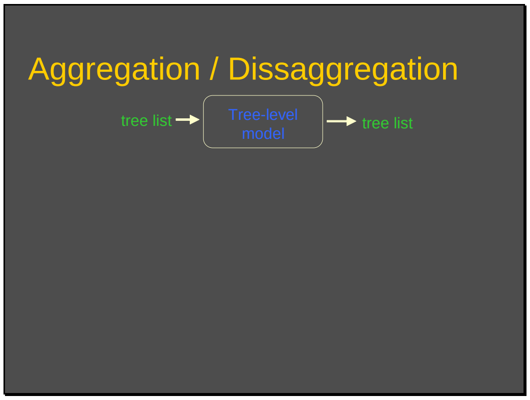
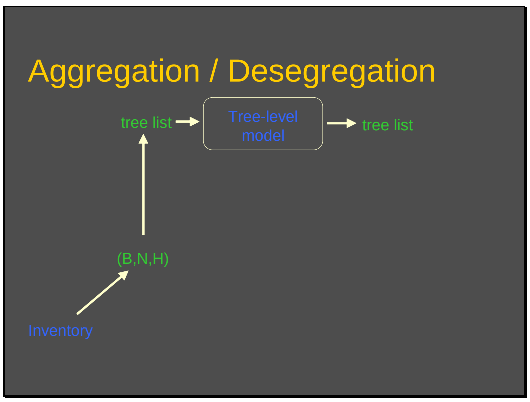
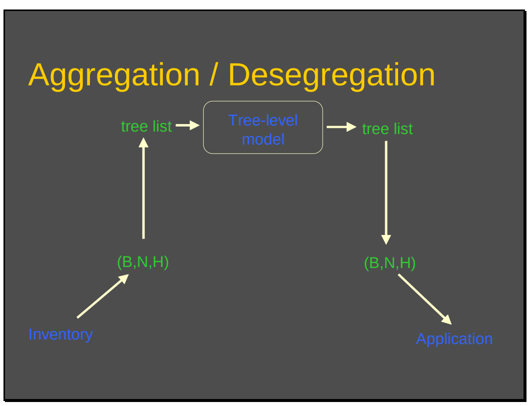


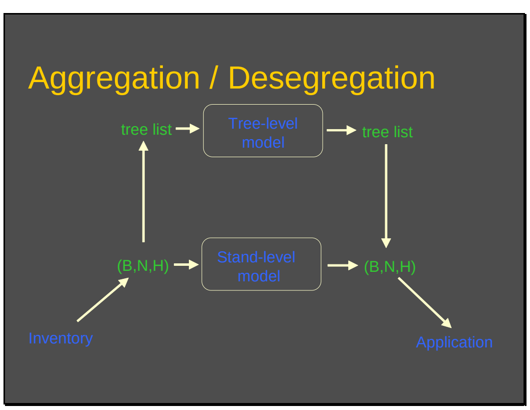
Individual-tree models require detailed initial state info.
E.g., full tree list.



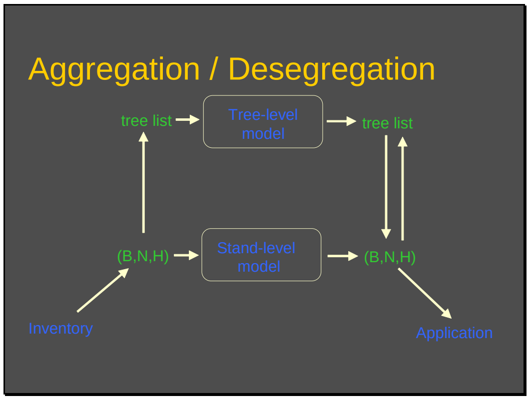
Often, only aggregate, stand level info is available.
Then, to use an individual-tree model, an artificial tree list or stem map needs to be generated somehow.



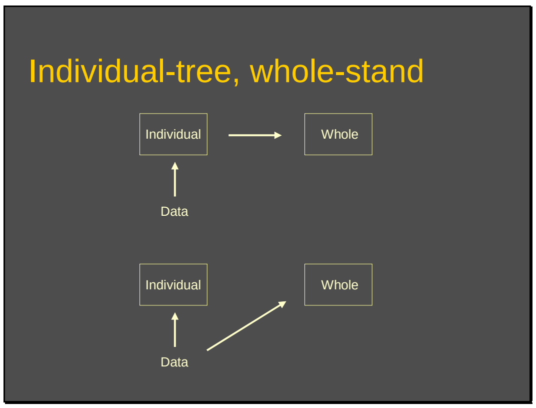
Often, the projected stand description is summarized to produce the stand-level variables actually needed for decision-making.
Conceptually, this is equivalent to a whole-stand model with a complicated transition function.



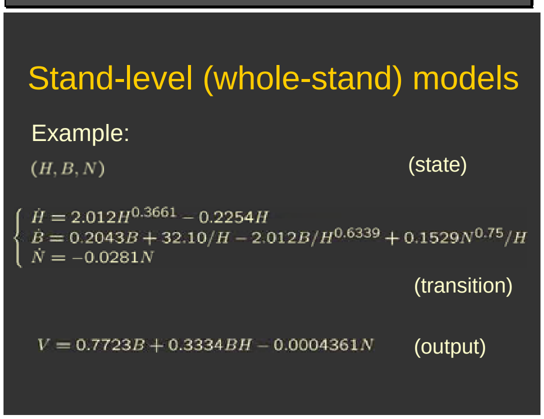
Alternative: whole-stand (stand-level) models.
Not feasible (yet) for many complex uneven-aged stands.
Not where detailed tree-level information is crucial, and cannot be reliably generated from stand-level variables.



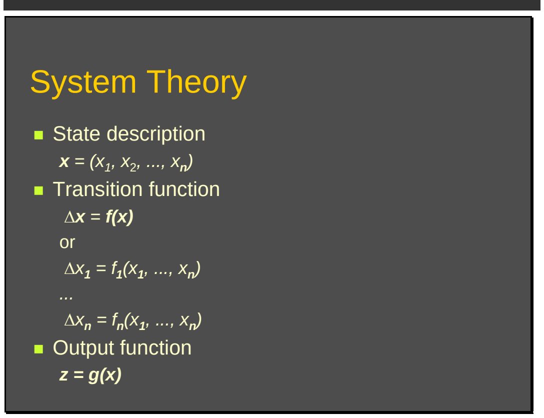
Often argued that tree-level model predictions provide more useful info.
But if “tree list generation” is OK for the initial state, then it could also be done at the end.



Lab examples and TADAM (below) showed whole-stand models derived from individual-tree models. That is (very) unusual. Most whole-stand models are obtained independently of any individual-tree model. That is, it is not necessary to have an individual-tree model first. Averaging/aggregation biases are only an issue when mathematically deriving a model from another.



From *Forest Ecology and Management* 173, 49-62, 2003.



General, for any system evolving in time.
Vector notation shorthand. Vector equation = system of (scalar) equations.
Local transition function: rate of change in state variables (either finite differences, shown, or derivatives). Accumulation (sum) or integration of local transition function produces the *global* transition function $\mathbf{x}(t) = \mathbf{F}[\mathbf{x}(t_0), t-t_0]$, giving change between any two times.
Individual-tree $\rightarrow$ large *n*
Whole-stand $\rightarrow$ small *n*

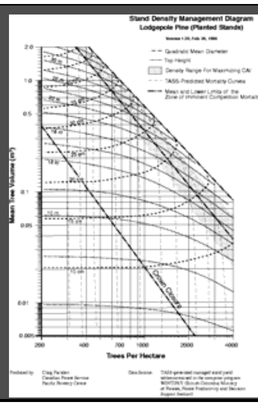
BC whole-stand models

- STIM (individual-tree + whole-stand)
- VDYP7
- Stand density management diagrams
- TADAM-df, TADAM-p, TADAM-s
- SBS spruce – Scube

Alternative “stand-level” term is sometimes used with reference to all growth models, as opposed to “forest-level” planning models (forest estate models), rather than as opposite of “tree-level”.
STIM combines both approaches (http://www.for.gov.bc.ca/hre/gymodels/STIM/)
VDYP7 released in 2008 (http://www.for.gov.bc.ca/hts/vdyp).

SDMDs

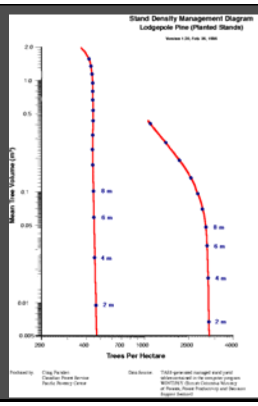
- Craig Farnden, CFS
- From TASS
- State variables: *v* and *N* (logs)
- + independent model for *H*
- Eichhorn
- Others: *V*, *D* or *B* instead of *v*



http://www.for.gov.bc.ca/hre/gymodels/SDMD/index.htm.
Various species.

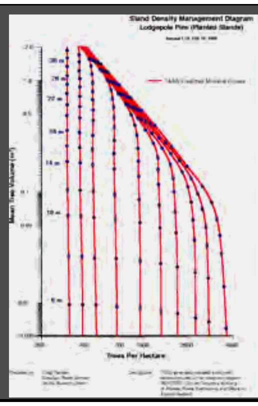
Two stands

- Different initial densities
- Dots: top heights
- State-space trajectories (“phase portraits”)



More stands

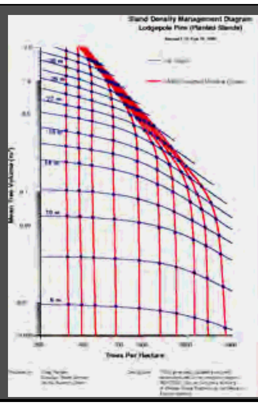
- -3/2 self-thinning rule:
  $\log v \approx a - (3/2) \log N$
- Other, Reineke:
  $\log D \approx a - (1/1.6) \log N$



Logarithmic axes used for historical reasons, related to self-thinning theories.

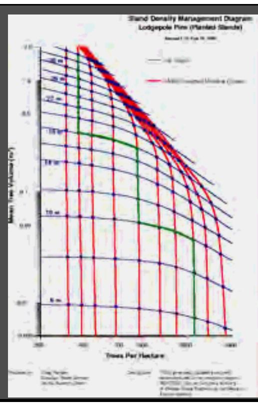
Isolines

- Lines of equal height (iso = equal)
- Unthinned



Thinning

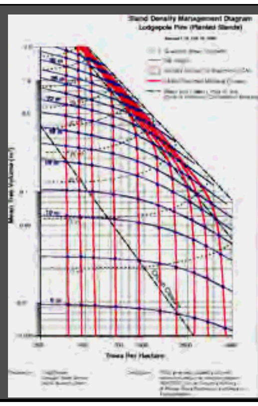
- Assumes that thinning change follows the isoline (why??)
- Otherwise...



No biological justification, logical flaw. Although might be close enough.

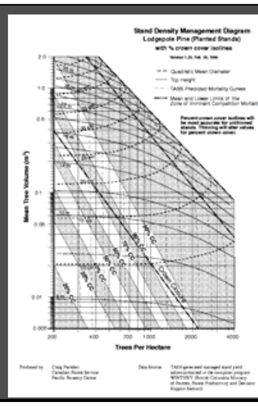
Outputs

- E.g., *D = f(v, N)*
- Level curves
- Other info:
  - Crown closure
  - Limits of "Zone of Imminent Competition Mortality"
  - Best CAI
  - Etc.

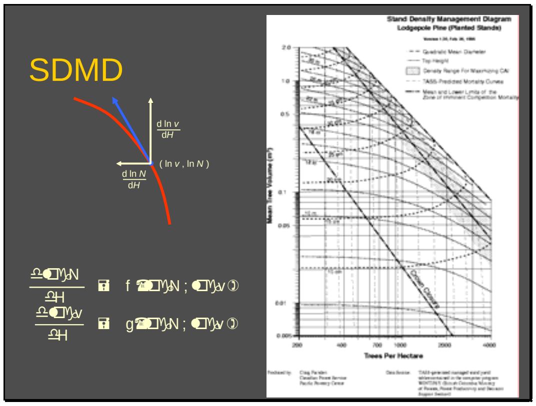


Etc.

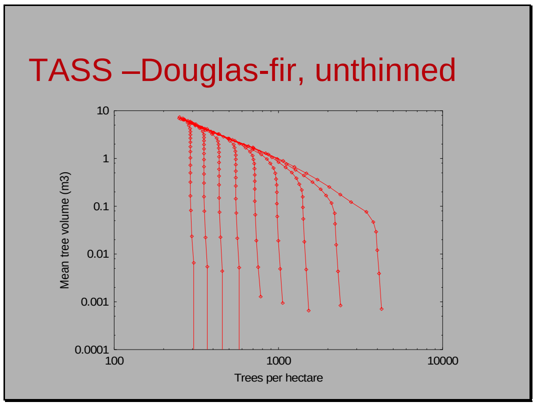
- E.g., % crown cover isolines



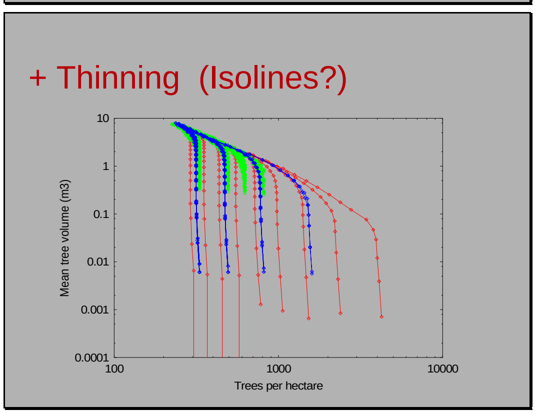
Other outputs (functions of the state). Variants.



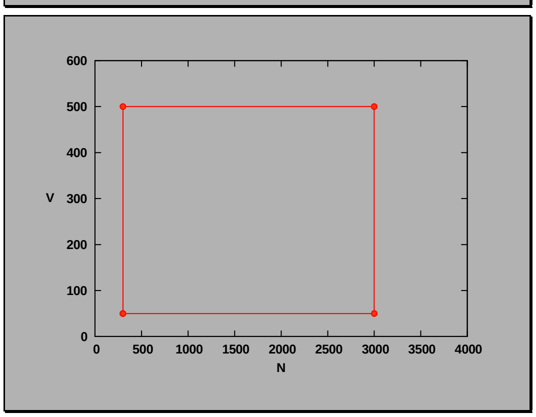
Conceptually, a two-dimensional state space.
Transitions given graphically.
A third variable, *H*, is completely determined by *v* and *N* (through the isolines). Not an independent state variable.



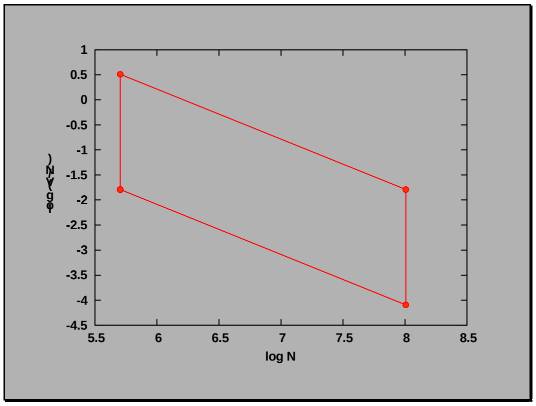
Testing. Simulations from the TIPSY yield table database.



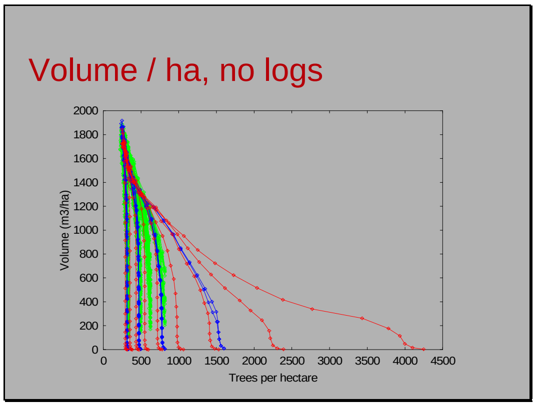
Pre-commercial (blue) and commercial thinning (green).
Thinned simulations somewhat out of phase; thinning did not follow the isolines exactly.
Many superimposed simulations: combinations of initial density, thinning age and intensity.
Most interesting data crowded at top of graph due to logarithmic scale.



Both the use of log scales, and having *N* on both axes (mean volume is *V/N*, where the volume per hectare *V* is probably a more meaningful variable), can cause self-thinning lines to look better than they really are (D.E. Weller, *Ecological Monographs 57*, 23-43, 1987).
Example: Four points, no *V* – *N* relationship (or imagine a cloud of points inside the box) ...

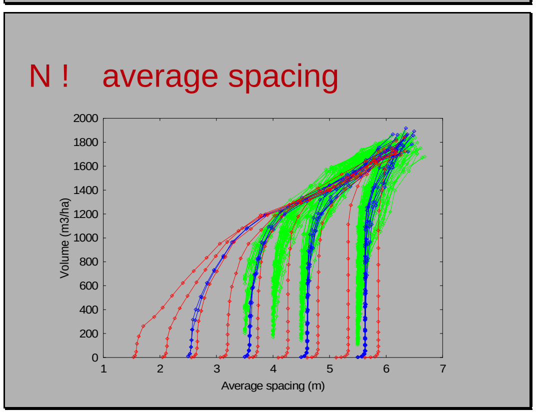


... Apparent trend after the usual transformation!



Removing log transforms. Also volume/ha instead of tree average.

Curves crowded on left, might have been better to keep log *N*.



With average spacing, 100/sqrt(*N*), instead of *N* or log *N*

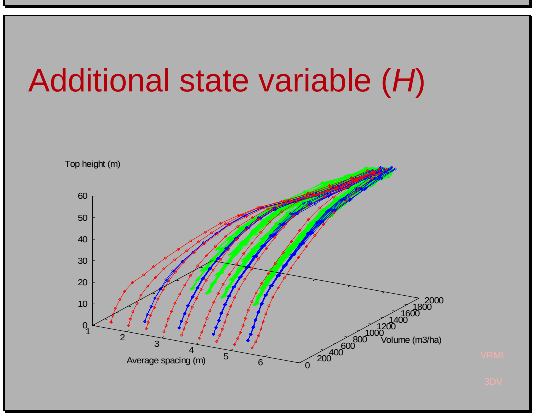
Clearer view of stand development.

Two state variables OK for no thinning, and possibly good enough for pre-commercial thinning.

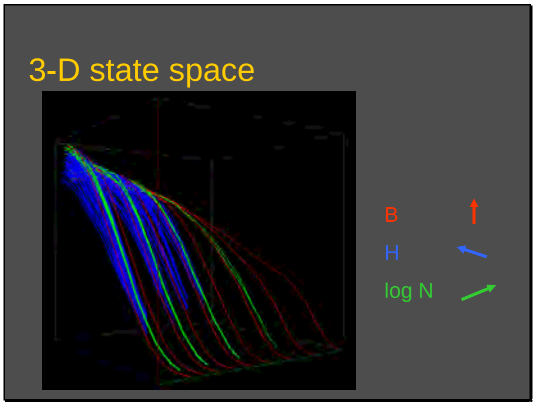
Not sufficient for commercial thinning: stands with equal states evolve differently.

Farnden (1996) warned that the SDMDs might be unreliable for commercial thinning.

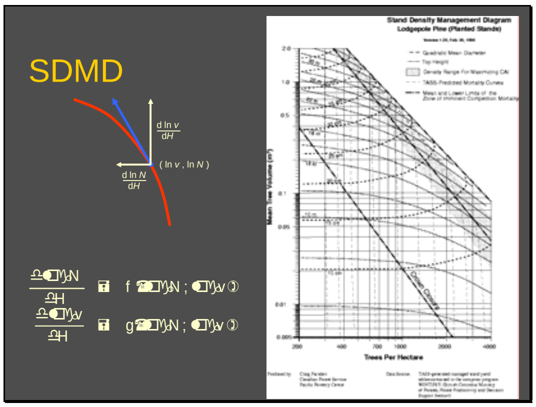
Need a third state variable? Top height?



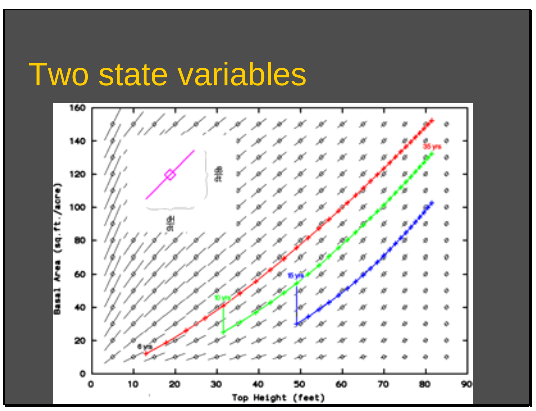
Thinned stands fall off the SDMD surface (seen more clearly on next slide). Third variable might help.



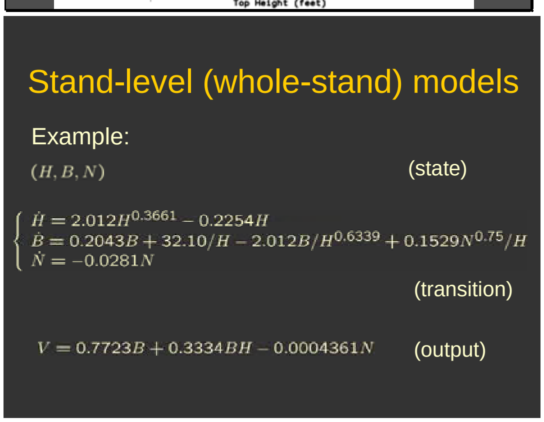
Dynamic 3D graph omitted (see www.unbc.ca/forestry/forestgrowth/tadam).
Different but equivalent set of state variables.
Also, different colour coding: here green is pre-commercial, and blue commercial thinning.



One state variable (*H*) sufficient for looking at top height development. Site index models.
Two state variables OK for unthinned, and perhaps pre-commercially thinned stand growth and mortality (pure, even-aged).
Individual-tree models use from dozens to thousands of state variables.
In all instances the fundamentals are the same: An *n*-dimensional state vector. A rate (transition) function for each of the components. These define a trajectory in the state space. Thinnings cause an instantaneous jump.



Another 2-D example, based on Clutter's 1963 model (*Can.J.For.Res. 24*:1894-1903, 1994).
In principle, the more state variables, the more accurately the real system behaviour (trajectories) could be described. On the other hand, more information about the starting point is needed. If that information is not accurate we can be worse off.



3-D example, already shown. Geometrical interpretation?
Can you visualize what happens with more state variables?

BC stand-level models

- STIM (partially)
- VDYP7
- Stand density management diagrams
  - Approximating TASS in 2-D
- TADAM
  - Approximating TASS in 3-D
- SBS spruce - Scube (Zhengjun Hu)
- Aspen (in progress)

Farnden's SDMDs and TADAM, both derived from TASS. TADAM uses an additional third state variable.

TADAM-s

$$\frac{\mathrm{d}\ln N}{\mathrm{d}H} = -a_1 N \mathrm{e}^{-a_2 H} - a_3 N^{a_4} \mathrm{e}^{a_5 H^{a_6}}$$

$$\frac{\mathrm{d}BH}{\mathrm{d}H} = a_7 H^{a_8} N^{a_9} \frac{1 - \exp(-b_{10} H^{a_{11}} N^{a_{12}})}{1 + a_{13} H^{a_{14}} N^{a_{15}}} + a_{16} BH^{a_{17}} \frac{\mathrm{d}\ln N}{\mathrm{d}H}$$

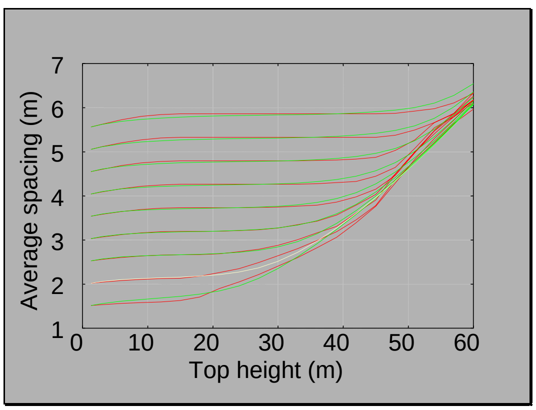
Basic rate equations.

BH is a proxy for volume per hectare. Avoids dependence on particular volume equations, utilization standards, etc.

Unlike in the SDMD, the rates vary with all three variables.

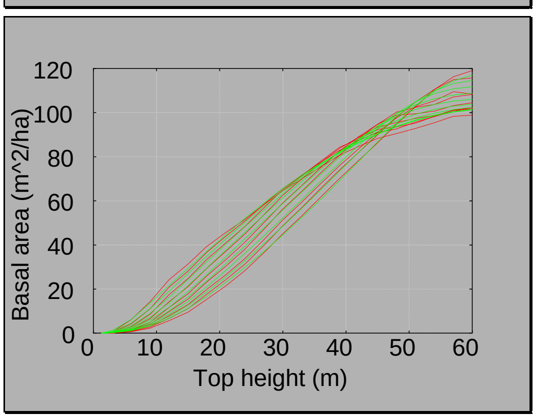
Rates relative to *H* are just for conveniently dealing with site quality, following Eichhorn. Time rates can be obtained from (d ln N / dH) (dH / dt) = d ln N / dt, where dH/dt comes from the site index model.

BH increment modelled as a gross increment minus mortality.

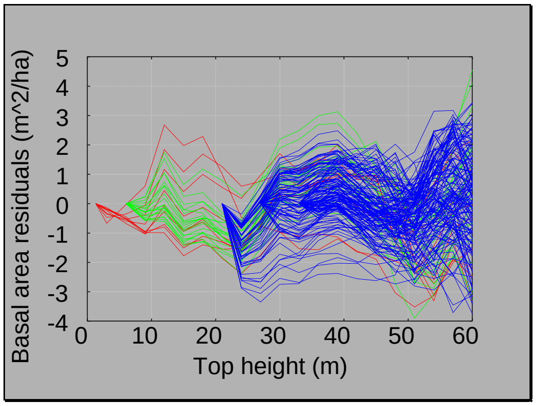


Red: unthinned TASS simulations in TIPSY. Green: TADAM-df.

Note irregularities due to stochastic simulation (TADAM is deterministic).

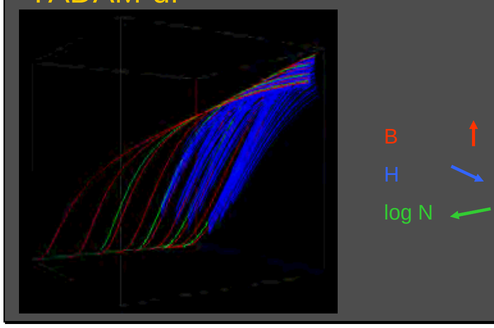


Note irregularities due to stochastic simulation.



Basal area differences in previous slide. Mostly between +/- 2 sq.m/ha.
Much of the variation probably due to the random numbers. Both TADAM and the TIPSY yield table differ from what would be an average TASS prediction.
Use of a common random seed caused similar random deviation patterns for all the simulations in the database.

TADAM-df



The TADAM predictions. 3-D graph in http://forestgrowth.unbc.ca/tadam/vrml.htm.

Ancillary relationships

- Thinning: basal area ↔ trees per hectare
- Stand total volume equation (output)
  V = f (B, N, H)
- Merchantable volume, limit diameter *d*
  Vm / V = g (d, B, N, H)
- Site index – height – age calculations
- Distributions?

Size distributions could be estimated, for instance, through a regression of tree dbh variance as a function of the state variables. This, together with the known mean, can determine a Weibull or some other distribution function. Thus, detail close to that in individual-tree models could be generated, if necessary.

TADAM

- Coastal Douglas-fir, Interior lodgepole pine, Interior white spruce
- Implementation: Palm, Excel add-in, C functions, dll

http://forestgrowth.unbc.ca/tadam

Scube

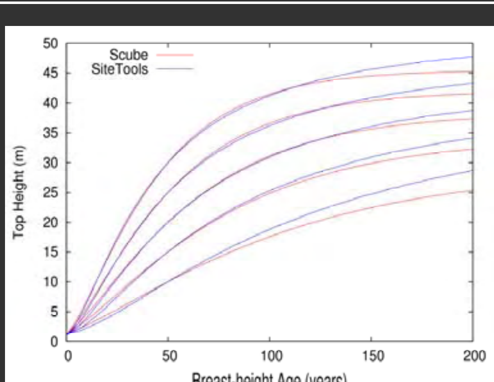
- Stand growth model for spruce in the SBS
- PSP data from natural and planted stands
- Plus stem analysis for heights
- Zengjun Hu's MSc
- New mortality and basal area submodels
- Excel implementation in *forestgrowth.unbc.ca/scube*

3 components, 4 state variables

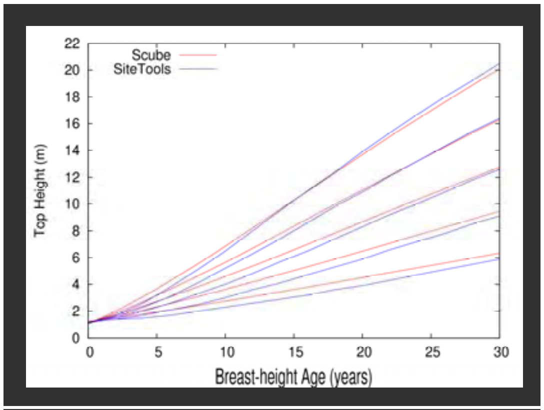
- Top height, site quality
  $dH/dt = f_1(H)$ (for a given site quality)
- Mortality
  $dN/dt = - f_2(H, N)$
- Basal area and "occupancy"
  $dBH/dt = \Omega f_3(H, N) + k \ (BH / N) \ dN/dt$
  $d\Omega/dt = f_4(\Omega)$
- Thinning, outputs (volumes, carbon, etc.)
  $V = - g(H, N, B)$

Data (MoF)

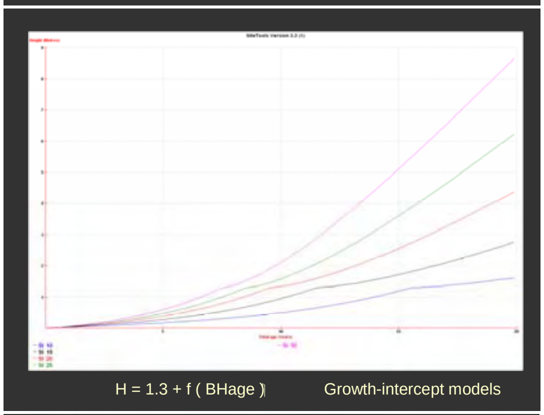
- Permanent sample plots, natural
  - Spruce-dominated (> 70% basal area)
  - Reasonably even-aged
  - Age > 25 years
- Planted, experiment EP660
  - 3 locations
  - Age < 25 years
- Stem analysis (for height)



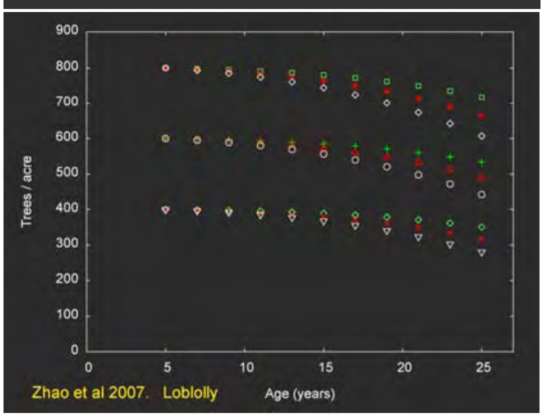
Height growth / site-index sub-model, and currently recommended model.
Similar for mature stands within the range of available data.
(data shown in class not cleared by MoF for wider use; exercise your memory!)



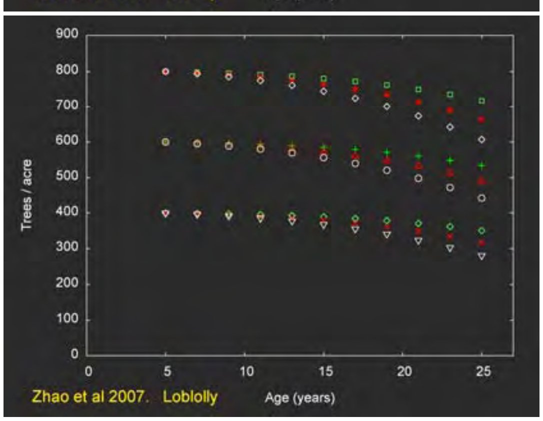
Substantial differences for young stands, however.



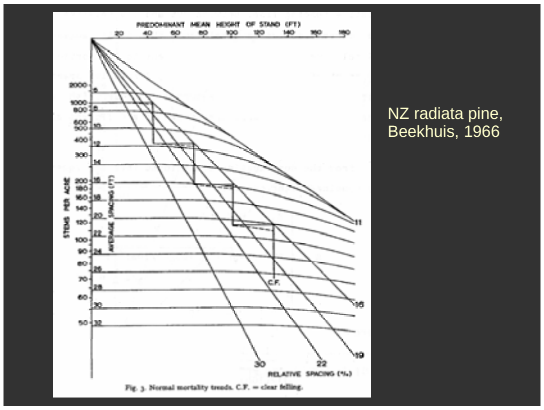
SiteTools model predicts zero growth at breast height, due to a flaw in a commonly used modelling approach. For details see around Fig.2 in Salas & Garcia (2006), Forest Ecology and Management 229: 1-6.



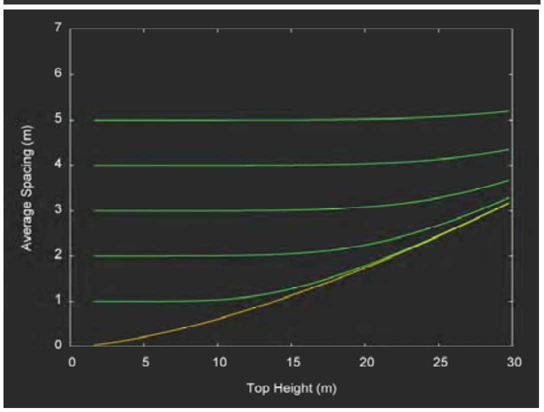
The mortality sub-model uses height instead of age to reduce the effect of site (Eichhorn).
Demonstrated here, with a model for loblolly pine. Trees/acre predictions for three site indices, over age.



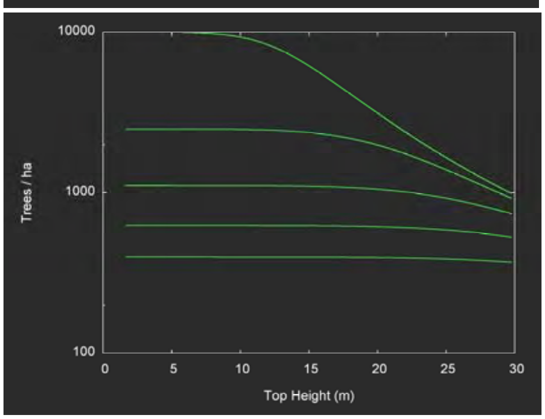
Same, plotted over top height.



Mortality model is a generalization of a relative spacing approach.



Curved relative spacing limit (data omitted).



Basal area, and comparison with other models, in Assignment 3.

Different basal area parameters for planted and natural stands.