A site index model for lodgepole pine in British Columbia

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Abstract

A height growth and site index model was developed combining permanent sample plot (PSP) and stem analysis (SA) data from the Sub-Boreal Spruce biogeoclimatic zone of British Columbia, Canada. SA data were screened for indications of early suppression, virtually eliminating that source of bias. Estimation was based on a stochastic differential equation form of the Bertalanffy-Richards growth curve, including environmental and measurement/sampling variation. Methods involve the inclusion of mixed effects, and obtaining simultaneous maximum likelihood estimates for all the parameters. Differences in error structure between PSP and SA data were taken into account. The model fits the data well, and compares favorably to existing alternatives.

Keywords: Forest growth and yield, site productivity, *Pinus contorta*, stem analysis

1. Introduction

Lodgepole pine (*Pinus contorta* var. *latifolia*) has been the most important commercial timber species in British Columbia. Despite a mountain pine beetle (*Dendroctonous ponderosae* Hopkins) epidemic that killed much of the mature lodgepole pine in the Central Interior over the last decade, it is still the preferred species for planting. Reliable tools for forecasting pine stand development are made even more desirable by an extensive area of younger pine coming on-stream, the challenges of managing and predicting the dynamics of beetle-damaged stands, and a tight future wood supply. Stand top height growth models, or site index models, are an important component of forest growth prediction systems, and are also used for site quality estimation and classification.

Permanent sample plots (PSPs) are the most reliable source of top height growth data (Clutter et al., 1983), but large representative samples are not always available, and the observations are subject to considerable sampling and measurement variability. Stem analysis (SA) reconstructs the past height development of a tree from growth-ring counts at various points up the stem. It was already in use by Reventlow in the late 18th Century (Reventlow, 1960) [1]. SA provides long low-noise height series, without the wait for re-measurements. However, the values are only representative of stand top or dominant height if a selected dominant tree has been equally dominant in the past, otherwise the slopes of the height-age curves are overestimated (Dahms, 1963; Magnussen and Penner, 1996). In fact, SA was deemed unsuitable for site index modelling (e.g., Bruce, 1926), until Curtis (1964) and others felt that the possible error might be of minor importance compared with the advantages of the SA approach. More recently, the issue has been largely ignored, and most modern site index models are derived from SA data (e.g. Hann, 1995).

Current site index curves in British Columbia were entirely derived from stem analysis (Ministry of Forests and Range, 2009). It has been observed that site indices estimated from old stands appear lower than those obtained from younger stands in similar sites (Nigh, 1998; Nussbaum, 1998; Stearns-Smith, 2001). This has been interpreted as an underestimation in mature stands, and yield projections for allowable cut calculations or other applications routinely use an *Old Growth Site Index Adjustment* (OGSI) to adjust estimates upwards. Nigh (1998) pointed out, however, that the bias discussed by Magnussen and Penner (1996) could cause overestimation in younger stands, providing a possible alternative explanation for the observed differences. It would be important to clarify this matter.

García (2005) showed how to combine PSP and SA data to make the most of the information available, and to reduce the impact of SA bias (see also Hu and García, 2010). We follow that methodology to produce a new lodgepole pine site index model, less biased than the ones currently available and based on a more extensive data set. We introduce also a new data screening approach that reduces further the effect of changes in dominance.

2. Data

All the data came from the Sub-Boreal Spruce (SBS) biogeoclimatic zone, which occupies the northern half of the Interior Plateau of British Columbia, and which accounts for the bulk of lodgepole pine timber production. Full details on sources and data processing are given by Batho (2011). Data reduction methods are described also in Hu and García (2010). A brief summary follows.

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2.1. *Permanent sample plots*

Two types of permanent sample plots were used: 1) Provincial PSPs (PPSP), which are isolated sample plots located within existing stands of natural origin; 2) experimental plots (EP), that are part of designed experiments established in young plantations. We used only the control plots from EP trials that involved fertilizing or unusual soil preparation treatments. All plot measurements were required to contain at least 80% pine by basal area, to have an age range not exceeding 20 years, and to have a size of at least 0.04 ha.

Breast-height age, defined as number of growth rings at breast height (1.3 m), was obtained from numbers of rings in increment core samples. If necessary, ring counts were harmonized to be consistent with the known measurement dates. In detail, a common breast-height date for each plot was first estimated by subtracting ring counts from their dates and averaging; the resulting value was then subtracted from each measurement date to produce the stand age estimate (Hu and García, 2010). The breast-height age definition implies an age of 0.5 years at breast height, a convention officially adopted in British Columbia in 2004 (Ministry of Forests and Range, 2009).

Top height corresponds to the largest-diameter suitable tree in 0.01 ha (Forest Productivity Council, 1988). Tree heights were calculated from standard height-diameter regressions on height sample trees. Top heights for sample plots are conventionally calculated as the mean tree height from a proportion corresponding to the largest 100 trees per hectare. However, this is always larger or equal than the mean from a subdivision into 0.01 ha subplots. The U-estimator of García and Batho (2005) was therefore used to avoid this source of bias. Data statistics are shown in Table 1. All the data are graphed later together with final model results.

Table 1: Plot statistics, after screening.

	Mean	Range	S.D.
PSP natural (PPSP, 244 plots, 813 meas.)			
Measurements	3.33	2 – 4	0.703
First b.h. age (yrs)	40.4	5.5 – 142	16.8
Last b.h. age (yrs)	63.6	15.5 – 144	19.3
First top height (m)	16.5	3.9 – 27.6	4.76
Last top height (m)	21.3	8.7 – 30.8	4.78
Est. site index	19.5	12.0 – 25.0	2.58
PSP planted (EP, 30 plots, 149 meas.)			
Measurements	4.97	4 – 5	0.183
First b.h. age (yrs)	3.22	0.53 – 6.70	2.61
Last b.h. age (yrs)	22.0	19.0 – 25.7	2.65
First top height (m)	2.79	1.67 – 4.59	1.09
Last top height (m)	11.8	10.8 – 13.8	0.789
Est. site index	20.8	17.8 – 21.9	1.03
SA natural (sections, 61 plots, 695 meas.)			
Measurements	11.4	6 – 12	0.862
Last b.h. age (yrs)	99.6	50.8 – 192	27.6
Last top height (m)	25.0	11.8 – 35.0	5.84
Est. site index	19.3	7.2 – 26.3	4.46
SA planted (nodes, 20 plots, 272 meas.)			
Measurements	13.6	9 – 19	2.33
Last b.h. age (yrs)	13.0	8.8 – 19.0	2.24
Last top height (m)	7.80	5.76 – 9.81	1.08
Est. site index	20.6	18.1 – 23.9	1.48

2.2. *Stem analysis*

These data were of two types: 1) node data, and 2) section data. In *node data*, young plantation trees were split longitudinally along the pith, providing an accurate determination of tree height at each age. Sampling selected 3 trees within circular plots of 0.03 ha, following procedures described by Nigh (1996) designed to be representative of the top height based on the largest tree in 0.01 ha. *Section data* employed the more conventional method of counting growth rings on cross-sections at fixed positions along the stem (Reventlow, 1960). Section data were obtained from 3 trees in 0.03 ha plots, or 4 trees in 0.04 ha plots, all of natural origin.

In section data, tree heights are only known to lie between two consecutive cross-sections, and a number of methods have been proposed for interpolating the cross-section heights (Dyer and Bailey, 1987). Following Milner (1992) and J. Goudie (unpublished, cited by Nigh, 1995), we instead kept the lower cross-section height and adjusted the cross-section age by subtracting 0.5 years. We used a single age-height pair for each section, avoiding the common practice of generating artificial values for each intermediate year through linear interpolation (Salas and García, 2006; Hu and García, 2010).

All SA trees were evaluated graphically, and some obviously inconsistent measurements or complete trees were rejected. A single age-height sequence was produced combining all the trees in each plot through matching nearest sections (Batho, 2011). Table 1 gives summary data statistics.

2.3. *Screening for suppression*

Graphical examination indicated that some SA plots had growth patterns suggestive of past suppression. That is, low height growth rates at young ages, compared to later growth rates and to the heights achieved at maturity. We quantified these trends by computing the ratio of height at age 50 years to height at age 25 years, interpolating available height-age observations when necessary. A threshold of 1.75 for the ratio seemed reasonable for discriminating the most extreme patterns, and 14 plots exceeding this ratio were removed from the database (Figure 1). The node data were excluded from this analysis because they did not contain long term trends.

It should be noted that in these data the effects of suppression were masked to some extent by the process of combining the 3 or 4 trees in a plot. In retrospect, a screening of the individual trees might have been preferable. Nevertheless, the results below suggest that the data selection was effective; in contrast to previous studies, no appreciable differences were found between PSP and screened SA height-age trends. A more striking example is observed in the classical data set of von Guttenberg (1915), that was produced by stem analysis of individual Norway spruce trees (Figure 2, data kindly provided by the late Prof. Boris Zeide).

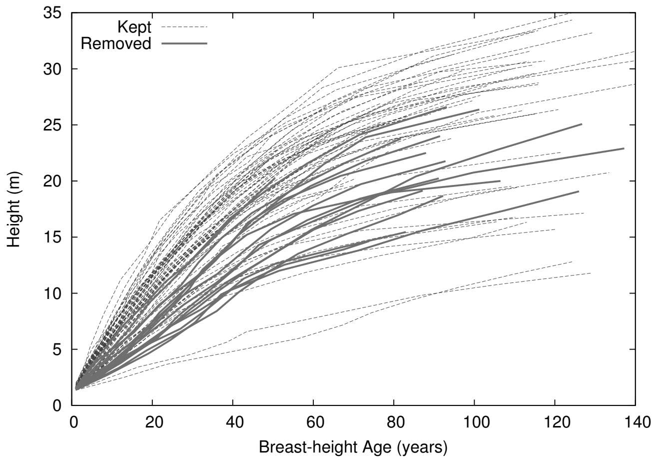

Figure 1: Stem analysis data suspected of early height growth suppression were removed.

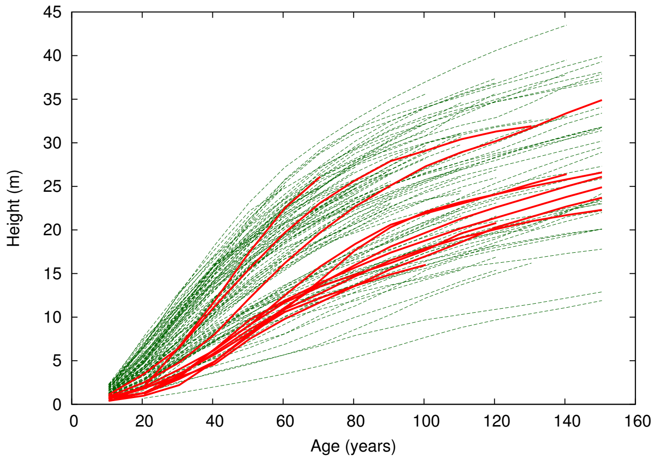

Figure 2: Indications of changing dominance in Norway spruce stem analysis data from von Guttenberg (1915). The thick curves are for trees where the ratio of height at age 70 to height at age 30 exceeds 3.5.

3. Models and parameter estimation

We used the Bertalanffy-Richards growth model (von Bertalanffy, 1938, 1949; Richards, 1959), also known as Richards or Chapman-Richards (Vanclay, 1994; García, 2008), one of the most commonly used height growth equations. Its differential form can be conveniently linearized as

$$\frac{\mathrm{d}H^c}{\mathrm{d}t} = b(a^c - H^c) \;, \tag{1}$$

which integrates to

$$H = a\{1 - [1 - (H_0/a)^c] \exp[-b(t - t_0)]\}^{1/c} \;, \tag{2}$$

where H is top height (m), t is time or age (yrs), and H_0 is a known top height at a known time or age t_0. Each of a, b and c may be common to all plots (*global* parameters), or may depend on stand site quality through a *local* site-specific parameter q (García, 1983, 2011). Four versions of site-dependence were tested, specified in Table 2, with a_1, a_2 denoting additional global parameters.

Table 2: Site-dependency alternatives for equations (1) and (2)

Model Id	a	b	c
a-local	q	global	global
b-local	global	q	global
Linear	$a_1 + a_2 q$	q	global
Power	$a_1 q^{a_2}$	q	global

Equation (2) can be used to estimate the height H at any time t, given a height H_0 at some other time t_0. With a fixed origin $t_0 = 0.5$, $H_0 = 1.3$, the equation describes a family of site index curves, each curve corresponding to a particular value of the local parameter. More conventionally, curves are labelled by a site index S, defined as the predicted height at some reference age (50 years breast-height age in British Columbia). The relationship between site index and the site-dependent parameter q is obtained by substituting 50 for t and S for H in equation (2).

Parameter estimation followed the methods of García (1983) (see also Seber and Wild, 2003, Section 7.5.3), implemented in the software package *EasySDE* (`http://forestgrowth.unbc.ca/sde`). It is a hierarchical mixed-effects approach, where environmental perturbations act through a stochastic process added to equation (1), integrating to temporally correlated random variation characterized by a parameter σ. Measurement and sampling error is an independent error component parametrized by σ_m. The plot-level parameter q is treated as a fixed effect, since the data were not a simple random sample from the target population (García, 2011). Given the error model (equations (6)–(7) of García (1983)), all the parameters are estimated simultaneously by maximum likelihood.

The maximized log-likelihood can be used to compare model formulations, if necessary penalizing more complex models by subtracting from the log-likelihood between 1/2 and 2 units for each additional parameter (the often-used AIC criterion is equivalent to subtracting 1 unit). After this adjustment, differences of about two units may be considered as "significant". Alternatively, hypotheses can be evaluated through a likelihood ratio test, based on the fact that twice the log-likelihood difference is asymptotically distributed as a χ^2 with degrees-of-freedom equal to the difference in the number of parameters (García, 2005).

Two ways of fitting a model with all the PSP and SA data were used (García, 2005; Hu and García, 2010). The simplest was to estimate the parameters by pooling the data, refered to as *Pooled* in the Results. However, the error structure of the PSP and SA may be expected to differ. Specifically, PSP data tend to have higher measurement and sampling variability, represented by a larger σ_m. On the other hand, SA is subject to changes in dominance and other factors that may contribute variation correlated over time, increasing σ. Therefore, a second way of combining the data was used that allowed the PSP and SA to have different values of σ and σ_m, giving the *Combined* estimates.

4. Results

4.1. Parametrization and data sources

For planted stands, the penalized log-likelihood differences among the 4 model formulations in Table 2 were small, and not consistent across PSP, SA, pooled or combined data. This might be expected because the narrow age ranges makes it difficult to discriminate curve shapes. In natural-origin plots, and when pooling natural and planted, the *a*-local model was consistently better than the *b*-local, and both the *Linear* and *Power* versions were significantly better than the other two. *Power* had generally a slightly better fit than *Linear*, although the differences were not statistically significant. We show only results for the *Power* parametrization in what follows.

Table 3 gives the maximized log-likelihoods by data sources and aggregates. 'Total' are sums that correspond to fitting separate models. Convergence was not achieved for the combined PSP + SA planted plots. The data labelled 'Natural' and 'Planted' differ also in other important ways: In the PSPs, *Natural* are isolated plots established in existing stands, while *Planted* plots came from designed experiments. In SA, *Natural* and *Planted* correspond to section and node data, respectively.

Table 3: Log-likelihoods (number of global parameters in parenthesis).

	Natural	Planted	Total	Pooled
PSP	1227.6 (5)	376.2 (5)	1603.8 (10)	1540.9 (5)
SA	1290.5 (5)	890.9 (5)	2181.4 (10)	2127.5 (5)
Total	2518.1 (10)	1267.1 (10)	3785.2 (20)	3668.4 (10)
Pooled	2375.3 (5)	1129.8 (5)	3505.0 (10)	3447.5 (5)
Combined	2505.8 (7)	No conv.	–	3645.6 (7)

Penalized likelihoods and likelihood ratio tests indicate statistically significant differences in favor of separate *Natural* and *Planted* models, compared to single models that pool both data sources (see García (2005) or Hu and García (2010) for calculation details). However, the *Planted* data includes only young ages, and cannot be expected to extrapolate reliably to mature stands. In fact, graphing indicates that extrapolations from the models fitted separately to the planted data are not realistic. Pooling the *Natural* and *Planted* data sources is the only practical option, and such a solution intermediate between plots of natural origin and those from carefully maintained experiments may be more representative of future operational stand conditions.

Allowing different error parameters for PSP and SA data (*Combined*) gave much higher penalized likelihoods than simply pooling the data with common parameter values. As expected, the measurement error component was higher in PSPs than in SA.

Comparing *Total* to *Combined* indicates statistically significant differences between the PSP and SA data. However, differences between top heights predicted by separate PSP and SA models are practically negligible. For site index 20, the maximum difference below 50 years is 12 cm at age 31, or 0.9% at age 25 (slightly larger heights in PSP, as might be expected). These differences are much lower than the 11% found by Hu and García (2010), or the even larger theoretical calculations of Magnussen and Penner (1996), and of Feng et al. (2006), suggesting that the dominance screening was effective in controlling SA bias.

4.2. Model

From the preceding analysis, we chose a *Power* combined model using all the data. Equation (2) becomes

$$H = a_q\{1 - [1 - (H_0/a_q)^{0.8297}]\exp[-q(t - t_0)]\}^{1/0.8297}\ , \quad (3)$$

with

$$a_q = 12313q^{1.645}\ , \quad (4)$$

giving the transition between any two time-height points (t_0, H_0) and (t, H). The equation is time-invariant, i.e., independent of the time origin, so that it makes no difference if t and t_0 are ages or chronological time.

As mentioned before, site index equations are obtained by substituting 0.5 for t_0 and 1.3 for H_0. The conventional site index S is obtained from q as the predicted height at $t = 50$:

$$S = a_q\{1 - [1 - (1.3/a_q)^{0.8297}]e^{-49.5q}\}^{1/0.8297}\ . \quad (5)$$

Given S, q can be obtained by solving (5) with any root-finding algorithm. More generally, equation (3) can be solved for q knowing any pair of points (t_0, H_0) and (t, H). A simple method is to use the iteration

$$q \leftarrow \left[(\frac{H^{0.8297} - H_0^{0.8297}\exp[-q(t - t_0)]}{1 - \exp[-q(t - t_0)]})^{1/0.8297}/12313\right]^{1/1.645} \quad (6)$$

(the iterative procedure from Hu and García (2010) does not converge in this instance).

Code and software implementing this model are available from `http://forestgrowth.unbc.ca/site`.

4.3. Evaluation

Site index curves are shown with the data in Figure 3. Figure 4 shows in more detail the younger ages, where site index estimates are often required for forest planning. There is good agreement between the model and the data over the whole range of ages and site qualities.

Table 4 indicates the differences that can be expected between individual plot measurements and predicted heights. Heights were predicted from breast height using the local site quality parameter estimated for each plot. The residuals (observed minus predicted) are plotted in Figure 5; the curve is a nonparametric local regression produced by the R *loess* procedure, with the default smoothing parameters in *scatter.smooth* (R Development Core Team, 2009).

The predictions shown in this Section are forecasts starting from breast height. When forecasting future heights for an existing stand, equation (3) would be used with (t_0, H_0) being the most recent observation. Errors would then vary with the starting point and the projection interval, and in general the predicted top height at age 50 will differ from the nominal site index (García, 2011).

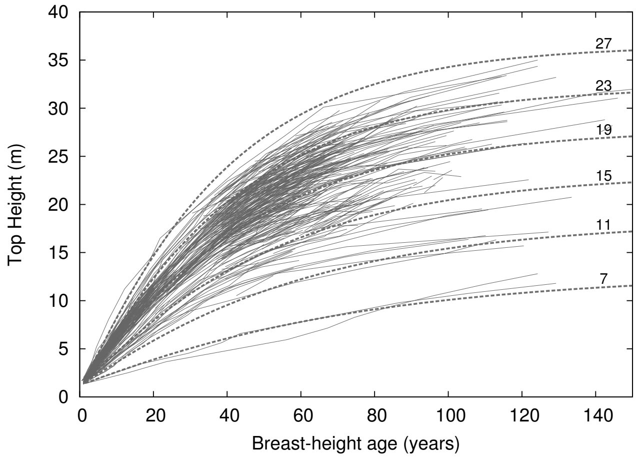

Figure 3: Data and site index curves for the final model.

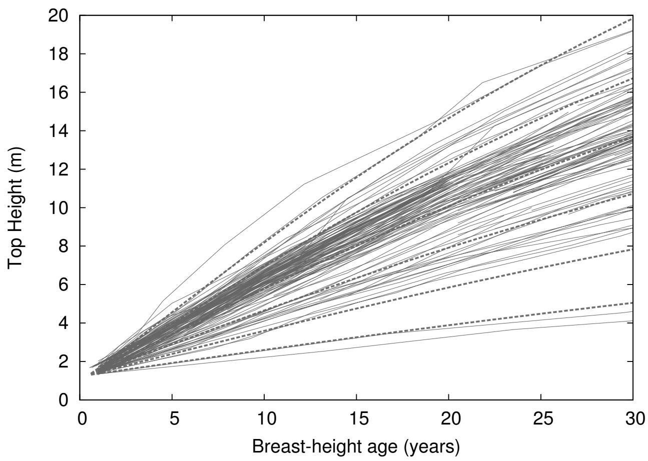

Figure 4: Detail of Figure 3 for younger ages.

Table 4: Residual statistics for height predicted from breast height, predicted minus estimated (m). PPSP: Provincial PSPs, natural. EP: Experimental Plots, planted.

	PPSP	EP	Section	Node	Total
Mean	0.00357	0.0392	-0.0145	-0.0892	-0.0133
S. D.	0.540	0.194	0.654	0.149	0.533
Minimum	-1.83	-0.555	-2.16	-0.406	-2.16
Maximum	2.91	0.471	2.23	0.237	2.91

The currently recommended model in British Columbia is that of Thrower and Associates (1994), developed from stem analysis data (Ministry of Forests and Range, 2009). Compared to the new model, Figure 6 shows that the Thrower and Associates's model over-estimates site index in young stands, more severely for lower site indices. Apart from possible dominance bias (Dahms, 1963), low height predictions at young ages can be expected because the equation forces a horizontal slope for the height-age curves at breast height (Salas and García, 2006). Formulations that produce such zero growth rate at breast-height are commonly used (Monserud, 1984; Carmean et al., 1989; Milner, 1992).

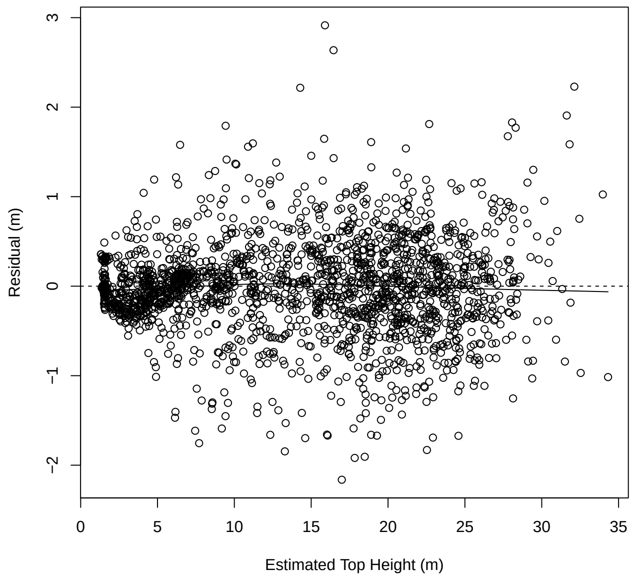

Figure 5: Residuals for top height forecasts starting at breast height. Continuous curve: nonparametric local regression. Dashed: zero line.

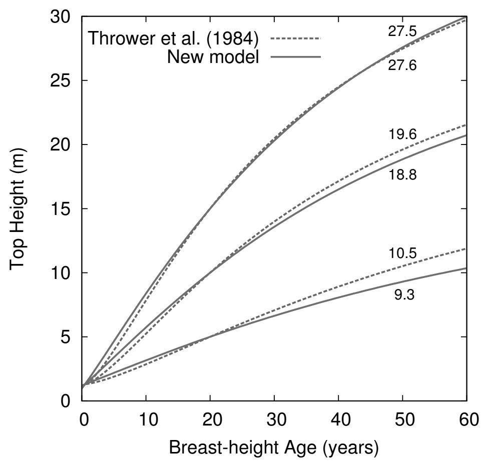

Figure 6: Comparison between the new model and that of Thrower and Associates (1994). The curves were chosen to pass through common heights (5, 10, and 15 m) at 20 years breast-height age. The site indices are shown (heights at age 50).

5. Conclusions

Combining data from permanent sample plots and from stem analysis, taking into account differences in error structure, ensured an efficient use of the available field information. Screening stem analysis data for suspected changes in dominance greatly reduced bias. Differences between height-age trends fitted separately to permanent sample plots and to the screened stem analysis data were negligible, compared to previous studies (García, 2005; Hu and García, 2010) and to theoretical calculations (Magnussen and Penner, 1996; Feng et al., 2006).

The model describes accurately the observed height-age relationships. Previously available models underestimate early growth rates, causing site index calculations in young stands to be too high. This may partly explain the apparently lower site indices observed in older stands (Nigh, 1998; Nussbaum, 1998; Stearns-Smith, 2001), although further study would be needed to reach definite conclusions.

The model presented here is recommended for lodgepole pine height growth forecasting and site index estimation in British Columbia.

Endnotes

[1] Count Reventlow (1748–1827), Lord Chancellor of the Danish Crown, wrote his treatise between the years 1800 and 1827, first in Danish and later in German. The manuscripts were only published long after his death, the Danish version in 1879 and the German in 1934. The reference cited contains an English translation based mainly on the Danish edition. Other notable contributions include Reventlow's use of spatially explicit individual-tree growth modelling, and forest economics analyses that anticipated the work of Faustmann.

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