$$
\begin{array}{lcl}
\mathit{stat} & \rightarrow & \texttt{if}\ \mathit{bool\_expr}\ \texttt{then}\ \mathit{stat}\ \texttt{else}\ \mathit{stat} \\
 & | & \mathit{stat}\ \texttt{;}\ \mathit{stat} \\
 & | & \mathit{variable}\ \texttt{:=}\ \mathit{expr} \\
\mathit{variable} & \rightarrow & \texttt{x} \mid \texttt{y} \mid \texttt{z} \mid \texttt{w} \mid \texttt{u} \mid \texttt{v} \\
\mathit{expr} & \rightarrow & \mathit{expr}\ \texttt{+}\ \mathit{expr} \\
 & | & \mathit{expr}\ \texttt{-}\ \mathit{expr} \\
 & | & \mathit{expr}\ \texttt{*}\ \mathit{expr} \\
 & | & \mathit{variable} \\
 & | & \texttt{0} \mid \texttt{1} \\
\mathit{relation} & \rightarrow & \texttt{<} \mid \texttt{=} \mid \texttt{>} \\
\mathit{bool\_expr} & \rightarrow & \mathit{expr}\ \mathit{relation}\ \mathit{expr}
\end{array}
$$

Figure 1: Abstract grammar for Semantics Worksheet

Homework Assignment #4

For the sake of concreteness in this assignment assume that we have the abstract grammar shown in Figure 1. Suppose also that the set of locations is given by $\mathcal{L} = \{\texttt{x}, \texttt{y}, \texttt{z}, \texttt{w}, \texttt{u}, \texttt{v}\}$, that the set of values that expressions can take is $\mathbb{Z}$. Let S_0 be the memory state $\mathcal{L}\times\{0\} = \{(\texttt{x}, 0), (\texttt{y}, 0), (\texttt{z}, 0), (\texttt{w}, 0), (\texttt{u}, 0), (\texttt{v}, 0)\}$.

1. Suppose that S_1 is the state $\{(\texttt{x}, 2), (\texttt{w}, 3), (\texttt{z}, 1), (\texttt{y}, 4), (\texttt{u}, 0), (\texttt{v}, 0)\}$, and that p is the program

   ```
   x := y+1+1 ; y := 1+1+1+1+1 .
   ```

   What is $\mathcal{C}[\![p]\!](S_0)$? What is $\mathcal{C}[\![p]\!](S_1)$?

2. Discuss the claim that for any statement p in our language, we can write

   $$\begin{aligned}\mathcal{C}[\![p]\!](s) = s[\texttt{x} \mapsto \phi_{\texttt{x}}(s)][\texttt{y} \mapsto \phi_{\texttt{y}}(s)][\texttt{z} \mapsto \phi_{\texttt{z}}(s)]\\ [\texttt{w} \mapsto \phi_{\texttt{w}}(s)][\texttt{u} \mapsto \phi_{\texttt{u}}(s)][\texttt{v} \mapsto \phi_{\texttt{v}}(s)]\end{aligned}$$

   for suitable functions $\phi_{\texttt{x}}$, $\phi_{\texttt{y}}$, $\phi_{\texttt{z}}$, , $\phi_{\texttt{w}}$, $\phi_{\texttt{u}}$, and $\phi_{\texttt{v}}$.

3. Suppose that we extend the abstract syntax of statements so that we can write empty statements, for instance, `;;;x:=1+1;;`. What should the meaning of the empty statement be?

4. Completely write out the formal denotational semantics for the programming language shown in Figure 1 using functions $\mathcal{C}$, $\mathcal{B}$, and $\mathcal{E}$, where the domains and co-domains are specified as follows:

function	domain	co-domain
$\mathcal{C}$	*stat*	$\mathcal{S} \mapsto \mathcal{S}$
$\mathcal{E}$	*expr*	$\mathcal{S} \mapsto \mathbb{Z}$
$\mathcal{B}$	*bool_expr*	$\mathcal{S} \mapsto \{\mathsf{T}, \mathsf{F}\}$

This assignment is due in class at the beginning of class 2007-10-23.