

First Name:
Last Name:

Student Number:

MATH 200 – Calculus III

MIDTERM I – OCT 7,

TOTAL MARKS : 50

Show all working. You may use non-programmable, non-graphing calculators

1. (18 points) Find the following quantities, when

$$\vec{a} = \langle 2, 3, -1 \rangle, \quad \vec{b} = \langle 1, -2, 2 \rangle .$$

(i). $\|\vec{b}\|$

(ii). $3\vec{a} - 2\vec{b}$

(iii). $\vec{a} \cdot \vec{b}$

(iv). $\vec{a} \times \vec{b}$

(v). A unit vector in the direction of \vec{b} .

Vector $\vec{a} = \langle 2, 3, -1 \rangle$ is decomposed into a vector \vec{u} parallel to and a vector \vec{v} orthogonal to $\vec{b} = \langle 1, -2, 2 \rangle$.

(vi). Find \vec{u}

(vii). Find \vec{v}

(viii). How could you check that \vec{v} really is orthogonal to \vec{b} ?

2. (6 points) Give a parametric equation for the line through the points $A(1, 2, 3)$ and $B(-1, -2, 5)$. Where does this line cross the xy -plane?

3. (3 points) What is the volume of the parallelepiped with corners $O(0, 0, 0)$, $P(1, 0, -1)$, $Q(0, 3, 5)$ and $R(2, -1, 2)$ and adjacent edges OP , OQ and OR ?

4. (4 points) Find the angle between the planes

$$x - y + \sqrt{2}z = 2, \quad x + y + \sqrt{2}z = 4.$$

5. (6 points) The position of a particle after t seconds is given by

$$\vec{r}(t) = \langle 3 \sin t, 3 \cos t, 4t \rangle$$

(a) Find the velocity of the particle after t seconds.

(b) What is the speed of the particle after t seconds?

(c) What was the distance travelled by the particle along its path during the first π seconds?

6. (6 points) Give the equation of the plane containing the three points $A(1, 0, -1)$, $B(3, 0, 5)$ and $C(2, -1, 3)$.

7. (7 points) Suppose S is the surface given in cylindrical coordinates by

$$z = 2r$$

(a) Write the equation of this surface in rectangular coordinates.

(b) What is the equation of this surface in spherical coordinates?

(c) Sketch the surface S .