Review questions

1. Compute the following:

   (a) $\lceil -13/5 \rceil$

   (b) $-\lceil -\lfloor -\pi \rfloor \rceil$

   (c) $\displaystyle\sum_{i=1}^{3}\left(\sum_{j=1}^{3} i\right)$

   (d) $\displaystyle\left(\sum_{i=1}^{13} i\right) - \left(\sum_{i=1}^{12} i\right)$

   (e) $\displaystyle\left\lfloor \sum_{i=1}^{8} \pi \right\rfloor - \sum_{i=1}^{8} \lfloor \pi \rfloor$

   (f) Compute the greatest common divisor of

   $$866313186631319096185 \quad \text{and}$$
   $$866313186631310433054$$

   Show your work. (With a little patience it is possible to do this with pencil and paper alone.)

   (g) Compute the greatest common divisor of

   $$2^{11}3^{400}5^{19} \qquad \text{and} \qquad 2^{400}3^{19}5^{11}$$

   Show your work.

   (h) Compute the least common multiple of

   $$2^{11}3^{400}5^{19} \qquad \text{and} \qquad 2^{400}3^{19}5^{11}$$

   Show your work.

2. Create a truth table that demonstrates the validity of the "Law of the Syllogism" inference rule.

3. How many functions are there from $\emptyset$ to $\{ a \}$?

4. How many functions are there from $\{ a \}$ to $\emptyset$?

5. How many relations are there between $\{ a \}$ and $\emptyset$?

6. Explain why $\{ \emptyset \}$ cannot be a binary relation.

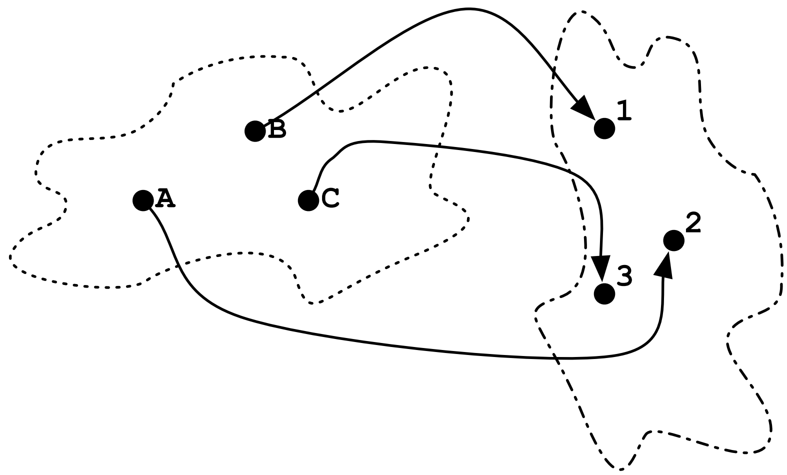

Figure 1: Puddle Diagram #1

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7. Explain why $\emptyset$ is a binary relation.

8. Compute the inverse of the function shown in Figure 1.

9. Given the sets $A = \{\ a, b, c\ \}$ and $B = \{\ 1, 2, 3, 4\ \}$.

   (a) How many one-to-one functions are there from A to B?

   (b) How many onto functions are there from A to B?

   (c) How many one-to-one functions are there from B to A?

   (d) Give an example of an onto function from B to A.

10. Which of the following statements are true?

    (a) $\neg\,\forall x[p(x) \downarrow q(x)] \Leftrightarrow (\exists x\, p(x)) \downarrow (\exists x\, q(x))$
    (b) $\neg\,\forall x[p(x) \uparrow q(x)] \Leftrightarrow (\exists x\, p(x)) \uparrow (\exists x\, q(x))$
    (c) $\neg\,\forall x[p(x) \downarrow q(x)] \Leftrightarrow (\forall x\, \neg p(x)) \uparrow (\forall \neg\, x q(x))$
    (d) $\neg\,\forall x[p(x) \downarrow q(x)] \Leftrightarrow (\exists x\, p(x)) \uparrow (\exists x\, q(x))$

11. All of the following statements are true, but three are true by definition, whereas the fourth requires the use double negation law $(\neg\,\neg\, p \Leftrightarrow p)$ to deduce. Circle the letter beside statement that requires the double negation law:

(a) The inverse of the converse is the contrapositive.
(b) The inverse of the contrapositive is the converse.
(c) The converse of the inverse is the contrapositive.
(d) The contrapositive of the converse is the inverse.

12. Prove that

$$\sum_{i=0}^{n-1} \frac{i}{3^i} = \frac{3}{4}\left(1 - \frac{2n+1}{3^n}\right). \tag{‡}$$

13. Consider the functions from to defined by

(a) $f(x) = (x-1)(x+1)x$
(b) $f(x) = e^x$
(c) $f(x) = (e^x - e^{-x})/2$
(d) $f(x) = (e^x + e^{-x})/2$

Which of these functions are one to one? Which of these functions are onto?

14. (a) In this question the universe of discourse (set of allowable substitutions) is people living in Prince George.

$$\begin{aligned} \phi(x,y) &\Leftrightarrow x \text{ is a friend of } y \qquad \Leftrightarrow y \text{ has } x \text{ as a friend.} \\ r(x) &\Leftrightarrow x \text{ speaks Russian.} \\ s(x) &\Leftrightarrow x \text{ is a student.} \\ t(x) &\Leftrightarrow x \text{ reads Tolkien.} \end{aligned}$$

Use the letters before the mathematical formulas to fill in the blanks before English language statements in Figure 2 on the following page. Some mathematical formulas do not correspond to any of the English language statements.

(b) Translate the unmatched formulæ into English.

15. Consider the finite state machine shown in Figure 3.

(a) What is the input alphabet?
(b) What is the output alphabet?
(c) What are the states of the machine?
(d) For the input "`1212012120`", what is the output?

____ No Russian-speaking student reads Tolkien.	**A.** $\exists x[r(x) \to s(x)]$
____ Every Russian-speaking person who reads Tolkien is a student.	**B.** $\forall x\, \exists y[\phi(x,y) \wedge s(y) \to t(x)]$
____ Every student has a Russian-speaking friend.	**C.** $\forall x[r(x) \wedge t(x) \to s(x)]$
____ Every Russian-speaking student has a friend.	**D.** $\forall x\, \exists y[s(x) \wedge \phi(x,y) \wedge \neg t(y)]$
____ There is a Russian-speaking student who has no friends who read Tolkien.	**E.** $\forall x[s(x) \to \exists y[r(y) \wedge \phi(x,y)]]$
____ Every student is a friend of someone who does not read Tolkien.	**F.** $\forall x[r(x) \to t(x)]$
____ Every person who speaks Russian reads Tolkien.	**G.** $\forall x\, \exists y[r(x) \wedge s(x) \to \phi(x,y)]$
____ Every friend of a student reads Tolkien.	**H.** $\neg \exists x[r(x) \wedge s(x)]$
	I. $\exists x[s(x) \wedge \forall y[\phi(x,y) \to \neg t(y)]]$
	J. $\forall x[s(x) \wedge \exists y[r(y) \to \phi(x,y)]]$
	K. $\exists x[s(x) \wedge \forall y\{\phi(x,y) \wedge t(y)\}]$
	L. $\exists x\, \exists y[s(x) \wedge r(y) \wedge \neg\phi(x,y)]$

Figure 2: Statements and formulæ for Question 14.

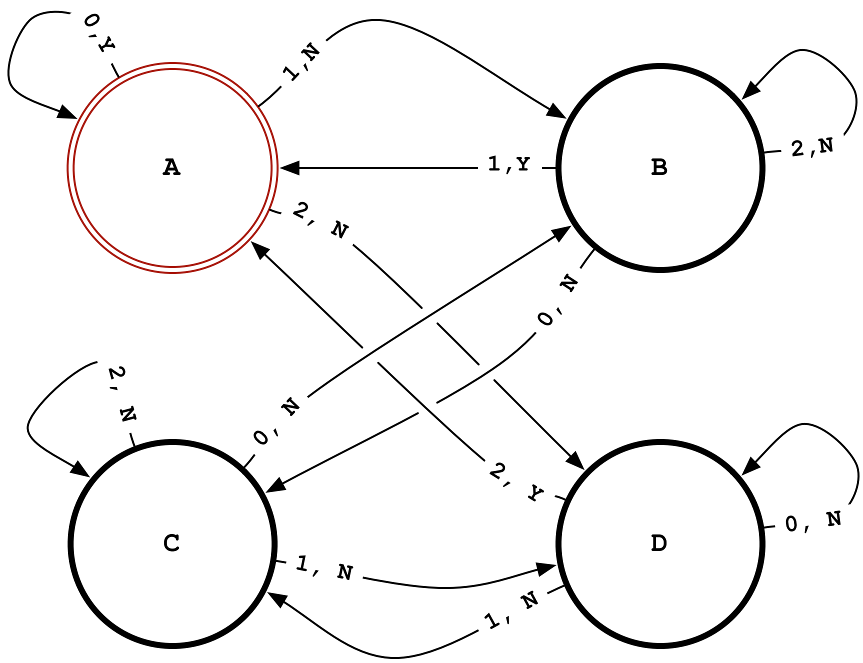

Figure 3: Finite State Machine