Two other sorting algorithms that have running time characteristics something like Insertion Sort, are Selection Sort and Bubble. Both are slightly easier to spacify with a helper routine `swap`, given next.

```
3   public static void swap(long [] data, int i, int j)
4       {
5       final long temp = data[i] ;
6       data[i] = data[j] ; data[j] = temp ;
7       }
```

The following version of Selection Sort can be improved slightly. For instance, the condition `i<n` can be changed to `i<n-1`, and the `swap` can be made conditional on `i!=k`. It is nonetheless correct.

Selection Sort

```
30  public static void
31  selectionSort(long [] data)
32      {
33      final int n = data.length ;
34      for (int i=0;i<n;++i)
35          {
36          int k=i ;
37          for (int j=k+1;j<n;++j)
38              {
39              if (data[j]<data[k])
40                  k=j ;
41              }
42          swap(data,i,k) ;
43          }
44      }
```

1. What loop invariant holds at line 35?
2. What loop invariant holds at line 38?
3. Does the running time depend on `data`?

For reasons not entirely clear to me, bubble sort seems to be the best known and most taught sorting algorithms. Simpler variants of bubble sort drop the `done` variable, or remove the `runNumber` variable.

Bubble Sort

```
10  public static void bubbleSort(long [] data)
11      {
12      final int n = data.length ;
13      boolean done = false ;
14      for (int runNumber=1;runNumber<n && !done;++runNumber)
15          {
16          done = true ;
17          for (int i=0;i<n-runNumber;++i)
18              {
19              if (data[i]>data[i+1])
20                  {
21                  swap(data,i,i+1) ;
22                  done = false ;
23      }   }   }   }
```

1. What loop invariant holds at line 15?
2. What loop invariant holds at line 18?
3. What is the running time like when the initial `data` is already sorted?
4. What is the running time like when the initial `data` is in reverse order?
5. Find a circumstance where the number of *inversions* is $O(n)$ but the running time is $\Omega(n^2)$. reverse order?