TERM TEST ONE

CPSC 241, Winter 2000

Wednesday, Feb. 9, 2000

NAME AND STUDENT NUMBER:

1. Write down all your work.
2. Calculators are allowed, but not needed (and not recommended).
3. Maximum Possible Score = 50. The marks for each question are given in square brackets.

1. [1] (a) Define a language over the alphabet $\sum$.

[3] (b) Find two languages A and B such that $|AB| < |A||B|$.

[3] (c) Let A be a language. Define A^n, where $n \geq 0$ is an integer, A^+ and A^*.

2. The language A on the alphabet $\{a, b\}$ is recursively defined by

(a) ab is in A

(b) for any x, y in A we have

(i) $xy \in A$

(ii) $axb \in A$

[3] 1) Show that $aabaabbb$ is in A.

[2] 2) Prove that $aababbb$ is not in A.

[5] **3**. Let $\sum = \{a, b, c, d, e, f, g\}$. Determine the number of strings in $\sum^*$ with length six and exactly three d's.

4. [3] (a) Define a finite state machine.

[5] (b) Construct a finite state machine with input alphabet $I = \{o, p, q\}$ that recognizes each occurrence of *pop* in an input string $t \in I^*$ with an output of 1, and 0 otherwise. (A state diagram suffices.)

[5] **5**. Given an alphabet $\sum$, let $A, B, C \subseteq \sum^*$. Show that $BA \cup CA \subseteq (B \cup C)A$.

[9] **6**. Given are relations on the set $A = \{a, b, c\}$:
$R_1 = \{(a, a), (b, b)\}$;
$R_2 = \{(a, a), (b, b), (c, c)\}$;
$R_3 = \{(a, b), (b, a), (a, a), (b, b), (c, c)\}$;
$R_4 = \{(a, b), (b, c), (a, a), (b, b), (c, c)\}$;
$R_5 = A \times A$;
$R_6 = \{(a, c), (b, a), (b, c), (a, a), (b, b), (c, c)\}$

Fill in the table with Yes=1 or No=0.

	Reflexive	Symmetric	Antisymmetric	Transitive	Partial order	Equivalence relation
R_1						
R_2						
R_3						
R_4						
R_5						
R_6						

[5]**7**. Let $\mathcal{R}$ be the relation on $Z^+ \times Z^+$ defined by $(a, b)\mathcal{R}(c, d)$ if and only if $ad = bc$. Prove that $\mathcal{R}$ is an equivalence relation.

8. Let $A = \{1, 2, 4, 5, 10, 14, 20, 28\}$. Define a relation $\mathcal{R}$ on A by $x\mathcal{R}y$ if and only if x divides y.

[1](a) Draw the Hasse diagram for the poset $(A, \mathcal{R})$

[1](b) Is this poset a lattice? Explain.

[1](c) Let $B \subseteq A$. Define $lub\ B$.

[1](d) Find $glb\{10, 20, 28\}$.

[1](e) Find $glb\{20, 28\}$.

[1](f) Find $lub\{2, 10, 14\}$.