Guards! Guards!

- Using

$$\sin x = \begin{cases} -\sin(-x) & \texttt{if } x < 0 \\ \sin(x - 2\pi) & \texttt{if } x \geq 2\pi \\ -\sin(x - \pi) & \texttt{if } x \geq \pi \\ \sin(\pi - x) & \texttt{if } x \geq \pi/2 \\ 3y - 4y^3 & \texttt{if } y = \sin(x/3) \end{cases}$$

  and, Equation (1),

$$\sin x \approx x\big(1 - \frac{x^2}{6}(1 - \frac{x^2}{20}(1 - \frac{x^2}{42}))\big) \qquad \texttt{for } 0 \leq x < 0.01 \ \texttt{(}x\texttt{ in radians)} \qquad (1)$$

  write a function (or functions) ***using guards*** to compute $\sin x$.

  Call your function something other than `sin` (perhaps `mySin`?) to avoid accidentally confusing your functions results with the Prelude function
  `sin :: Floating a => a -> a`.

Bonus

If you are interested in how computer hardware actually does trig, you may find below interesting. (However, calculation of $\sin x$ to several hundred digits is best done by a different method (See the Wikipedia entry on the Arithmetic Geometric Mean for a starting point).)

- Compare your function against the builtin `sin` function on the range $[0, \pi/2]$. Where does your function differ most from the builtin function?

- Equation (1) is an approximation of the Taylor series

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$$

  factored as

$$\sin x = x(1 - \frac{x^2}{2 \cdot 3}(1 - \frac{x^2}{4 \cdot 5}(1 - \frac{x^2}{6 \cdot 7}(1 - \cdots)))).$$

  In Equation (1), the (worst) error is $\sim x^7$. By carrying the series out to the $x^{21}/21!$ term, the error also becomes $\sim x^{21}$. This means that you can use it for $x < 0.1$ (rather than $x < 0.01$) and still get an answer accurate to 20 digits or so.

  The equation

$$\sin x = 3y - 4y^3 \qquad \texttt{where } y = \sin(x/3)$$

  is mathematically exact, but rounding errors creep in every time that you use it.

\+ Write a helper function to compute $x(1 - \frac{x^2}{2\cdot 3}(1 - \frac{x^2}{4\cdot 5}(1 - \frac{x^2}{6\cdot 7}(1 - \cdots))))$, and see if you get a more accurate sin function.