1. (a) Solve the equation

$$e^{2+\ln x} = 1.$$

$$e^{2+\ln x} = 1 = e^{0}$$

So $2 + \ln x = 0$

$\ln x = -2$

$x = e^{-2}.$

(b) Find

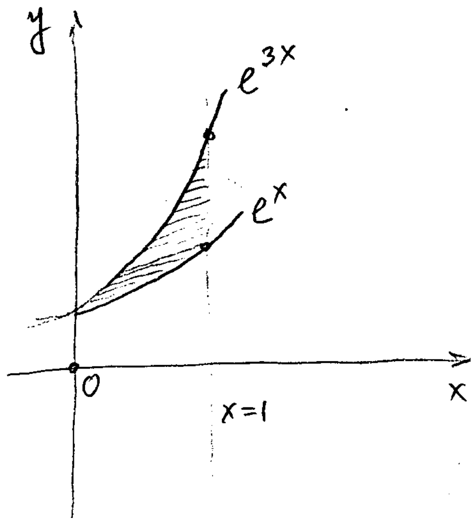
$$\lim_{x \to 2^-} e^{\frac{1}{x-2}}.$$

$$\lim_{x \to 2^-} (x-2) = 0^-$$

$$\Rightarrow \quad \lim_{x \to 2^-} \frac{1}{x-2} = -\infty$$

$$\Rightarrow \quad \lim_{x \to 2^-} e^{\frac{1}{x-2}} = 0$$

2. Find the area bounded by the curves $y = e^x$, $y = e^{3x}$, and $x = 1$.



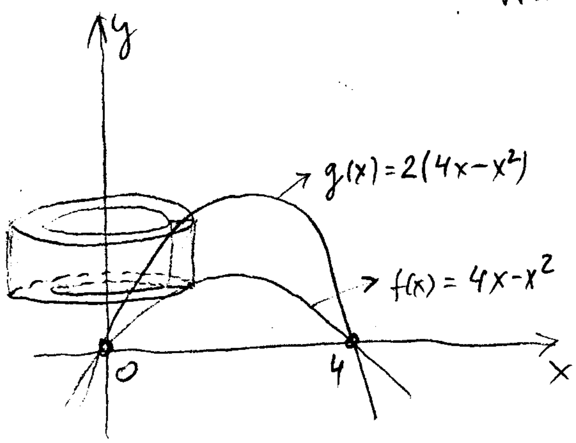
The area is $\int_0^1 (e^{3x} - e^x)\,dx = \int_0^1 e^{3x}\,dx - \int_0^1 e^x\,dx$

$= \frac{1}{3} e^{3x} \Big]_0^1 - e^x \Big]_0^1 = \frac{1}{3}(e^3 - e^0) - (e^1 - e^0)$

$= \frac{1}{3} e^3 - e + \frac{2}{3}$

3. Find the volume of the solid generated by rotating the region bounded by $y = 4x - x^2$ and $y = 2(4x - x^2)$ about the y-axis.

We use shells.



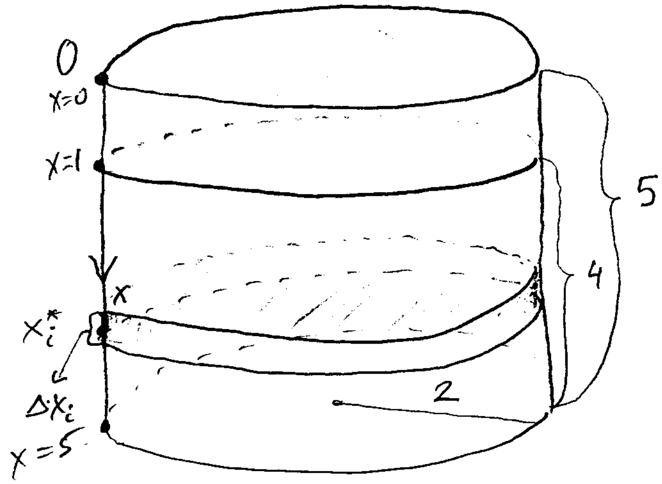
Solving $y = 4x - x^2 = f(x)$
$y = 2(4x-x^2) = g(x)$
we obtain $4x - x^2 = 0$,
so $x=0$ and $x=4$ are the first coordinates of the intersection points, and $y=0$ is the second coordinate.

Then the volume of rotation is

$$\int_0^4 2\pi x\,[g(x)-f(x)] = \int_0^4 2\pi x\,(4x-x^2) = 2\pi \int_0^4 (4x^2-x^3)\,dx$$

$$= 2\pi\left(\frac{4x^3}{3} - \frac{x^4}{4}\right)\Big]_0^4 = 2\pi\left(\frac{256}{3} - 64\right) = \frac{128}{3}\pi .$$

4. A tank has the shape of a right cylinder with height 5m and base radius 2m. It is filled with water to a height of 4m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The acceleration due to gravity is 9.8 m/s^2.)



Partition $[1,5]$ by points $1 = x_0 < x_1 < \dots < x_n = 5$ and choose x_i^* in the i-th interval $[x_{i-1}, x_i]$

Consider the i-th layer of water. It has

volume: $V_i = \pi 2^2 \Delta x_i = 4\pi \Delta x_i \quad m^3$

mass: $m_i = \text{density} \times \text{volume} = 1000 \cdot 4\pi \Delta x_i \quad kg$

The force required to raise this layer must overcome the force of gravity, so

$$F_i = m_i g = 4000\pi \Delta x_i (9.8) = 39{,}200\pi \Delta x_i \quad N$$

Each particle of the layer must travel a distance of approximately x_i^*. So the work done to raise this layer to the top is approximated by

$$F_i x_i^* = 39{,}200 \pi \Delta x_i \, x_i^* \quad N\text{-}m \ (J)$$

The total work is the limit of

$$\sum_{i=1}^{n} F_i x_i^* \quad \text{as } \|P\| \to 0, \text{ that is}$$

$$W = \lim_{\|P\| \to 0} \sum_{i=1}^{n} 39200\pi \, x_i^* \Delta x_i = \int_1^5 39200\pi \, x \, dx$$

$$= 39200\pi \int_1^5 x \, dx = 39200\pi \left. \frac{x^2}{2} \right]_1^5 = 470400\pi \ J$$

5. Evaluate

$$\int e^{3x} cosx \, dx.$$

$$I = \int e^{3x} \cos x \, dx = \int \underbrace{e^{3x}}_{u} \, d\underbrace{(\sin x)}_{v} \quad \text{(Integration by parts)}$$

$$= e^{3x}\sin x - \int \sin x \, d(e^{3x})$$

$$= e^{3x}\sin x - \int 3e^{3x}\sin x \, dx$$

$$= e^{3x}\sin x - 3\int e^{3x}\sin x \, dx \qquad \left(d(\cos x) = -\sin x \, dx\right)$$

$$= e^{3x}\sin x + 3\int \underbrace{e^{3x}}_{u} \, d\underbrace{(\cos x)}_{v} \quad \text{(Again Integration by Parts)}$$

$$= e^{3x}\sin x + 3\left[e^{3x}\cos x - \int \cos x \, d(e^{3x})\right]$$

$$= e^{3x}\sin x + 3\left[e^{3x}\cos x - \int 3e^{3x}\cos x \, dx\right]$$

$$= e^{3x}\sin x + 3e^{3x}\cos x - 3(3)\int e^{3x}\cos x \, dx$$

$$= e^{3x}\sin x + 3e^{3x}\cos x - 9I$$

So, $I = e^{3x}\sin x + 3e^{3x}\cos x - 9I$

Solving for I, we obtain

$$I = \frac{1}{10}\left(e^{3x}\sin x + 3e^{3x}\cos x\right) + C$$