Individual-tree

- Distance-dependent

  Tree growth depends of its size, and size and position of others:

  $$\Delta z_i = f(z_1, p_1, z_2, p_2, \ldots)$$

  $$\Delta z_i = f(z_i, c_i) \quad c_i = g(z_1, p_1, z_2, p_2, \ldots)$$

- Distance-independent

  Tree growth depends of its size, and size of others:

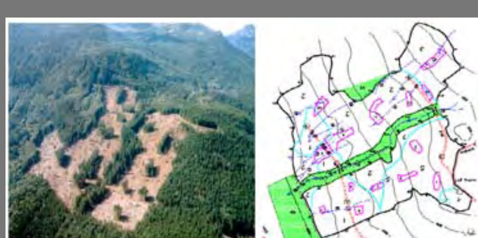
  $$\Delta z_i = f(z_1, z_2, \ldots)$$

  $$\Delta z_i = f(z_i, c_i) \quad c_i = g(z_1, z_2, \ldots)$$

z: tree “size”, possibly a vector. E.g., dbh, or dbh and height.
p: tree position (x-y coordinates)
c: competition index

Not a clearcut ;-)
Distance-independent requires less state information. But can only model well stands that are spatially homogeneous.

Spatial heterogeneity



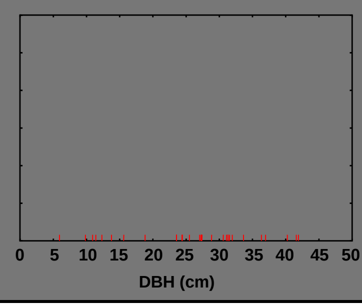
Tree locations are essential for some purposes.
Figure from presentation by Jim Goudie on variable retention simulation with TASS, at http://westernforestry.org/wmens/m2003/m2003_agenda.htm.

Tree lists

- List: ***d*** = (37.0, 18.8, 27.3, 25.5, 31.3 ,32 .0, 41.6, 33.6, 9.8, 41.9, 23.6, 11.4, 28.8, 27.4, 31.5, 12.3, 27.0, 23.5, 24.4, 15.6, 30.6, 31.1, 5.9, 31.3, 10.9, 27.2, 24.4, 36.3, 40.2, 13.8)
- Expansion factor (weights): ***w*** = (20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 20)

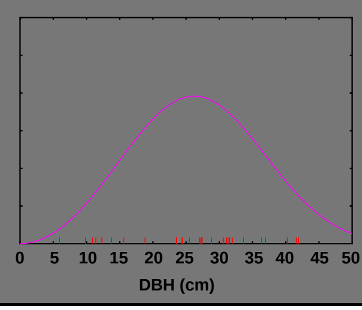
d and *w* define the state of the stand.

Lists and Distributions



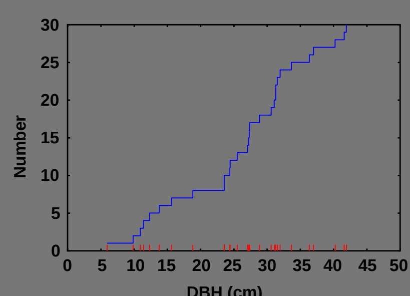
Tree list is essentially equivalent to a discrete distribution. Weights (bar heights) could vary.

Lists and Distributions



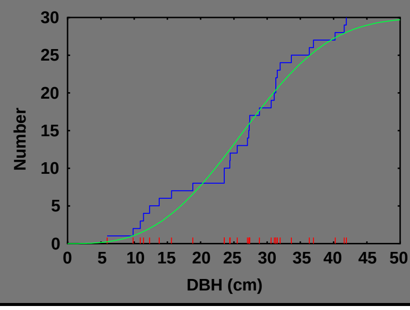
Example 40 trees were selected at random from this distribution density.

Lists and Distributions



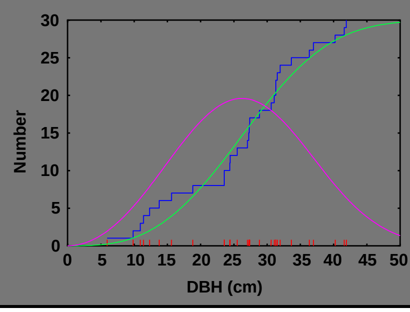
The cumulative sample distribution (aka *empirical distribution function*).

Lists and Distributions



Compared to the “true” population distribution. Variability in the observed distribution shape can be large.

Lists and Distributions



Distance-independent (aspatial)

- Size: usually dbh (or tree basal area)
  - Tree list: *(d_1, d_2, d_3, ...)*
  - Expansion factors: *(w_1, w_2, w_3, ...)*
- Density / competition: *N, B, CCF, BAL, etc.*
- Growth: $\Delta d_i = f(d_i, N, B, \ldots)$
- Mortality: Prob{*i* dies} = $g(d_i, N, B, \ldots)$
  ! Δw_i
- Regeneration / ingrowth

State: $\{d_i, w_i\}$

N, *B*, etc., summarize effect of the other trees. Similar to tree competition indices in distance-dependent.

Density measures / indices

- Simple: *N, B*
- Stand density indices:
  - Relative spacing *s / H* / *1 / H* $N^{1/2}$
  - Reineke's density index / $ND^{1.605}$
  - Crown competition factor (CCF)
- Tree competition indices:
  - Rank
  - Basal area of largest trees (BAL)

Density indices attempt to summarize in a single number the degree of crowding, independently of age, etc.

One could use a pair or triple of numbers instead.

CCF: Crown Competition Factor, ratio of estimated open-grown tree crown projection area to the average space per tree in the stand (stand level). There are also individual-tree CCFs, i.e., values calculated for each tree (tree competition index).

BAL: Basal area of all trees larger than the given tree.

BC distance-independent models

- Prognosis^{BC}
  - Southern Interior, “complex” stands
  - USFS FVS (ex Prognosis, Stage 1973)
  - Northern Idaho variant, adaptation underway
- MGM (Mixewoods Growth Model)
  - Steve Titus, U. of Alberta
  - Aspen + white/black spruce, aspen + lodgepole/jack pine
  - Alberta, BC, Saskatchewan, Manitoba
- STIM

http://www.for.gov.bc.ca/hre/gymodels

FVS / Prognosis^{BC}

- Initial conditions
  - Site quality: habitat, slope, aspect, elevation
  - Tree list: *d, w*, species. Optional *h*, Δd, Δh
  - Crown ratio?
  - Estimate missing *h*’s
    $h = 1.3 + \exp[\, \beta_0 + \beta_1 / (d+1)\,]$
  - “Calibrate” if Δd or Δh given

So-called “height dubbing” estimates *h* from a fixed *h-d* relationship. Ignores effects of stand density on dbh for a given height.

FVS / Prognosis^{BC}

- Growth. Small ($d < 3$”), large *trees* (*d* ¸ 3”)
  - Tree basal area
    $\ln \Delta b = \beta_0 + \beta_1 D + \beta_2 D^2 + \beta_3 CCF + \beta_4 BAL + \beta_5 c + \beta_6 c^2$ + random (large trees)
    c = crown ratio = $f(d, h, rank, B, CCF)$
  - Height
    $\ln \Delta h = \beta_7 + \beta_8 \ln d + \beta_9 \ln h + \beta_{10} h^2 + \beta_{11} \ln \Delta b$ (large)
- Mortality
  Probability of dying is a (complicated) function of the state variables. Changes *w*.

Convention: lower-case variables are tree-level, upper case stand-level (usually).
Different relationships for “large” and “small” trees.
Coefficients vary with species and site.
The model crown ratio relationships are adjusted (“calibrated”) if initial values are given. But then at each step the ratio is estimated from the other variables, rather than being projected (output, not state).

FVS / Prognosis^{BC}

- Regeneration / ingrowth
  Not available in Prognosis^{BC}
- Thinning / partial cutting
- Summaries, outputs (volume, etc.)

Discrete (10-yr steps)
Stochastic (only in Δb)
User interface
- FVS: Command language
- Prognosis^{BC}: GUI

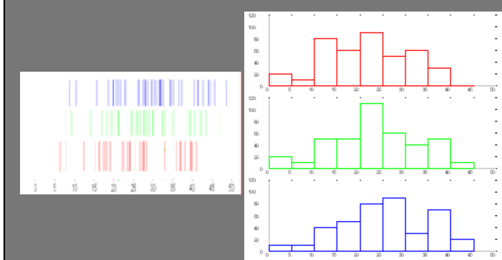
Stochastic (in *b*-increment), discrete time (10-year steps).

MGM

- Boreal aspen
- Deterministic. DBH increment = f(DBH)
- Implemented as Excel macros

STIM can be used as distance-independent or as whole-stand model.

Size-class models



Trees grouped in classes (histograms). In mixed stands, by species or species groups.

Tree lists change through displacement of the individual trees, and possibly changes in the weights. In size-class models, some proportion of trees moves from one class to the next.

Intermediate level of detail between individual-tree and whole-stand.

Stochastic *vs.* Deterministic

- Random (or pseudo-random) numbers
  - Start with *random seed*
  - Next number: e.g. multiply by a constant and ignore leading (binary) digits
- Random? Probability? Exist in “reality”?
- Model to represent ignorance
- Unpredictable, given what *we* know

Stochastic models use random numbers.

Stochastic = random = probabilistic.

Probability best seen as a way of modelling a state of knowledge or ignorance.

Random numbers are perfectly predictable if we know the generator formula, unpredictable otherwise.

Stochastic *vs.* Deterministic

- Appearance of “realism”
- Assessing variability, uncertainty
  - Repeated simulations
  - Uncommon in practice
  - Sometimes the random seed is *hardwired*
- Handling issues of averaging / aggregation

Realism not necessarily a good thing.

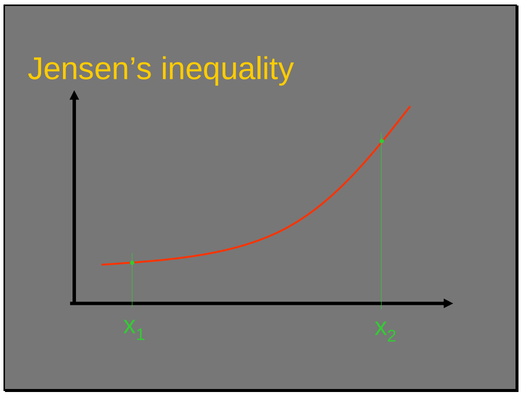
Managers often only need or can use a single “most likely” value.

Stochastic elements can be an easy way of properly approximating averages of nonlinear functions.

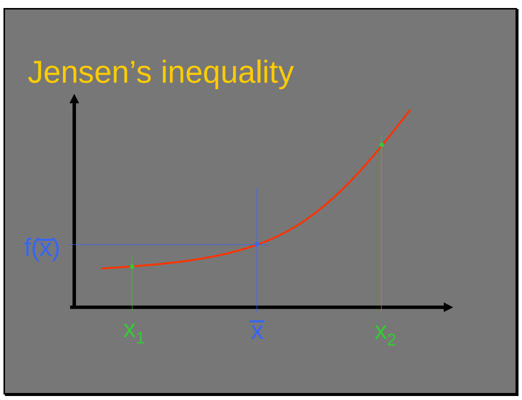
Example: mean tree basal areas

- Method 1: d → k d^2 → Mean(k d^2)
  (4, 8, 10) →
- Method 2: d → Mean(d) → $[\text{Mean}(d)]^2$

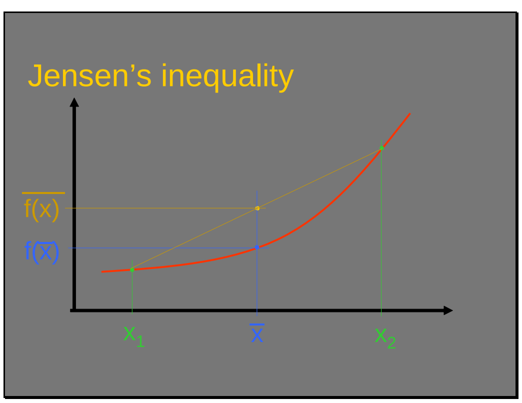
mean(f(x)) ≠ f(mean(x))



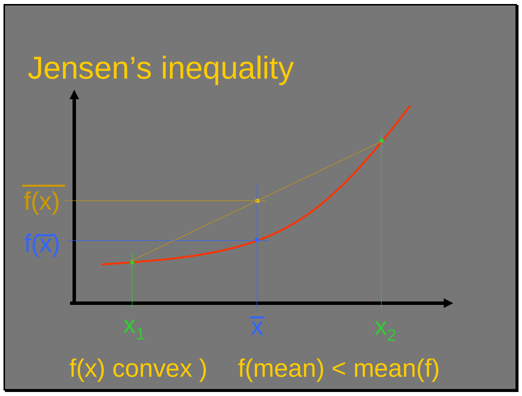
Averaging is common in many modelling applications. Jensen's inequality deals with its effects.



Function of an average.



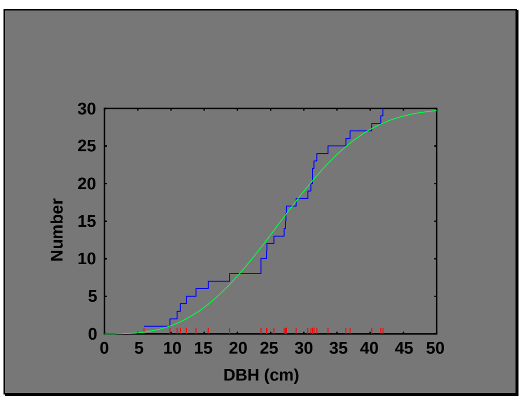
Average of a function.



Introducing stochastic components is a simple way of coping with consequences of Jensen's inequality. There are others.

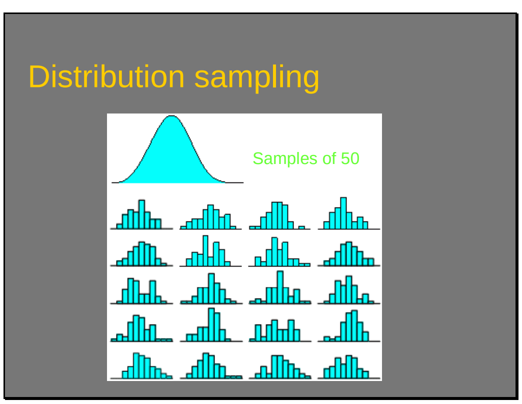
Advantages of individual-based

- "Easy" to understand
  - Low level of abstraction
- "Simple"
  - Substitute aritmetic for higher math
  - Less aggregation issues
- Good research tools
  - Synthesise knowledge, formulate hypotheses
  - "The sky is the limit"



A limitation is the often imprecise knowledge of the highly detailed starting state (initial conditions).

E.g., in distance-independent models, tree lists or size distributions have a high sampling error.



Simulated 20 samples of 50 trees each. Means OK, but shape is uncertain.

Other problems and issues with individual-based models in http://forestgrowth.unbc.ca/warsaw.pdf