UNBC

CPSC 141 Fall 2003

Midterm I— 8 October 2002

Name(Printed) : _____

Signature : _____

StudentNumber : _____

ACRE	AREA	BALE	BAND	BARD	BASS
BETA	BIRD	BLOT	BOOK	BREW	CAMP
CHIP	CLAN	COAT	COIL	CORN	CROW
CURL	DARK	DEER	DOSE	DROP	DUCK
DUSK	FARE	FILM	FLAX	GAZE	GIFT
GOLD	GULF	HINT	HORN	HULL	IBOU
INCH	IRIS	ISLE	KERN	KILN	KITE
LANE	LARK	LENS	LOFT	LURE	MALT
MANX	MESH	MINK	MOTH	MOVE	MUSK
NAVY	NEWT	NOON	OATS	OBOE	OPAL
PARK	PINE	POET	RAFT	REED	RING
RUBY	RUFF	SEAM	SEED	SHOP	SILK
SINE	SNIP	SOAP	STUB	TASK	TAXI
TEAM	TELL	TEXT	TIDE	TILT	TOIL
TOME	TOUR	TURN	VANE	VISA	WALL
WICK	WOLF	WRIT	YARD		

Question	Score
1	/5
2	/3
3	/2
4	/2
5	/2
6	/6
7	/10
8	/5
9	/6
10	/6
11	/3
12	/0
Total	/50

- Write the word marked above on each page of your exam. Do not put any other identifying marks on any page of your exam. Failure to put the circled word on a page of your exam may result in no marks being awarded for that page.
- *Read each question carefully. Ask yourself what the point of the question is. Check to make sure that you have answered the question asked.*
- This is a **50** minute exam. This exam contains **8** pages of questions not including this cover page. Make sure that you have all of them.
- Answer all questions on the exam sheet. If you do some of your work on the back of a page, clearly indicate to the marker what work corresponds with which question.
- Partial marks shall be awarded for clearly identified work.
- This exam counts as **20%** of your total grade. There are **50** points total on the exam.

(5) **1.** Fill in the following truth table.

p	q	$p \wedge q$	$p \vee q$	$p \leftrightarrow q$	$p \oplus q$	$\neg p$	$p \rightarrow q$	$p \downarrow q$
0	0							
0	1							
1	0							
1	1							

(3) **2.** Use a truth table to decide whether $p \rightarrow (q \vee r)$ is logically equivalent to $[\neg(p \rightarrow q)] \rightarrow r$. Be sure to state and justify your answer!

You may find it easier to answer the following questions if you label the primitive propositions with p and q, and write down the logical formula for the statement you wish to find. If nothing else, you can earn part marks for showing your work.

(2) **3.** Circle the *negation* of "Harry is a wizard if Harry plays Quidditch."

(a) Harry is a wizard and Harry doesn't play Quidditch.
(b) Harry is not a wizard and Harry doesn't play Quidditch.
(c) Harry plays Quidditch and Harry is not a wizard.
(d) Harry doesn't play Quidditch if Harry is not a wizard.
(e) *None of the above.*

(2) **4.** Circle the *contrapositive* of "If Hallowe'en is a holiday, then happiness is hardly possible."[1]

(a) If happiness is hardly possible, then Hallowe'en is a holiday.
(b) If happiness is a holiday, then Hallowe'en is hardly possible.
(c) If happiness is easily possible, then Hallowe'en is not a holiday.
(d) If Hallowe'en is not a holiday, then happiness is easily possible.
(e) If Hallowe'en is a holiday, then happiness is hardly possible.

(2) **5.** Circle the *converse* of "You are a New Yorker if you like Ani diFranco."

(a) You are a New Yorker if you do not like Ani diFranco.
(b) You like Ani diFranco if you are a New Yorker.
(c) You are not a New Yorker if you do not like Ani diFranco.
(d) You do not like Ani diFranco if you are not a New Yorker.
(e) *None of the above.*

[1] For this question take "easily possible" to be the negation of "hardly possible".

(6) **6.** Compute the following:

(a) $\lfloor -2.5 \rfloor$

(b) $\sum_{i=1}^{3} (i/2)$

(c) $38!/37!$

(d) $\prod_{i=-3}^{1} i$

(e) $\binom{6}{2}$

(f) $\sum_{i=1}^{5} \left\lceil \frac{31}{2^i} \right\rceil - \sum_{i=1}^{4} \left\lceil \frac{31}{2^i} \right\rceil$

2 each **7.** Simplify the following statements by using the Laws of Logic:

(a) $(p \uparrow q) \vee \big[(p \uparrow q) \wedge r\big]$.

(b) $\neg\big[p \rightarrow \neg r]$.

(c) $[\exists x\ p(x)] \vee [\forall x\, \neg p(x)]$.

(d) $\forall x\, [p(x) \vee \neg q(x)] \wedge \forall x\, [p(x) \vee q(x)]$.

(e) $\neg\forall x\ [\exists y p(x, y) \vee \exists y(\neg r(x, y))]$.

(5) **8.** (a) Explain how to form the dual of a statement.

(b) Show that the dual of $p \uparrow q$ is $p \downarrow q$ and that the dual of $p \downarrow q$ is $p \uparrow q$.

(c) What logical equivalence can we derive from $(p \uparrow q) \downarrow (p \uparrow q) \Leftrightarrow p \wedge q$ using the principle of duality?

(2) **9.** (a) Give the definition of a well-ordered set.

(1 each) (b) Decide which of the following sets are well-ordered, and explain why:

i. $\{\ 1, \sqrt{2}, \pi, 5, 11\ \}$,

ii. $\{\ -1, -1/2, -1/4, -1/8, \ldots\ \}$,

iii. the positive integers.

iv. the positive real numbers.

(6) **10.** In this question, let the universe of discourse $\mathcal{U}$ be the set of all integers. Determine whether the following statements are true. Write down "TRUE" or "FALSE". Briefly justify each of your answers.

(a) $\forall x \exists y\ x = y^2$.

(b) $\exists x\ 5 + x = 10$.

(c) $\exists x \forall y\ x + y = y$.

(d) $\forall x \exists y\ x^2 = y$.

(e) $\forall x\ 5 + x = 10$.

(f) $\exists x \forall y\ [(x = y + 1) \vee (x = 0)]$.

(3) **11.** Use a truth table to decide whether $(p \vee q) \rightarrow r$ logically implies $(p \wedge q) \rightarrow r$. Be sure to state and justify your answer!

Do not attempt the following question until you have checked and re-checked your work on the previous pages.

[BONUS] **12.** Make a statement from the three primitive statements p, q, and r and logical connectives that has the following properties: (i) its dual is logically equivalent to its negation, and (ii) its truth table has exactly two ones. Your statement must use all of p and q and r.