Heap Sort Notes

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2007-11-12 / CPSC 200 Lecture

Outline

The Notion of a Heap

A heap is a binary tree

- where the left and right subtrees are heaps, and
- the root of the tree is larger than everything below it.

The Notion of a Heap
Array Based Heaps

- often implement heaps using trees and pointers
- can use arrays when the shape doesn't change often
- for 0-indexed arrays:
  - we store the left subnode of node i in node $2i + 1$
  - we store the right subnode of node i in node $2i + 2$

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- for 0-indexed arrays:
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  - we store the right subnode of node i in node $2i + 2$
  - the parent of node i is in node $(i - 1)/2$.

Building a Heap

How to form a heap

1
6
9
13
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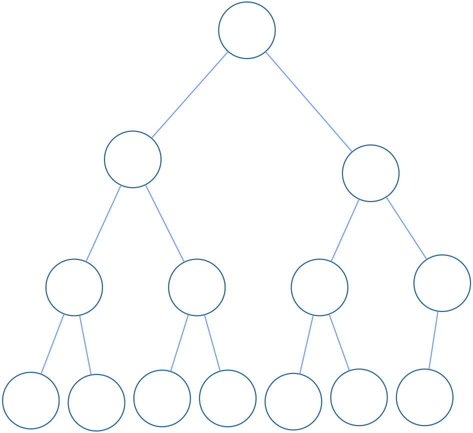

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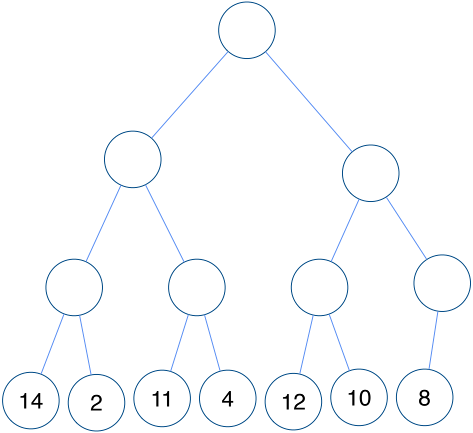

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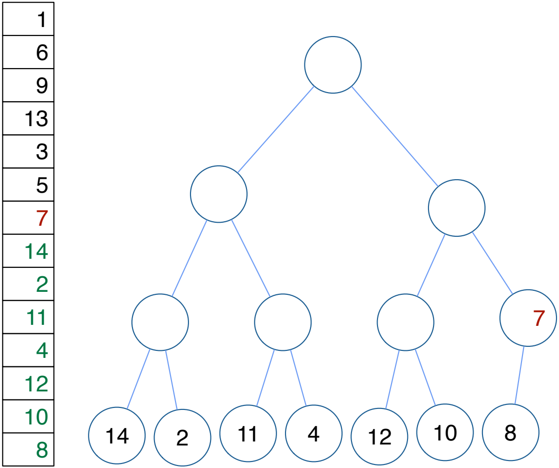

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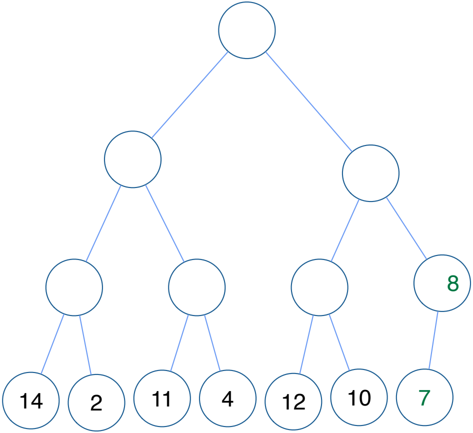

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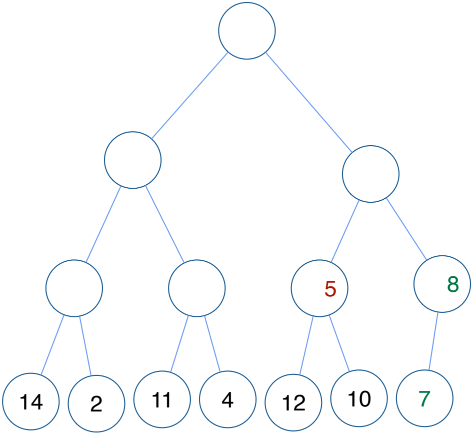

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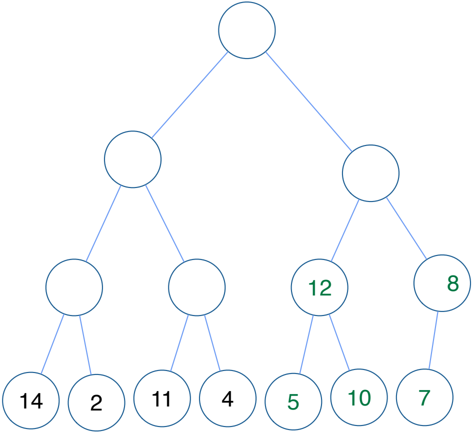

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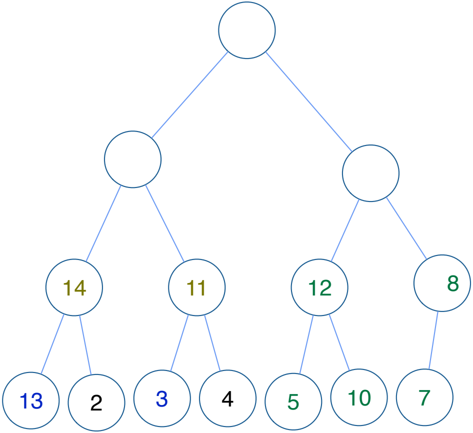

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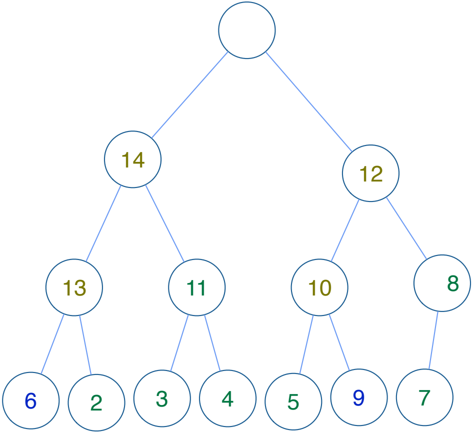

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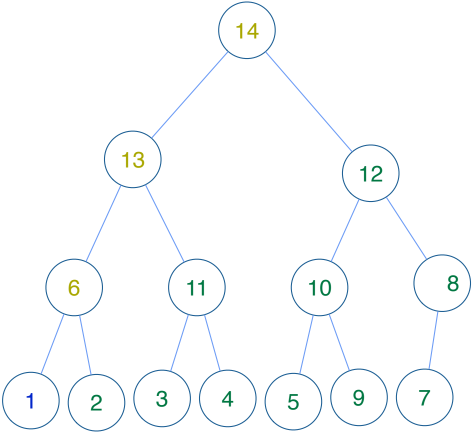

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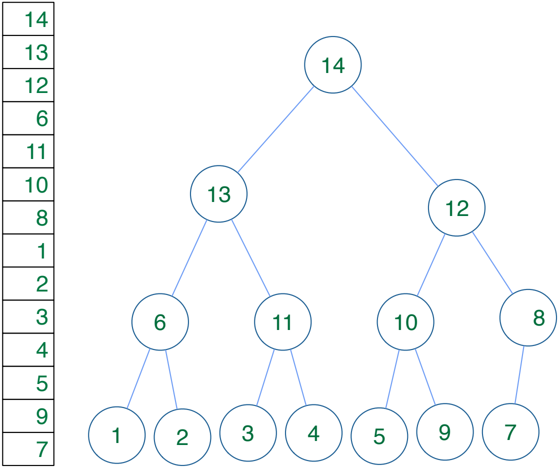


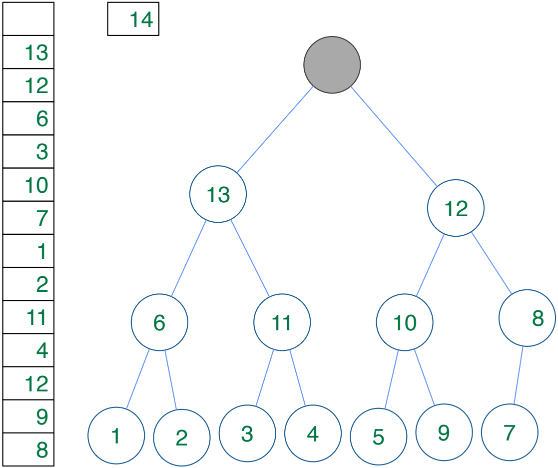
Building a Heap

How to form a heap

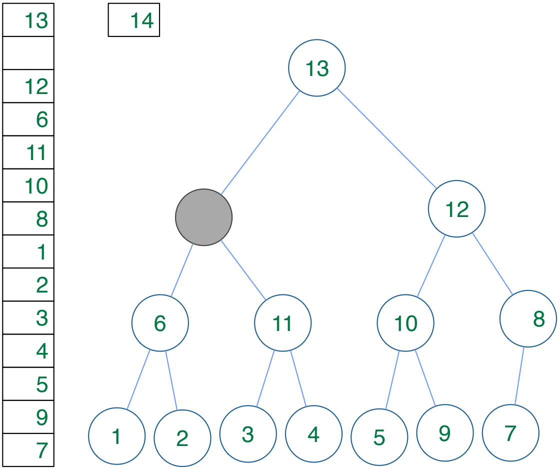
- Building a heap from the bottom up costs $\Theta(n)$-time.
- Building a heap from the bottom up costs $\Theta(1)$-space.
- Building a heap is not stable.

Unbuilding the Heap

Making a hole



Unbuilding the Heap
Moving the Hole Down



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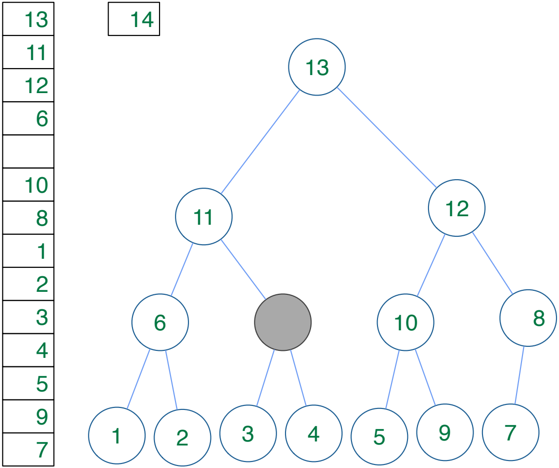

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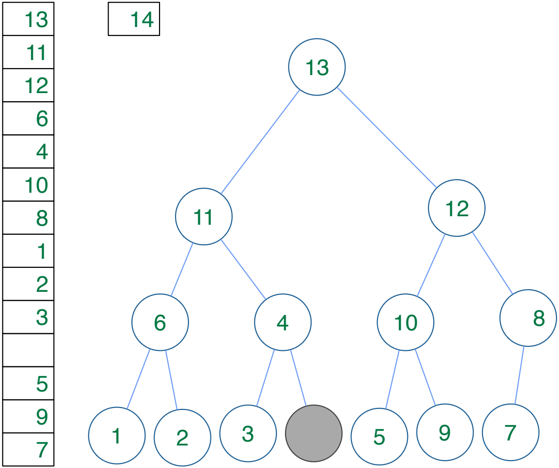
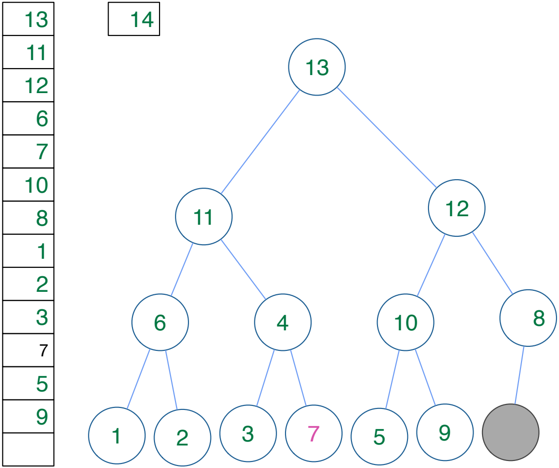


Unbuilding the Heap

Moving an Element Up



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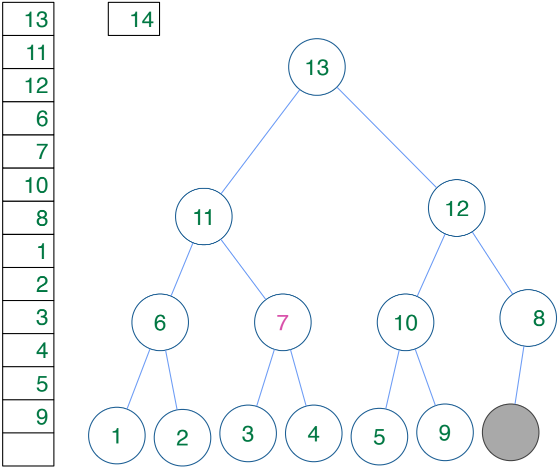

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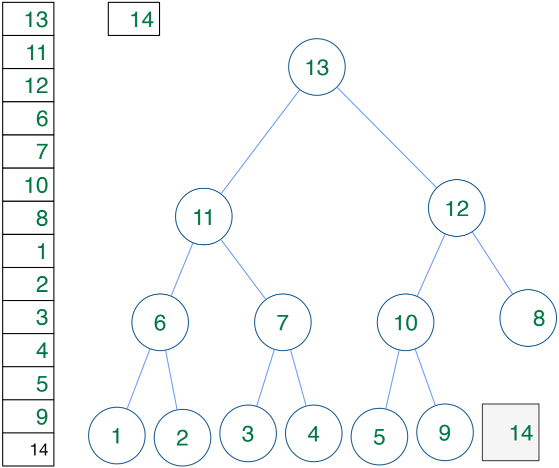
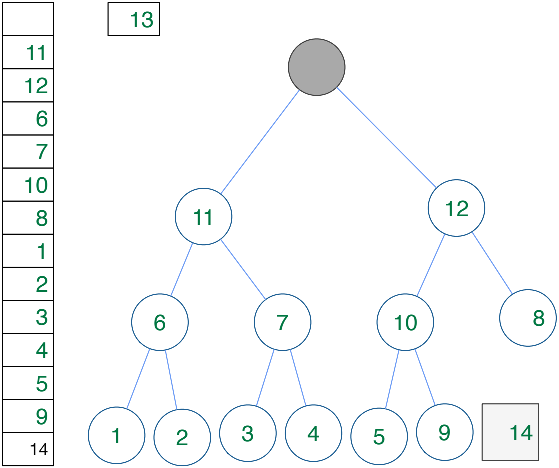


Unbuilding the Heap

The second element



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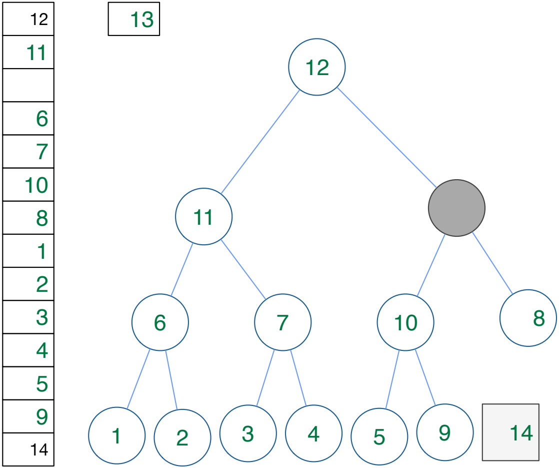

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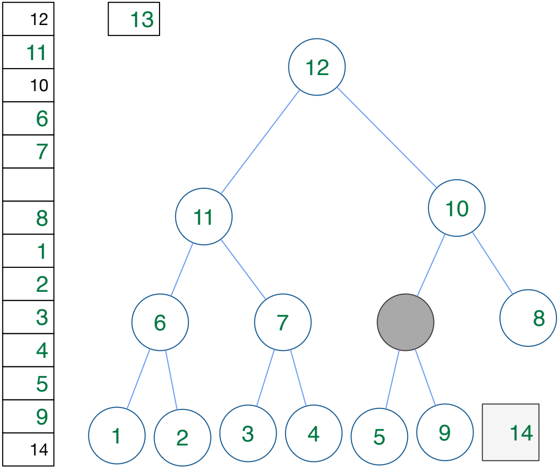

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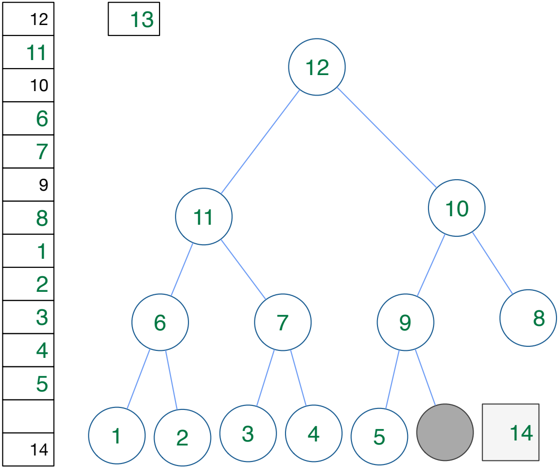

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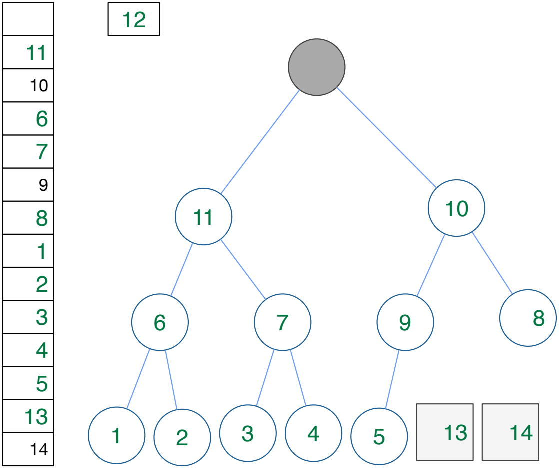

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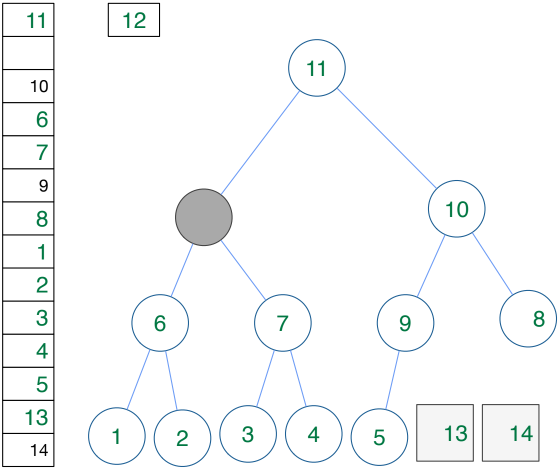

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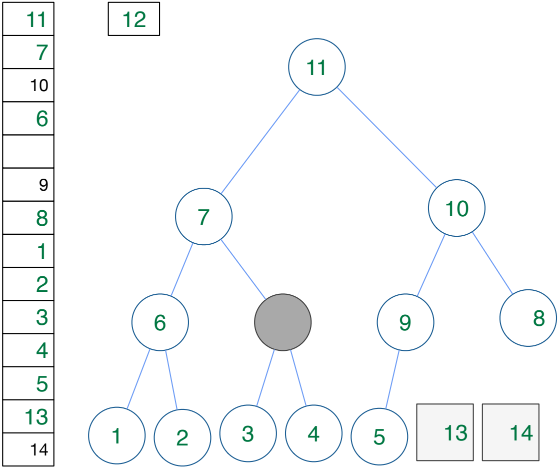

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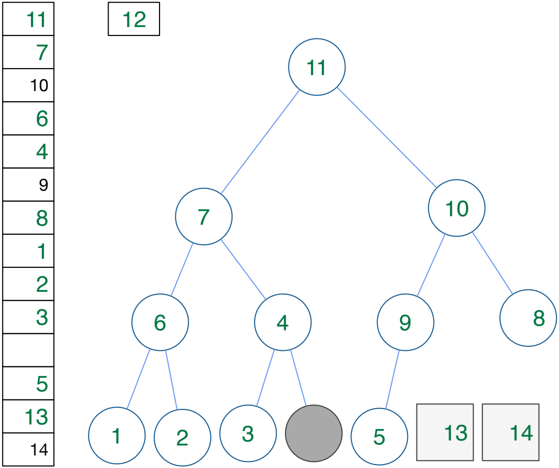

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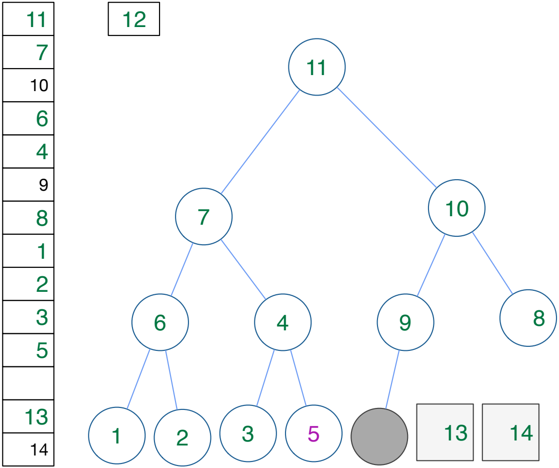

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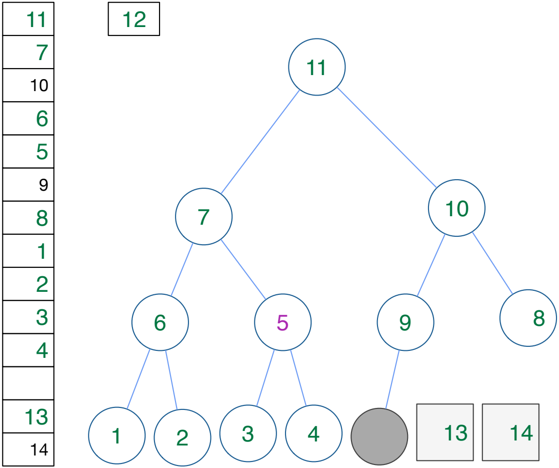

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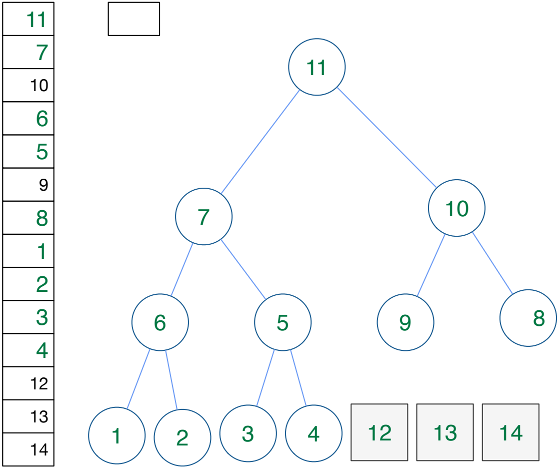

- It is faster to move a hole down, then an element up.
- Moving a hole down costs $\Theta(\log n)$-time.
- Moving a element up costs worst-case $\Theta(\log n)$-time.
- Moving a element up costs average-case $\Theta(1)$-time?
- Unbuilding the whole heap costs $\Theta(\log(n!))$-time.
- Unbuilding the whole heap costs $\Theta(1)$-space.
- Unbuilding the heap is not stable.

Heap Sort:

- uses a *heap*
- represents a binary tree in an array
- has worst-case running time of $\Theta(n \log n)$
- has constant extra space usage.

Quick Sort Summary

Report Card

- $T_{worst}(n) = \Theta(n \log n)$

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- not stable.

The End