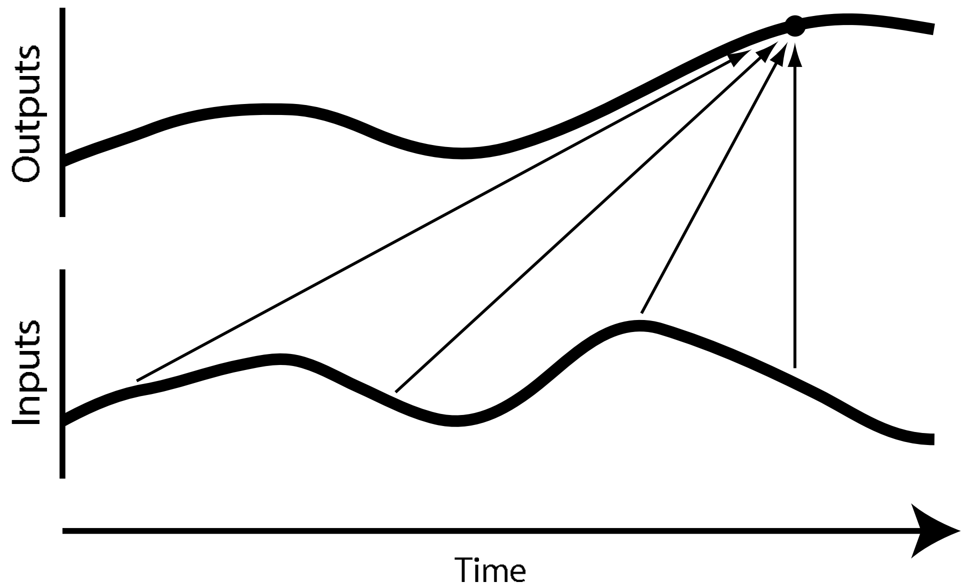

Figure 1: Input/output view.

acterization and behaviour of the system at a point in time, and perhaps also data that is not relevant in that sense. In a harvest planning problem, that *state* might include, among other things, the number of hectares in each forest type and age class. In a stand yield projection it might include the dimensions and relative locations of all trees, their nutritional status, the amount of moisture in the soil, etc.

Clearly, by definition, the state at some given time determines the output at that time. In other words, the current outputs are a (vector) function of the current state:

$$\text{outputs} = \boldsymbol{g}(\text{state}) \tag{1}$$

(I use $\boldsymbol{g}(\cdot)$ for the function to keep with tradition, reserving $\boldsymbol{f}$ for later). Given this *output function*, we "only" need to know the state at the time of interest.

Therefore, let us forget the outputs, and try to predict the state. Imagine again a process like Figure 1, but with "state" in place of "outputs". At first sight it appears that we have gained nothing, on the contrary, we seem to have made the prediction problem much worse. This is not the case.

State transitions

The crucial insight comes from realizing that any effect of the past on the future has to pass through the present. Past history can only affect the future through a changed present state (Figure 2). Causality requires mediation through tangible properties of the system at a point in time. Knowledge of the state makes history irrelevant.

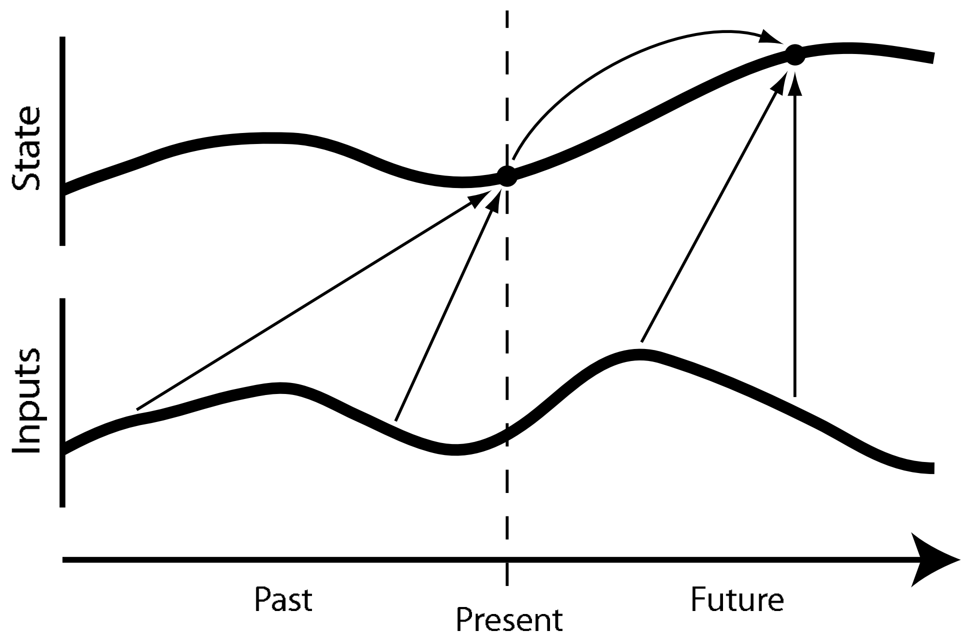

Figure 2: State projection.

It is important to understand that this is not an assumption; it is a consequence of the definition of state. If systems with the same state and future inputs behave differently, it means that more should have been included in the state description. "Roughly, the state of a system at any given time is the information needed to determine the behavior of the system from that time on" (Zadeh 1969), "a kind of information storage or memory or an accumulation of past causes" (Kalman et al. 1969). Of course, in a model we compromise, taking into account trade-offs between accuracy, complexity, information requirements, and other considerations.

From Figure 2,

$$\text{future state} = \boldsymbol{F}(\text{current state},\ \text{time interval},\ \text{interval inputs})\ . \quad (2)$$

This is called a (global) *transition function.* It is especially convenient and easy to use for time intervals without inputs. For example, in-between thinnings when modelling stand density management. A thinning causes an essentially instantaneous change of state.

The transition function (2) has reduced the problem to an input-state relationship over a smaller time interval than in the original formulation. Nevertheless, (2) must satisfy some consistency conditions. For instance, splitting the time interval in two, and projecting the state over the two intervals in sequence, should produce the same result as the one-step projection over the whole interval. Even without inputs, it is not obvious how to obtain a function with these properties. The solution is to model the change of state, or the rate of change per unit time, for small time steps, and to iterate or accumulate. With small enough steps, ignoring any input changes within the time-step,

$$\text{next state} = \boldsymbol{f}(\text{current state},\ \text{current inputs})\ , \tag{3}$$

Or, equivalently (different $\boldsymbol{f}$),

$$\text{rate of change of state} = \boldsymbol{f}(\text{current state},\ \text{current inputs})\ . \tag{4}$$

These are known as a *local transition function.* Given an initial state and any inputs over some time interval, the function can be repeatedly evaluated, and (3) iterated, or the increments from (4) accumulated, to obtain the state at the end of the interval as specified by (2).

Alternatively, the local transition function (4) can be defined as the limit when the step length tends to zero (instantaneous rates, Figure 3). The resulting differential equation can then be integrated, analytically or numerically, to produce (2).

State-space modelling

The approach should now be clear. Instead of attempting to model directly the input-output relationships of Figure 1, one first decides on an appropriate state description. Then, a rate equation (4) is obtained. Summation or integration can produce state predictions for any initial state, any projection interval, and any inputs. Finally, outputs are estimated with the output function (1). Unlike *ad hoc* input-output-based models, results are logically consistent.

The choice in (4) between discrete or continuous time (difference or differential equations), can be largely a matter of personal taste. Finite steps

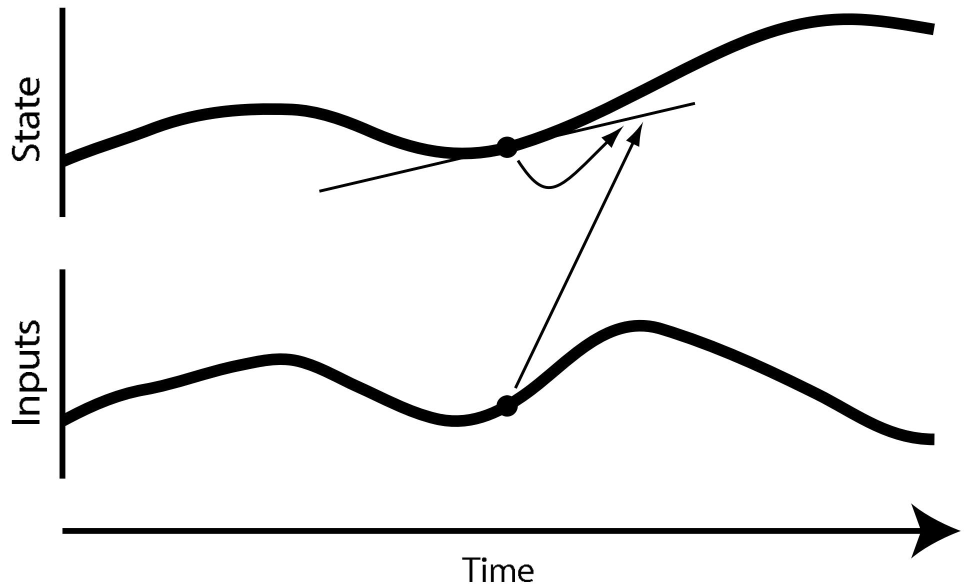

Figure 3: Rates.

can be less intimidating, and maybe easier to implement, although some awkward approximations may be needed when the projection interval is not a multiple of the basic step. Instantaneous rates are more general, more "accurate", and the model's mathematical properties are easier to study.

The most appropriate level of detail in a state description (the model scale or resolution level) can vary, depending of many factors. The classic Goulding-Munro classification of forest growth models reflects those differences: a few variables in whole-stand models, sizes of many trees in individual-tree distance-independent models, additional spatial coordinates in distance-dependent models (Goulding 1972, Munro 1974, Vanclay 1994). Fundamentally, the state must satisfy two conditions: (a) the rate of change must be predictable from the current state and current inputs, and (b) it must be possible to estimate the outputs of interest from the state. Other considerations include the model purpose (prediction or understanding), the data available for parameter estimation, and in predictive models the reliability of initial state estimates.

With research models the choices are less clear, due to the varied and sometimes ill-defined purposes that they serve, and to the fact that they do not need to be limited to variables that are economical or even feasible to measure. Computing advances have made it easier to produce complex models rather than simpler ones, ignoring the motto of Hamming (1962): "The purpose of computing is insight, not numbers". It has been suggested that, as a rough guide, one should model at the next scale below (more detailed than) the scale of interest. On the contrary, in predictive models for decision-making, the most direct link between actions and consequences should probably be used. Paraphrasing Einstein, "use as few state variables as possible, but not less".

An active and still incipient area of research, relates to the linking of different scales. See, for instance, Picard and Franc (2004), and references therein.

Conclusions

Thinking about systems that evolve over time can be confusing, and logical errors are often made. The theory of dynamical systems can help. It is taken for granted in many disciplines, but curiously, it has rarely been systematically applied in forestry.

Once the basic ideas are understood, it becomes natural to model any time-dependent system through rates of change.

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