UNBC Properties of Logarithms CPSC 200

Definitions

$$\log_a x = y$$
$$\text{means} \quad a^y = x$$

More formally,

$$\log_e x = \int_1^x \frac{\mathrm{d}t}{t},$$
$$\log_a x = \log_e x / \log_e a$$

Arithmetic Properties

$$\log a^s b^t = s \log a + t \log b,$$

in particular

$$\log ab = \log a + \log b,$$
$$\log a/b = \log a - \log b,$$
$$\log a^s = s \log a,$$
$$\log_a a = 1$$
$$\log_a 1 = 0.$$

Base Changes

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Calculus

$$\log_e(1+x) \approx x \quad \text{for } x \ll 1.$$

$$\lim_{x\to+\infty} \log x = +\infty$$

$$\lim_{x\to+\infty} \frac{\log x}{x^\varepsilon} = 0, \text{for } \varepsilon > 0 \text{ fixed.}$$

$$\left.\frac{\mathrm{d}(\log_e x)}{\mathrm{d}x}\right|_{x=1} = 1$$

Practical

$$0.6931472\ldots = \log_e 2$$
$$0.3010300\ldots = \log_{10} 2 \quad \text{so}$$
$$\log_2 1000 \approx 10$$
$$\log_2 1\,000\,000 \approx 20$$
$$\log_2 1\,000\,000\,000 \approx 30$$

Logarithms and Factorials

$$\log_e n! = n \log_e n + \log_e \sqrt{2\pi n} - n + \left(\frac{1}{12\,n} - \frac{1}{360\,n^3} + \frac{1}{1260\,n^5} + \ldots \right)$$

$$\text{or} \quad \lim_{n\to+\infty} \frac{\log_e n!}{n \log_e n} = 1$$

Convexity For $0 < f < 1$, $0 < a < b$,

$$\log[af + b(1-f)] > f \log a + (1-f) \log b$$