FSTY-405 — Lab 8

Aggregation

Name: . . . .

Objectives

Moving from trees to stand. A simple stand-level model derived from the tree-level model of last lab. Two implementations (Excel, Vensim): *simulator* $\neq$ *model*.

Model

We had used the following simple tree-level model:

$$\begin{aligned}\Delta d &= (1.42 d^{2/3} - 0.246 d)\mathrm{e}^{-0.0392 B}\\ p &= 1/[1 + exp(0.08 d\sqrt{N/B} - 6)]\ ,\end{aligned}$$

where d is tree dbh (cm), Δd the 5-year dbh increment, p the 5-year probability of mortality, B stand basal area, and N number of trees per hectare.

Suppose now that we do not know the individual tree diameters, and/or we are not interested in that level of detail. We use instead aggregated values, that is, stand-level, per hectare values.

For simplicity, here we just ignore the variability among trees, and substitute for d the mean dbh D (recall Jensen's inequality). Some refinements could be tried to achieve a closer agreement between both levels of modelling by approximating the effects of variability. We could even include the coefficient of variation as an additional stand-level state variable. But we won't get into that.

The second equation could have been written as a rate $\Delta n = -pn$, where n (or w) is the number of trees represented by a dbh in the tree list. Stop and think about this.

Substituting,

$$\begin{aligned} \Delta D &= (1.42D^{2/3} - 0.246D)\mathrm{e}^{-0.0392B} && (1) \\ \Delta N &= -N/[1 + exp(0.08D\sqrt{N/B} - 6)] \ . && (2) \end{aligned}$$

Convince yourself that if all trees were the same size this would be exact (assuming, of course, that the tree-level model is exact).

As is usual in forestry, by mean dbh we mean here the *quadratic mean* dbh ($\sqrt{\sum d^2/N}$) or, in other words, the dbh of the tree of mean basal area. D, B and N are therefore related by:

$$B = \pi(\frac{D}{200})^2 N = \frac{\pi}{40000} D^2 N \ .$$

Write D as a function of B and N:

. . . .

We have therefore only two non-redundant state variables: D and N, or B and N. Of course, other equivalent pairs are possible; we could, for instance, use the average spacing $100/\sqrt{N}$ instead of N. Use the pair (D, N).

Remember that we are dealing with an uneven-aged stand. Therefore, here "time" is not the same as "age".

Implementation in Excel

Set the initial values of the two state variables to the values at time 0 in the previous lab. Include your calculated B, and use the initial D as calculated from B and N, not the arithmetic mean dbh. Arrange the three values (t, D, N) either as columns, with time advancing to the right, or as rows, with time going down.

Look at your dbh calculations. How do the arithmetic and quadratic mean dbh's differ? Would this be always the case?

In fact, it can be shown that $(quadratic\ mean)^2$ - $(arithmetic\ mean)^2$ = *variance*. This fact should answer the questions (a variance cannot be negative). The difference may also be seen as a consequence of Jensen's inequality, with $f(x) = x^2$; work out the details later.

Use the transition functions (rates) above to project the state at 5 year intervals, up to year 30. Compare with the results from the previous lab (tree-level)[1]. Differences? How large? Why? (Hint: Jensen).

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Keep Excel open while going into the next part.

Implementation in Vensim

Make boxes for the state variables (aka *levels* or *stocks*). Add the rates (ΔD, ΔN: *Rate* tool). Add an *Arrow* for each dependency of the rates on state variables. Add B as an auxiliary variable and include the corresponding influence arrows.

Enter the rate equations, and the equation for B. Note that you have to give rates per unit time (year), so that (1) and (2) need to be divided by 5. Enter the initial values for the state variables.

[1]If your calculations in the last lab were wrong, typical results at age 30 were $N = 494, B = 37.4$.

Run the simulation. Use *Euler* and a 5-year step, so that we are computing finite differences (Δx), not derivative approximations ($\mathrm{d}x/\mathrm{d}t$) as we would with RK4 (*Model>Settings>Time Bounds*). Stop to think.

Compare with the Excel results (use the *Table* tool). If they do not agree, check your work.

One model, two simulators.

This is a *discrete* model, using 5-year finite increments Δx. It advances in 5-year steps, and results are available only at 5-year intervals. A *continuous* model would use infinitesimal increments $\mathrm{d}x/\mathrm{d}t$, advancing continuously in time. RK4 (a 4-th order Runge-Kutta numerical integration procedure) approximates well a continuous process. Run the simulation with RK4. Differences? Why?

In lab 7 and now in lab 8 you have used tree-lists (distance-independent individual-tree), and whole-stand approaches for projection, respectively. As concisely and legibly as possible, answer the following:
Detail the essential differences/similarities and pro's/con's of both methods (bullet form would be OK). Lastly, state one instance where either method would be most useful.

Answer here: