

# UNBC

CPSC 141 Fall 1999

Midterm I—08 October 1999

Name(Printed) : \_\_\_\_\_

Signature : \_\_\_\_\_

Student Number : \_\_\_\_\_

|      |      |      |      |      |      |
|------|------|------|------|------|------|
| ABED | ACRE | AFAR | AREA | BALE | BAND |
| BARD | BETA | BIDE | BIRD | BLOT | BREW |
| BUZZ | CAMP | CHIN | CHIP | CHOP | CLAN |
| CLOG | COAT | COIL | CORE | CORN | CRAM |
| DEAN | DISH | DOCK | DOSE | DROP | DUSK |
| ELSE | FARE | FIND | FLAX | FOOD | GAZE |
| GIFT | GOAD | GOLD | GULF | HELP | HINT |
| HULL | ISLE | KERN | KIND | KITE | LANE |
| LARK | LAVA | LOFT | LUCK | LURE | MALT |
| MESH | MOTH | MOVE | MUSK | NAVY | NEWT |
| NOON | OBOE | PARK | PINE | POET | REED |
| RIFT | RING | RUBY | SEAM | SEED | SHOP |
| SHUN | SILK | SINE | SLID | SNIP | SOAP |
| SOUR | STIR | STUB | TASK | TAXI | TEAM |
| TELL | TEXT | TIDE | TILT | TOIL | TOLD |
| TOUR | TURN | VANE | VERY | VISA | WALL |
| WICK | WINK | WRIT | YARD |      |      |

| Question | Score |
|----------|-------|
| 1        | /5    |
| 2        | /3    |
| 3        | /3    |
| 4        | /2    |
| 5        | /2    |
| 6        | /2    |
| 7        | /2    |
| 8        | /5    |
| 9        | /6    |
| 10       | /2    |
| 11       | /4    |
| 12       | /4    |
| 13       | /5    |
| 14       | /5    |
| 15       | /0    |
| Total    | /50   |

- Write the word circled above on each page of your exam. Do not put any other identifying marks on any page of your exam. Failure to put the circled word on a page of your exam may result in no marks being awarded for that page.
- *Read each question carefully. Ask yourself what the point of the question is. Check to make sure that you have answered the question asked.*
- This is a **50** minute exam. This exam contains **6** pages of questions not including this cover page. Make sure that you have all of them.
- Answer all questions on the exam sheet. If you do some of your work on the back of a page, clearly indicate to the marker what work corresponds with which question.
- Partial marks shall be awarded for clearly identified work.
- This exam counts as **20%** of your total grade. There are **50** points total on the exam.

## Short Answers

(5)

1. Fill in the following truth table.

| $p$ | $q$ | $\neg q$ | $p \vee q$ | $p \wedge q$ | $p \leftrightarrow q$ | $p \underline{\vee} q$ | $q \rightarrow p$ | $p \uparrow q$ |
|-----|-----|----------|------------|--------------|-----------------------|------------------------|-------------------|----------------|
| 0   | 0   |          |            |              |                       |                        |                   |                |
| 0   | 1   |          |            |              |                       |                        |                   |                |
| 1   | 0   |          |            |              |                       |                        |                   |                |
| 1   | 1   |          |            |              |                       |                        |                   |                |

(3)

2. Use a truth table to prove that  $(p \underline{\vee} q) \rightarrow r$  is logically equivalent to  $(p \rightarrow r) \leftrightarrow (q \rightarrow r)$ .

(3)

3. Use a truth table to decide whether  $r \rightarrow (p \wedge q)$  logically implies  $r \rightarrow (p \vee q)$ . Be sure to state and justify your answer!

You may find it easier to answer the following questions if you label the primitive propositions with  $p$  and  $q$ , and write down the logical formula for the statement you wish to find. If nothing else, you can earn part marks for showing your work.

- (2) 4. Circle the **negation** of “If  $b > 2$  and  $b$  is even then  $b$  is prime.”
- (a) If  $b > 2$  and  $b$  is even then  $b$  is not prime.
  - (b)  $b > 2$ ,  $b$  is even, and  $b$  is not prime.
  - (c) Either  $b > 2$  or  $b$  is even; and  $b$  is not prime.
  - (d) If  $b \leq 2$  and  $b$  is not even then  $b$  is not prime.
  - (e) If  $b$  is not prime then  $b \leq 2$  or  $b$  is not even.
  - (f) None of the above.
- (2) 5. Circle the **converse** of “If Thanksgiving is on Thursday, then you are American.”
- (a) You are American if Thanksgiving is on Thursday.
  - (b) If Thanksgiving is not on Thursday, then you are American.
  - (c) If you are American, then Thanksgiving is on Thursday.
  - (d) If Thanksgiving is not on Thursday, then you are not American.
  - (e) If you are not American, then Thanksgiving is not on Thursday.
- (2) 6. Circle the **contrapositive** of “If Smilla understands snow, then she is from Greenland.”
- (a) Smilla is from Greenland if she doesn't understand snow.
  - (b) Smilla doesn't understand snow if she is from Greenland.
  - (c) Smilla is not from Greenland if she doesn't understand snow.
  - (d) Smilla doesn't understand snow if she is not from Greenland.
  - (e) None of the above.
- (2) 7. Circle the **inverse** of “You are foolish if you are not heroic.”
- (a) You are foolish if you are heroic.
  - (b) You are heroic if you are not foolish.
  - (c) You are not heroic if you are foolish.
  - (d) You are not foolish if you are heroic.
  - (e) None of the above.

(5) 8. Compute the following:

(a)  $\lceil -15.6 \rceil$

(b)  $0! + 2!$

(c)  $\sum_{i=1}^2 i^3$

(d)  $\prod_{i=-2}^2 (i + 1)$

(e)  $\binom{6}{2}$

(6) 9. (a) Explain how to form the dual of a statement.

(b) What is the principle of duality?

(c) Find the dual of  $p \leftrightarrow q$ . [*Hint:*  $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$ .]

(2) 10. Circle the letter before each statement that is a tautology.

(a)  $\forall z[p(z) \wedge q(z)] \Leftrightarrow (\forall z p(z)) \wedge (\forall z q(z))$

(b)  $\forall z[p(z) \vee q(z)] \Leftrightarrow (\forall z p(z)) \vee (\forall z q(z))$

(c)  $\exists z[p(z) \wedge q(z)] \Leftrightarrow (\exists z p(z)) \wedge (\exists z q(z))$

(d)  $\exists z[p(z) \vee q(z)] \Leftrightarrow (\exists z p(z)) \vee (\exists z q(z))$

(4) **11.** Simplify the following statements by using the Laws of Logic:

(a)  $\neg[(p \leftrightarrow q) \rightarrow F_0]$ .

(b)  $[(p \vee q) \wedge (p \vee \neg q)] \vee r$ .

(4) **12.** Negate and simplify the following statements by using the Laws of Logic:

(a)  $\exists x [(\neg p(x)) \wedge \neg \forall y [q(y)]]$ .

(b)  $(p \wedge r) \vee \neg q$ .

- (5) **13.** In this question, let the universe of discourse,  $\mathcal{U}$ , be the set of all real numbers. Determine the truth value of the following statements and write down “TRUE” or “FALSE”. Briefly justify each of your answers.

(a)  $\forall x \exists x = 10$ .      *Example.* FALSE. Take  $x = 3$ .

(b)  $\exists x \exists x = 10$ .

(c)  $\exists x \forall y [(x = y + 1) \vee (x < -\pi)]$ .

(d)  $\forall x \exists y x^2 = y$ .

(e)  $\exists x \forall y xy = 0$ .

(f)  $\exists x \forall y x + y = 0$ .

- 14.** The following questions are about well ordered sets.

- (3) (a) Define a well-ordered set.

- (2) (b) Give an example of a set that has a least element, but is not well-ordered.

*Do not attempt this question until you have checked and re-checked your work on the previous pages.*

[BONUS]

15. (*Modified from the Mathematics Calendar 1999 by theoni pappas.*) Dudley Doright delves deeply into the devious declarations of three dastardly denizens of Dartmouth, NS, but is unable to deduce who is telling the truth. Their statements were:

Padraigh: Rusung is lying.

Quisling: Padraigh and Rusung are lying.

Rusung: Quisling is lying.

Letting  $p$ ,  $q$ , and  $r$  denote “Padraigh is telling the truth;” “Quisling is telling the truth;” and “Rusung is telling the truth;” respectively, show how to construct a truth table and use it to solve Doright’s dilemma.