

Homework Assignment #6

Due: 2006-11-24 (Friday)

- Which of the following sets are Cartesian products?
 - $\{(2, a), (1, a), (3, b), (3, a), (2, b), (1, b)\}$?
 - $\{(2, a), (1, a), (3, c), (3, b), (2, c), (1, b)\}$?
 - \emptyset ?
 - $\{\emptyset\}$?
 - $\{(1, b)\}$?
- How many functions are there from the empty set to the empty set?
- Which of the following sets are relations from the set $\{a, b\}$ to the set $\{1, 2, 3\}$?
 - \emptyset ?
 - $\{(1, a), (2, a)\}$?
 - $\{a, b\} \times \{1, 2, 3\}$?
 - $\{(a, 1), (a, 3)\}$?
 - $\{(a, 1), (b, 2)\}$?
- Which of the puddle diagrams shown in Figures 1–3 illustrate functions? For the ones that do not, explain why not.
- How many onto functions are there
 - from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$?
 - from $\{1, 2, 3\}$ to $\{1, 2, 3\}$?
- The hyperbolic sine and hyperbolic arc sine functions are defined from the reals to the reals by
$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \quad \text{arc sinh}(y) = \log_e(y + \sqrt{y^2 + 1}).$$
Prove that these functions are inverses of each other, and show that the hyperbolic sine function is onto.
- Compute the inverse of the function $\{(2, a), (3, b), (1, c)\}$.

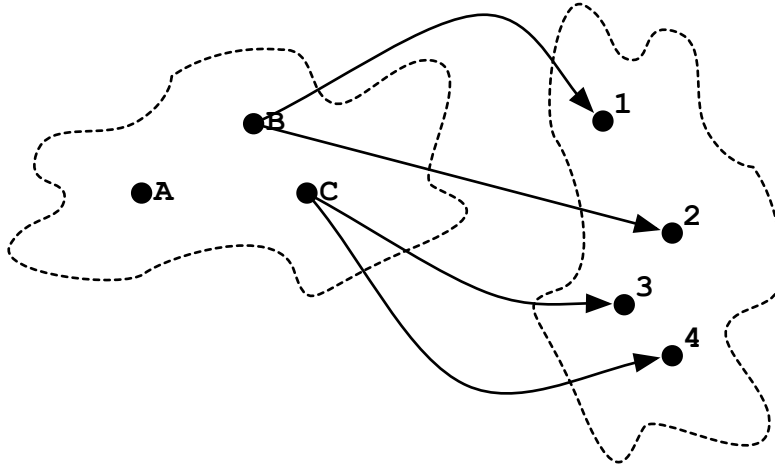


Figure 1: Puddle Diagram #1

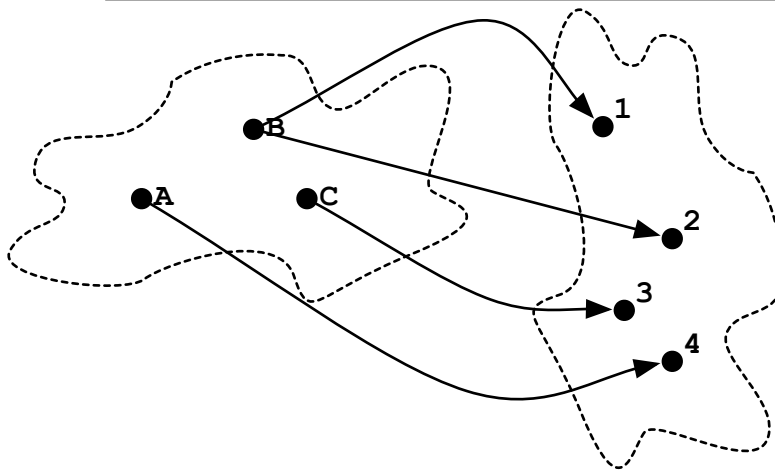


Figure 2: Puddle Diagram #2

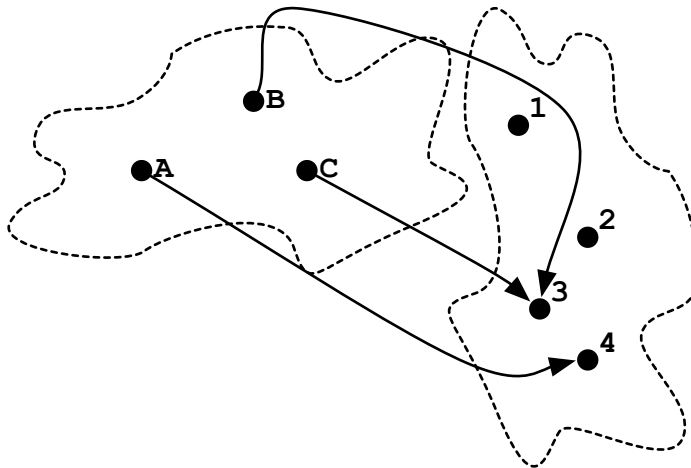


Figure 3: Puddle Diagram #3