

UNBC

CPSC 141 Fall 1999

Midterm II—12 November 1999

Name(Printed) : _____

Signature : _____

Student Number : _____

ABED	ACRE	AFAR	AREA	BALE	BAND
BARD	BETA	BIDE	BIRD	BLOT	BREW
BUZZ	CAMP	CHIN	CHIP	CHOP	CLAN
CLOG	COAT	COIL	CORE	CORN	CRAM
DEAN	DISH	DOCK	DOSE	DROP	DUSK
ELSE	FARE	FIND	FLAX	FOOD	GAZE
GIFT	GOAD	GOLD	GULF	HELP	HINT
HULL	ISLE	KERN	KIND	KITE	LANE
LARK	LAVA	LOFT	LUCK	LURE	MALT
MESH	MOTH	MOVE	MUSK	NAVY	NEWT
NOON	OBOE	PARK	PINE	POET	REED
RIFT	RING	RUBY	SEAM	SEED	SHOP
SHUN	SILK	SINE	SLID	SNIP	SOAP
SOUR	STIR	STUB	TASK	TAXI	TEAM
TELL	TEXT	TIDE	TILT	TOIL	TOLD
TOUR	TURN	VANE	VERY	VISA	WALL
WICK	WINK	WRIT	YARD		

Question	Score
1	/10
2	/5
3	/5
4	/5
5	/3
6	/4
7	/4
8	/5
9	/9
10	/0
Total	/50

- Write the word circled above on each page of your exam. Do not put any other identifying marks on any page of your exam. Failure to put the circled word on a page of your exam may result in no marks being awarded for that page.
- *Read each question carefully. Ask yourself what the point of the question is. Check to make sure that you have answered the question asked.*
- This is a **50** minute exam. This exam contains **6** pages of questions not including this cover page. Make sure that you have all of them.
- Answer all questions on the exam sheet. If you do some of your work on the back of a page, clearly indicate to the marker what work corresponds with which question.
- Partial marks shall be awarded for clearly identified work.
- This exam counts as **20%** of your total grade. There are **50** points total on the exam.

Number Theory

1. The following questions are related to number theory.

- (4) (a) How many positive integers are simultaneously multiples of 14 and divisors of 280? Draw a picture showing the divisibility relations between these numbers.
- (2) (b) Define what a prime number is.
- (2) (c) Use Euclid's algorithm to compute the greatest common divisor of 294 and 203.
- (2) (d) Find the least common multiple of $29^6 \cdot 31^2 \cdot 67^3 \cdot 71$ and $29^2 \cdot 31^5 \cdot 37^4 \cdot 71^2$. You may leave your answer as a product of powers of primes.

Induction, Miscellaneous

- (1) **2.** (a) What kind of complex logical statement are we proving when we start with the words “Suppose” or “Assume”?

- (2) (b) Fill in the blanks

$$\begin{array}{c} p(\quad) \\ \wedge \quad \forall k \left[\quad \right] \\ \hline \Rightarrow \quad \forall n p(n). \end{array}$$

- (2) (c) Write down the pattern for *strong* induction in terms of quantified statements like above.

- (5) **3.** Consider the claim that

$$\text{for } n \geq 1, \quad \sum_{i=0}^{n-1} 3 \cdot 4^i = 4^n - 1 \quad (**)$$

Prove Equation (**) using mathematical induction.

- (5) 4. Use mathematical induction to show that for $n \geq 1$,

$$\sum_{i=1}^n \frac{1}{i(i+1)} = 1 - 1/(n+1).$$

- (3) 5. Let $A = \{ n \in \mathbf{Z}^+ : 50 < n \leq 500 \text{ and } 5 \text{ divides } n \}$
and $B = \{ n \in \mathbf{Z}^+ : n \leq 90 \text{ and } 3 \text{ divides } n \}$.

What is $|A \cup B|$? Show your work.

Set Theory

- (4) **6.** Let the universe of discourse \mathcal{U} be the numbers $\{ 1, 2, 3, 4, 5, 6, 7 \}$, and let $R = \{ 4, 5, 6, 7 \}$, $S = \{ 2, 3, 6, 7 \}$, and $T = \{ 1, 3, 5, 7 \}$. Determine each of the following sets:

(a) $R - \overline{S} =$

(b) $S \Delta (T \cap R) =$

- (4) **7.** Let $B = \{ \{ 4 \}, 3 \}$.

(a) What is the cardinality of B ?

(b) Compute the powerset of B :

(c) What is the general formula for the cardinality of the powerset of a given set?

- 8.** Let $I = \mathbb{Z}^+$, and for $i \in I$ let $S_i = [1 - 1/i, \sqrt{i+3})$.

- (1) (a) Complete the following:

$$c \in \bigcap_{i \in I} S_i \quad \Leftrightarrow$$

- (2) (b) What is $\bigcup_{i \in I} S_i$?

- (2) (c) What is $\bigcap_{i \in I} S_i$?

9. Because of its high ranking in *Acclaim* magazine, enrollment at the College of Battlefield–North Umbria (CBNU) has grown dramatically. Here are the 1999 enrollment statistics. There are 50 students taking a Theology course, 40 students taking a Dentistry course, and 30 students taking a Law course. There are 75 students taking either Theology or Dentistry or both. There are 6 student taking Theology and Dentistry who are not taking Law, and there are 10 students taking Law that are not taking either Theology or Dentistry. There is 1 student taking Law and Theology, but not Dentistry.

Let L denote the students taking a Law course, D denote the students taking a Dentistry course, and T denote the students taking a Theology course. Two of the above facts can be written with set theory notation as

$$|L| = 30 \quad |T \cup D| = 75$$

- (3) a. Write down the remaining facts using set theory notation.
- (2) b. How many students are taking both Theology and Dentistry?
- (3) c. Draw a Venn diagram of the situation.
- (1) d. If there are 125 students at the College, how many students are not taking a course from any of Theology, Dentistry, or Law?

Bonus Question

Do not attempt this question until you have checked and re-checked your work on the previous pages.

- [BONUS] 10. Let S and T be sets, and suppose that $\mathcal{P}(S) \subseteq \mathcal{P}(T)$.
- (a) Show that $\mathcal{P}(S \cap T) = \mathcal{P}(S) \cap \mathcal{P}(T)$.

- (b) Find a small example to show that $\mathcal{P}(S \cup T) \neq \mathcal{P}(S) \cup \mathcal{P}(T)$ in general.