UNBC

CPSC 141 Fall 1998
Midterm II—13 November 1998

Name(Printed) : ______________________

Signature : ______________________

StudentNumber : ______________________

ABLY	AFAR	AREA	BAND	BARD	BEST
BETA	BIDE	BLOT	BREW	CAMP	CHIN
CHIP	CHOP	CLAN	CLOD	CORE	CORN
CRAM	DEAN	DOER	DREW	DYER	FARE
FIND	FLAX	FOOD	GAZE	GOLD	GULF
HELP	HIND	ITCH	KERN	KILL	KIND
LANE	LARK	LAVA	LOVE	LUMP	LURE
META	MOTH	MOVE	MUSK	NECK	NEWT
NOON	PARK	POET	REED	RIME	RING
RUIN	SEAM	SEED	SHOP	SHUN	SILK
SINE	SLID	SNIP	SOAP	SOUR	STIR
TAXI	TEEN	TEXT	THUG	TILT	TINY
TOUR	TURN	VANE	VISA	WALL	WICK
WINK	WRIT	YARD			

Question	Score
1	/4
2	/5
3	/4
4	/13
5	/7
6	/5
7	/7
8	/5
9	/10
10	/10
11	/0
Total	/70

- Write the word circled above on each page of your exam. Do not put any other identifying marks on any page of your exam. Failure to put the circled word on a page of your exam may result in no marks being awarded for that page.
- *Read each question carefully. Ask yourself what the point of the question is. Check to make sure that you have answered the question asked.*
- This is a **50** minute exam. This exam contains **6** pages of questions not including this cover page. Make sure that you have all of them.
- Answer all questions on the exam sheet. If you do some of your work on the back of a page, clearly indicate to the marker what work corresponds with which question.
- Partial marks shall be awarded for clearly identified work.
- This exam counts as **20%** of your total grade. There are **70** points total on the exam.

Set Theory

(4) **1.** Let the universe of discourse $\mathcal{U}$ be the numbers $\{ 1,2,3,4,5,6,7 \}$, and let $R = \{ 4,5,6,7 \}$, $S = \{ 2,3,6,7 \}$, and $T = \{ 1,3,5,7 \}$. Determine each of the following sets:

(a) $R - \overline{S} =$

(b) $S \,\Delta (T \cap R) =$

(5) **2.** Let $B = \{ p, \{ q \} \}$.

(a) What is the cardinality of B?

(b) List the elements of the powerset of B:

(c) Is the statements $\{ q \} \subseteq B$ true? Justify your answer.

3. For $i \in \mathbb{N}$ let $S_i = [2^{-i}, 2^{i+1})$.

(2) (a) What is $\bigcup_{i \in \mathbb{N}} S_i$?

(2) (b) What is $\bigcap_{i \in \mathbb{N}} S_i$?

4. Here are the latest enrollment statistics for the College of Battlefield–North Umbria (CBNU). There are 40 students taking a Theology course, 35 students taking a Dentistry course, and 30 students taking a Law course. There are 60 students taking either Theology or Dentistry or both. Every student taking Theology and Dentistry is also taking Law, but there are 10 students taking Law that are not taking either Theology or Dentistry. There are 3 students taking Law and Dentistry, but not Theology.

Let L denote the students taking a Law course, D denote the students taking a Dentistry course, and T denote the students taking a Theology course. Two of the above facts can be written with set theory notation as

$$|L| = 30 \qquad |T \cup D| = 60$$

(4) a. Write down the remaining facts using set theory notation.

(2) b. How many students are taking both Theology and Dentistry.

(3) c. Draw a Venn diagram of the situation.

(2) d. How many students are taking courses from exactly two of the disciplines?

(2) e. If there are 102 students at the College, how many students are not taking a course from any of Theology, Dentistry, or Law?

Induction, Miscellanious

5. Consider the claim that

$$\text{for } n \geq 1, \qquad \sum_{i=0}^{n-1} (2i+1) = n^2 \qquad (**)$$

If we write this as $\forall n\, p(n)$

(1) (a) What is the universe of discourse $\mathcal{U}$?

(1) (b) What is the least element of $\mathcal{U}$?

(2) (c) Fill in the blanks

$$\begin{array}{ll} & p(\quad) \\ \wedge & \forall k \left[\qquad\qquad\qquad\qquad \right] \\ \hline \Rightarrow & \forall n\, p(n). \end{array}$$

(1) (d) What is $p(n)$?

(2) (e) What are $p(k)$, $p(k+1)$?

(5) **6.** Prove Equation $(**)$ using mathematical induction.

(7) **7.** Use mathematical induction to show that for $n \geq 1$,

$$\sum_{i=1}^{n} i^2 \leq n^3.$$

(5) **8.** Let $A = \{\ n \in \mathbf{Z}^+ : n \leq 1000 \text{ and } 25 \text{ divides } n\ \}$
and $B = \{\ n \in \mathbf{Z}^+ : n \leq 1000 \text{ and } 20 \text{ divides } n\ \}$.

What is $|A \cup B|$? Show your work.

9. This question is about the general pattern of induction proofs. For each of these questions assume that the set of statements to be proved by induction is $\{\ S(n) \mid n \in \mathbf{Z}^+\ \}$

The induction step often begins "*Now suppose that the statement is true for $n = k$*".

(2) (a) Why is it important that we make no special assumptions when picking $n = k$?

(1) (b) Does the induction step by itself prove that *any* of the statements $S(k)$ are true?

(3) (c) Why are we allowed to assume something? Explain what the induction step is proving.

(2) (d) Write down the pattern for *strong* induction in terms of quantified statements.

(2) (e) What words should replace "*Now suppose that the statement is true for $n = k$*" in a proof by strong induction?

Well Ordering, Miscellanious

10. The following questions are about well ordered sets.

(3) (a) Define what it means for a set to be well-ordered.

(5) (b) Which of the following sets are well-ordered? Explain briefly why.

i. the set of real numbers greater than or equal to zero,

ii. the set of integers less than zero,

iii. the set $\{\ p \in \mathbf{Z}^+ : p \text{ is prime} \wedge p > 2000\ \}$. and

iv. the set $\{\ -2^{-n} \mid n \in \mathbf{Z}^+\ \}$. and

v. the set of commercially available C++-compilers.

(2) (c) Is every finite set of real numbers well-ordered? Explain your answer briefly.

[BONUS] **11.** Let S and T be sets, and suppose that $\mathcal{P}(S) \subseteq \mathcal{P}(T)$. Show that $S \subseteq T$.