UNBC

CPSC 141 Fall 2000
Midterm II—10 November 2000

$Name(Printed)$: _____

$Signature$: _____

$StudentNumber$: _____

ABED	ACRE	AFAR	AREA	BALE	BAND
BARD	BETA	BIDE	BIRD	BLOT	BREW
BUZZ	CAMP	CHIN	CHIP	CHOP	CLAN
CLOG	COAT	COIL	CORE	CORN	CRAM
DEAN	DISH	DOCK	DOSE	DROP	DUSK
ELSE	FARE	FIND	FLAX	FOOD	GAZE
GIFT	GOAD	GOLD	GULF	HELP	HINT
HULL	ISLE	KERN	KIND	KITE	LANE
LARK	LAVA	LOFT	LUCK	LURE	MALT
MESH	MOTH	MOVE	MUSK	NAVY	NEWT
NOON	OBOE	PARK	PINE	POET	REED
RIFT	RING	RUBY	SEAM	SEED	SHOP
SHUN	SILK	SINE	SLID	SNIP	SOAP
SOUR	STIR	STUB	TASK	TAXI	TEAM
TELL	TEXT	TIDE	TILT	TOIL	TOLD
TOUR	TURN	VANE	VERY	VISA	WALL
WICK	WINK	WRIT	YARD		

Question	Score
1	/9
2	/5
3	/2
4	/3
5	/8
6	/5
7	/2
8	/2
9	/2
10	/4
11	/3
12	/5
13	/5
14	/0
Total	/55

- Write the word circled above on each page of your exam. Do not put any other identifying marks on any page of your exam. Failure to put the circled word on a page of your exam may result in no marks being awarded for that page.

- *Read each question carefully. Ask yourself what the point of the question is. Check to make sure that you have answered the question asked.*

- This is a **50** minute exam. This exam contains **6** pages of questions not including this cover page. Make sure that you have all of them.

- Answer all questions on the exam sheet. If you do some of your work on the back of a page, clearly indicate to the marker what work corresponds with which question.

- Partial marks shall be awarded for clearly identified work.

- This exam counts as **20%** of your total grade. There are **55** points total on the exam.

Set Theory

1. Because of its high ranking in *Acclaim* magazine, enrollment at the College of Battlefield–North Umbria (CBNU) has grown dramatically, again! Here are the 2000 enrollment statistics. There are 55 students taking a Theology course, 45 students taking a Dentistry course, and 35 students taking a Law course. There are 85 students taking either Theology or Dentistry or both. There are 10 students taking both Law and Theology, and 9 student taking all three subjects, and there are 15 students taking Law that are not taking either Theology or Dentistry.

Let L denote the students taking a Law course, D denote the students taking a Dentistry course, and T denote the students taking a Theology course. Two of the above facts can be written with set theory notation as

$$|L| = 35 \qquad |T \cup D| = 85$$

(3)   a. Write down the remaining facts using set theory notation.

(2)   b. How many students are taking both Theology and Dentistry?

(3)   c. Draw a Venn diagram showing all of the information.

(1)   d. If there are 150 students at the College, how many students are not taking a course from any of Theology, Dentistry, or Law?

(5)   **2.** Let the universe of discourse $\mathcal{U}$ be the numbers $\{\ 1, 2, 3, 4, 5, 6, 7\ \}$, and let $R = \{\ 4, 5, 6, 7\ \}$, $S = \{\ 2, 3, 6, 7\ \}$, and $T = \{\ 1, 3, 5, 7\ \}$. Determine each of the following sets:

    (a) $S \cap T =$

    (b) $R \triangle T =$

    (c) $\overline{R} =$

    (d) $T - S =$

    (e) $|T| =$

(2)   **3.** Show that $\mathcal{P}(\{1\}) \cup \mathcal{P}(\{2\}) \neq \mathcal{P}(\{1\} \cup \{2\})$

   4. The following questions are about well ordered sets.

(2)   (a) Define a well-ordered set.

(1)   (b) Give an example of a set that has a least element, but is not well-ordered.

1 each   **5.** Circle the appropriate answer.

    (a) $\emptyset \in \emptyset$ **TRUE**   **FALSE**

    (b) the set $\{\ n \in \mathbf{Z}^{+} : n$ has at least 118 divisors $\}$ is well-ordered.
                                                   TRUE   **FALSE**

    (c) the set $\{\ -1/n \mid n \in \mathbf{Z}^{+}\ \}$ is well-ordered.   **TRUE**   **FALSE**

    (d) $\emptyset \subset \{\emptyset\}$.   **TRUE**   **FALSE**

Number Theory

(5)   **6.**   (a) Compute the gcd of 54 and 66 using Euclid's method.

(b) Compute the positive least common multiple of 437 and 667.

(2)   **7.** Define what a *prime* number is.

(2)   **8.** Carefully define what it means for c to be a *least common multiple* of a and b.

(2)   **9.** Compute a greatest common divisor of $2^1 3^{13} 5^{11} 7^2$ and $2^{19} 3^2 5^1 7^1$.
[You may leave the answer as a product of primes.]

(4)   **10.**   (a) Define $a \mid b$ using a quantified statement.

         (b) Show that if $a \mid b$ and $b \mid c$, then $a \mid c$.

Induction, Miscellanious

(3)   **11.** Let A be the set $\{$ Alliance, Bloc, Conservative, *Democrat*, Liberal, NDP, *Republican* $\}$

         (a) How many subsets does A have?

         (b) How many subsets of size 3 does a set of size 7 have?

Do EITHER Question 12 OR Question 13. Circle the number of the question that you wish to have marked.

(5)

12. Show, using mathematical induction, that $\sum_{i=1}^{n} \left[\frac{1}{i} - \frac{1}{i+1} \right] = 1 - \frac{1}{n+1}$.

(5)   **13.** Show, using strong induction, that every positive integer greater than 13 can be written as a sum of 3's and 8's.

Do NOT ATTEMPT the Bonus Question until after you have checked and re-checked your exam.

Bonus Question

[BONUS]   **14.**   (a) Show that for $m = apq + bp + c$ where $0 \leq b < q$ and $0 \leq c < p$ we have

$$\left\lfloor \frac{m}{pq} \right\rfloor = \left\lfloor \frac{\lfloor m/p \rfloor}{q} \right\rfloor .$$

(b) Show that $\lfloor m/pq \rfloor = \lfloor \lfloor m/p \rfloor / q \rfloor$ holds in general.

(c) Use (strong) induction to show that

$$n - \sum_{j=1}^{n} \left\lfloor \frac{n}{2^j} \right\rfloor = \text{the number of one bits in the binary rep-} \quad (1)$$
$$\text{resentation of } n.$$

Hints: (i) Explain why n and $2n$ have the same number of one bits; and (ii) Show why if (1) is true for k it is also true for $2k + b$ where b is either 0 or 1.