MATH 101 Lab 2

In this lab we will look at volumes of rotation. The introduction to the lab sets up graphs of two surfaces of revolution around the x-axis. Question 2 repeats the revolution around the y-axis.

Hand in a commented worksheet in lab class. Plots of surfaces of revolution should not be handed in.

Introduction

Define $f(x) = 1 - x^2$ and $g(x) = x^6 - x + 1$ with

```
> f:=x->1-x^2;
> g:=x->x^6-x+1;
```

Plot $y = f(x)$ and $y = g(x)$ in the xy-plane:

```
> plot({f(x),g(x)},x=-0.2..1);
```

Rotating a curve $y = f(x)$ around the x-axis creates circles around the x-axis with radius $f(x)$. The circles are in the yz-plane so have equation $y^2 + z^2 = (f(x))^2$ or $\sqrt{y^2 + z^2} = f(x)$. Define A1 to be the equation of the surface formed by these circles.

```
> A1:=sqrt(y^2+z^2)=f(x);
```

Define B1 to be the equation of the surface created by rotating $y = g(x)$ around the x-axis.

```
> B1:=sqrt(y^2+z^2)=g(x);
```

Use **implicitplot3d** to view the surface defined by A1. Use the plot features to view the graph with different colours, axis styles, patch styles etc. Click on the plot with the left mouse button and drag the box that appears. Click with the right mouse button to redraw the graph. Be sure to understand the shape of the surface before you go on.

```
> with(plots);
> implicitplot3d(A1,x=-0.2..1, y=-2..2,z=-2..2);
```

Now view the surface defined by B1 on the same x, y, and z intervals . Explore this surface as you did the last one.

```
> implicitplot3d(B1,x=-0.2..1, y=-2..2,z=-2..2);
```

In order to view the two surfaces together, define F1 and G1 as below. The same x, y, and z intertervals are used. NOTE that the colon (:) is used instead of the semicolon (;). The colon suppresses the output from a command. The **display** command causes maple to display one or more predefined plots.

```
> F1:=implicitplot3d(A1, x=-0.2..1, y=-2..2, z=-2..2, color=red,
```

```
> style=contour, light=[90,20,0.7,0.3,0.5], contours=60):
> display([F1]);
> G1:=implicitplot3d(B1, x=-0.2..1, y=-2..2, z=-2..2, color=blue,
> style=patch, ambientlight=[.2,.6,.6], light=[90,20,0.7,0.3,0.5]):
> display([G1]);
```

View the two plots together to see the region bounded by the two surfaces:

```
> display([F1,G1]);
```

Question 1

Use Maple to approximate the volume of revolution defined by rotating $y = 1-x^2$ and $y = x^6 - x + 1$ around the x-axis. Should you use the disk method or the cylindrical shell method? How do you know? Use the **fsolve** command to find the points of intersection of the two curves. **fsolve** is like **solve** but gives approximate answers when **solve** cannot give exact answers.

Question 2

Determine the equation of the surface of revolution given by rotating $y = f(x)$ around the y-axis. As in the Introduction, define A2 to be the equation of this surface. Determine the equation of the surface of revolution given by rotating $y = g(x)$ around the y-axis. As in the Introduction, define B2 to be the equation of this surface.

Use implicitplot3d to view the surfaces A2, and B2 on the intervals $-1 < x < 1$, $0.2 < y < 1.2$ and $-1 < z < 1$.

In order to view the two surfaces together, define F2 and G2 similarly to F1 and G1. In fact copy and paste the definitions of F1 and G1 and change the reference from A1 to A2, B1 to B2.

View the two plots together to see the region bounded by the two surfaces.

```
> display([F2,G2]);
```

Question 3

Use Maple to approximate the volume of revolution defined by rotating $y = 1-x^2$ and $y = x^6 - x + 1$ around the y-axis. Should you use the disk method or the cylindrical shell method? How do you know?