First Name: **Student Number:**
Last Name:

MATH 200 – Calculus III

MIDTERM II – NOVEMBER 18,

TOTAL MARKS : 50

Show all working. You may use non-programmable, non-graphing calculators

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1. (12 points)

$$F(x, y, z) = 2x^3 + xy - z, \quad P = (1, 0, 2)$$

(a) Find ∇F at P.

(b) Find the equation of the plane tangent to the surface $F(x, y, z) = 0$ at the point P.

(c) Find the rate of change of F at P in the direction of the vector

$$\vec{v} = < 3, 4, 12 > .$$

(d) What is the maximum rate of change of F at P?

2. (7 points) Use the method of Lagrange Multipliers to find the maximum value of

$$f(x, y) = 2x + y$$

subject to the constraint

$$x^2 + 2y^2 = 18.$$

(No points if you don't use Lagrange Multipliers).

3. (7 points) Find all the critical points (a, b) of the function

$$f(x, y) = x^3 - 12x + y^3 - 3y + 1.$$

Use the second derivative test to decide whether these correspond to local maxima, minima or saddle points.

4. (4 points) Give the value of the limit

$$\lim_{(x,y)\to(0,0)} \frac{y^4x^4}{x^8+y^8}$$

or say why the limit does not exist.

5. (8 points) D is the region bounded by the lines $y = x^2$, $y = 0$ and $x = 2$.

(a) Sketch D

(b) Write D as both a Type I and Type II region.

(c) Evaluate $\int\int_D 3x \, dA$.

6. (7 points) Find the surface area of the portion of the paraboloid

$$z = x^2 + y^2$$

that lies above the disc

$$D = \{(x, y) \ : \ x^2 + y^2 \leq 4^2\}.$$

(You might want to use polar coordinates).

7. (5 points) Evaluate

$$\iiint_B 5(x^2 + y^2 + z^2)\ dV$$

where B is the solid sphere

$$B = \{(x, y, z)\ :\ x^2 + y^2 + z^2 \leq 1\}.$$