# THE THERMODYNAMIC EFFECTS OF SUBLIMATING, BLOWING SNOW IN THE ATMOSPHERIC BOUNDARY LAYER

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Abstract. A seasonal snowcover blankets much of Canada during wintertime. In such an environment, the frequency of blowing snow events is relatively high and can have important meteorological and hydrological impacts. Apart from the transport of snow, the thermodynamic impact of sublimating blowing snow in air near the surface can be investigated. Using a time or fetch-dependent blowing snow model named 'PIEKTUK' that incorporates prognostic equations for a spectrum of sublimating snow particles, plus temperature and humidity distributions, it is found that the sublimation of blowing snow can lead to temperature decreases of the order of 0.5 °C and significant water vapour increases in the near-surface air. Typical predicted snow removal rates due to sublimation of blowing snow are several millimetres snow water equivalent per day over open Arctic tundra conditions. The model forecast sublimation rates are most sensitive to humidity, as well as wind speed, temperature and particle distributions, with a maximum value in sublimation typically found approximately 1 km downstream from blowing snow initiation. This suggests that the sublimation process is selflimiting despite ongoing transport of snow by wind, yielding significantly lower values of blowing snow sublimation rates (nearly two-thirds less) compared to situations where the thermodynamic feedbacks are neglected. The PIEKTUK model may provide the necessary thermodynamic inputs or blowing snow parameterizations for mesoscale models, allowing the assessment of the contribution of blowing snow fluxes, in more complex situations, to the moisture budgets of high-latitude regions.

Keywords: Blowing snow, Mackenzie Basin, Saltation, Sublimation, Suspension.

# 1. Introduction

High-latitude regions of Canada, as well as many other parts of the world, are prone to frequent snowstorms or even ground blizzards during their lengthy winters (Stewart et al., 1995). These storms are often associated with sub-freezing temperatures and high winds, conducive to the development of blowing and drifting snow which may seriously reduce optical visibilities and impede the activities of local inhabitants and fauna. Blowing snow can also strongly influence the water budget in regions with seasonal snowcovers, through the transport and redistribution of snow by wind and the sublimation of airborne snow while in motion. This process



Boundary-Layer Meteorology 89: 251–283, 1998. © 1998 Kluwer Academic Publishers. Printed in the Netherlands. of snow sublimation may then act as a significant source of moisture and a sink of sensible heat in the high-latitude atmospheric boundary layer (ABL).

Although much research on the movement and properties of blowing and drifting snow has been conducted (e.g., Kind, 1981; Schmidt, 1982a), it is only recently that the potentially significant impact of blowing snow sublimation on the water budget of high-latitude regions has attracted wider interest. Dyunin et al. (1991) state, for instance, that the deforestation of northern lands may lead inevitably to their aridization as the wind is now capable of transporting snow out of the region while simultaneously stimulating the sublimation of blowing snow. Pomeroy et al. (1997), using a 'Distributed Blowing Snow Model', claim that as much as 28% of the winter snowfall in a small northern basin in the Arctic tundra was sublimated. In addition, Pomeroy and Gray (1994, 1995) have argued that the transport of snow in prairie fields may remove as much as 75% of the annual snowfall over a one kilometre long fallow field, with about half of this amount sublimating into the ABL. This implies that both the transport and sublimation of blowing snow are factors that can no longer be neglected in assessing the wintertime water budgets of wide, frozen areas such as the Arctic tundra and the Antarctic ice sheet (King and Turner, 1997; Bintanja, 1998; Cullather et al., 1998). In addition, blowing snow sublimation may have substantial effects on the heat and water vapour fluxes within the ABL, as suggested by King and Anderson (1994) and King et al. (1996) for Antarctica.

The purpose of this paper is to describe initial developments of a numerical model named "PIEKTUK" (an Inuktituk word for blowing snow, also spelt "PIQ-TUQ"), which incorporates particle size, temperature and moisture distributions for a column of blowing snow, in either a time or fetch-dependent mode. This investigation of blowing snow is conducted within the context of the ongoing Mackenzie GEWEX Study (MAGS), the Canadian contribution to the international Global Energy and Water Cycle Experiment (GEWEX), which closely examines the water budget of the entire Mackenzie River Basin (Krauss, 1995). Due in part to its northern location and the duration of its seasonal snowcover, the hydrology of the Mackenzie River Basin may be sensitive to the impacts of blowing snow (Lawford, 1993, 1994).

# 2. Numerical Model-PIEKTUK

A number of numerical models of blowing snow have been developed recently. Some have focused on the erosion, transport and deposition of snow in surface flows, perhaps more appropriate for engineering purposes (e.g., Liston et al., 1993; Uematsu, 1993; Moore et al., 1994). Others, meanwhile, have attempted to determine the significance of blowing snow as a hydrometeorological feature of open, windswept and snow-covered regions (e.g., Pomeroy, 1988; Pomeroy et al., 1993; Mobbs and Dover, 1993; Bintanja, 1998). For this study we have developed two versions of PIEKTUK: one is a time dependent (PIEKTUK-T), and the other a fetch-dependent (PIEKTUK-F), numerical model for a column of sublimating, blowing snow. PIEKTUK is innovative (cf., for instance, with the steady-state Prairie Blowing Snow Model or PBSM of Pomeroy, 1988) in that it considers the thermodynamic feedbacks of sublimation on all the predictive quantities, namely the temperature, moisture and particle distributions. Although some details of the PIEKTUK algorithm are found in Déry and Taylor (1996) and elsewhere, there have been modifications and a brief review of the blowing snow sublimation and diffusion equations follows.

#### 2.1. SUBLIMATION

Dyunin (1959) was one of the first scientists to emphasize the thermodynamic effect of wind on ice or snow particles which may undergo, in part, a phase change to water vapour. Using the results of Thorpe and Mason (1966), Schmidt (1972) derived a model describing this process, now known as the sublimation of blowing snow. The process, essentially analogous to diffusional growth or evaporation of water drops (see, for example, Rogers and Yau, 1989, or Pruppacher and Klett, 1997), is controlled by two principal factors: (1) the rate at which water vapour is removed from the snow particle, and (2) the amount of thermal energy being delivered to the same particle. The change in mass m (kg) of a blowing snow particle of radius r (m) due to sublimation is given by (Thorpe and Mason, 1966):

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \left(2\pi r\sigma - \frac{Q_r}{KN_{\mathrm{Nu}}T_a} \left[\frac{L_s}{R_v T_a} - 1\right]\right) / \left(\frac{L_s}{KN_{\mathrm{Nu}}T_a} \left[\frac{L_s}{R_v T_a} - 1\right] + R_v \frac{T_a}{N_{\mathrm{Sh}} De_i}\right), \quad (1)$$

where  $\sigma$  (dimensionless and negative) is the water vapour deficit with respect to ice  $(e - e_i)/e_i$ , where *e* and  $e_i$  are the vapour pressure and its value at saturation over ice,  $T_a$  the ambient air temperature (K), *K* the thermal conductivity of air  $(2.4 \times 10^{-2} \text{ W m}^{-1} \text{ K}^{-1})$ ,  $L_s$  the latent heat of sublimation  $(2.838 \times 10^6 \text{ J kg}^{-1})$ ,  $R_v$  the gas constant for water vapour (461.5 \text{ J kg}^{-1} \text{ K}^{-1}), and *D* the molecular diffusivity of water vapour in air  $(2.25 \times 10^{-5} \text{ m}^2 \text{ s}^{-1})$ . Dependence of the sublimation process on the ventilation velocities  $(V_r)$  and the kinematic viscosity of air  $(\nu)$  occurs through the Nusselt and Sherwood numbers, denoted respectively by  $N_{\text{Nu}}$  and  $N_{\text{Sh}}$ , both equal to  $1.79 + 0.606N_{\text{Re}}^{0.5}$ , with  $N_{\text{Re}} (= 2rV_r/\nu)$  being the Reynolds number. We take  $V_r$  to be equal to the terminal velocities of the individual suspended particles in still air (Schmidt, 1982b; Déry and Taylor, 1996). Horizontal particle velocity components are assumed equal to the horizontal wind speed. The net radiation transferred to the particle,  $Q_r(W)$ , is expressed by Schmidt (1991) as:

$$Q_r = \pi r^2 (1 - \alpha_p) Q_*, \tag{2}$$

where  $\alpha_p$  represents the shortwave particle albedo and  $Q_*$  the total incident radiation (W m<sup>-2</sup>).

From the rate of mass loss by sublimation (Equation (1)), the total sublimation rate  $Q_{subl}$  (kg m<sup>-2</sup> s<sup>-1</sup>) for a column of blowing snow over a unit horizontal land surface area is then found by (Equation (12) from Schmidt, 1982b):

$$Q_{\text{subl}} = \int_0^\infty \int_0^\infty N(z) f(r, z) \frac{\mathrm{d}m}{\mathrm{d}t} \mathrm{d}r \,\mathrm{d}z \tag{3a}$$

or

$$Q_{\rm subl} = \int_0^\infty q_{\rm subl} \, \mathrm{d}z,\tag{3b}$$

where N is the number of suspended snow particles in a unit volume (m<sup>-3</sup>), f is the relative frequency of a snow particle of radius r and  $q_{subl}$  is the local sublimation rate per unit volume (kg m<sup>-3</sup> s<sup>-1</sup>). Although by definition both  $Q_{subl}$  and  $q_{subl}$  are negative quantities, for simplicity we report sublimation rates with positive values in Section 3.

## 2.2. THERMODYNAMICS

Since the sublimation of blowing snow acts as a source of water vapour and a sink of sensible heat in the atmosphere, we take into account the heat required for the sublimation of blowing snow as well as the moisture added when proceeding forward from the time or point of blowing snow initiation.

The amount of heat dQ (J) per unit volume involved in sublimating an amount dw (mixing ratio change, dimensionless and positive) of airborne snow in time dt (= dx/U(z) in the fetch-dependent case) is given by:

$$\mathrm{d}Q = \rho_a L_s \,\mathrm{d}w,\tag{4a}$$

where

$$\rho_a \,\mathrm{d}w = -q_{\mathrm{subl}} \,\mathrm{d}t,\tag{4b}$$

is the corresponding change in water vapour mass. Some of the heat required is provided by radiation absorbed by the particles while the remainder is taken from both the air at temperature  $T_a$  and the snow particle, which, in the updated version of our model, is assumed to be at the ice bulb temperature,  $T_{wi}$ , so that

$$\mathrm{d}Q = -\rho_a c_p \,\mathrm{d}T_a - \rho_s c_i \,\mathrm{d}T_{wi} + \int_0^\infty Q_r Nf(r,z) \mathrm{d}r \,\mathrm{d}t, \tag{5a}$$

where  $c_p$  and  $c_i$  (J kg<sup>-1</sup> K<sup>-1</sup>) are the specific heats of dry air at constant pressure and of ice respectively, and  $\rho_a$  and  $\rho_s$  (kg m<sup>-3</sup>) are the dry air and snow drift densities. Following Dorsey (1940) and Langham (1980),  $c_i$  will vary with temperature as:

$$c_i = 2115 + 7.79(T_a - T_0), \tag{5b}$$

where  $T_0 = 273.15$  K. We use  $c_i$  at  $T_{wi}$  for the ice particles. We also use the ideal ice bulb relationship (Rogers and Yau, 1989), ignoring possible differences of the psychrometer constant, so that,

$$T_{wi} = T_a - \left(\frac{L_s}{c_p}\right) [w_s(T_{wi}) - w]$$
(6)

and the differential increment needed in Equation (5a) is

$$dT_{wi} = \frac{\left[dT_a + \left(\frac{L_s}{c_p}\right) dw\right]}{\left[1 + \left(\frac{L_s}{c_p}\right) \partial w_s / \partial T_{wi}\right]}.$$
(7)

The saturation mixing ratio  $w_s$  can be determined from

$$w_s = 0.622 \frac{e_i}{(P - e_i)} \approx 0.622 \frac{e_i}{P},$$
 (8a)

where P (Pa), the atmospheric pressure ( $\gg e_i$ ), is assumed constant. We use the approximate result (Rogers and Yau, 1989) that

$$e_i(T_a) = A \exp\left(-\frac{B}{T_a}\right),$$
(8b)

where  $A = 3.41 \times 10^9$  kPa and  $B = 6.13 \times 10^3$  K to obtain  $w_s(T_{wi})$  and  $w_s(T_a)$ ; the latter will be needed in order to compute the ambient relative humidity (RH<sub>a</sub>). [Note that in this study we apply the term 'relative humidity' implicitly referring to the relative humidity with respect to ice (RH<sub>a</sub>), although standard meteorological practice has the relative humidity computed with respect to water (RH), even at temperatures < 0 °C (King and Turner, 1997).]

To summarize this section, the sublimation of blowing snow is computed by integrating the change of mass over the spectrum of ice particles, then evaluating the corresponding change in both water vapour and temperature, as well as the vertically integrated sublimation rate. As such, it is, in theory, an improvement from the PBSM, which assumes a constant (with fetch and height) temperature and a constant (with fetch only) moisture profile (see Déry and Taylor, 1996, for a discussion on the sub-saturation profile assigned to low-level air in the PBSM). The Mobbs and Dover (1993) model is similar to PIEKTUK in that it considers the

effects of sublimation on the moisture profile, but different in that it neglects any feedbacks to the air temperature.

#### 2.3. SUSPENSION OF BLOWING SNOW PARTICLES

For the suspension of aeolian sediments such as sand or dust, one would consider a balance between the settling and turbulent diffusion processes only. However, sublimation of blowing snow will result in a particle distribution that is continually shifting towards smaller particle sizes. Thus, for snow blowing at time t above homogeneous terrain, we will assume that the snow particles of radius r(characterized by a bin subscript, i) satisfy (Shiotani and Arai, 1967):

$$\frac{\mathrm{d}F_i}{\mathrm{d}t} = \frac{\partial}{\partial z} \left[ K_s \frac{\partial F_i}{\partial z} \right] + S_i,\tag{9a}$$

with

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + V\frac{\partial}{\partial y} + W\frac{\partial}{\partial z},\tag{9b}$$

where  $F_i$  (=  $Nf_i$ , if  $f_i$  is the normalized distribution) is the absolute number density of particles (of average radius  $r_i$ ) in a size bin (m<sup>-3</sup> bin<sup>-1</sup>). Note that x and yare the horizontal directions and z the vertical direction associated with particle velocities U, V, and W respectively. In the steady-state, two-dimensional (2D) fetch-dependent case, we set  $\partial/\partial t = \partial/\partial y = 0$  whereas in the one (space) dimensional (1D), time-dependent case,  $\partial/\partial x = \partial/\partial y = 0$ . Note that if the vertical air velocity is zero we still have particle velocities,  $W = -\omega$ , the terminal velocities of the particles.

Other quantities in expression (9a) are  $K_s$  for the eddy diffusivity and the source/ sink term  $S_i$  associated with the transfer of particles through the spectrum as a result of sublimation. This can be approximated with finite differences as:

$$S_i \approx \left[ -F_{i+1} \frac{\mathrm{d}r_{i+1}}{\mathrm{d}t} + F_i \frac{\mathrm{d}r_i}{\mathrm{d}t} \right] / \Delta r,\tag{10}$$

where particle bins are of constant size  $\Delta r(m)$  and dr/dt (negative) is the time rate of change in particle radius due to sublimation  $(m s^{-1})$ . In the steady-state advective case dr/dt is replaced by U(z) dr/dx. The calculation of dr(i) is done by simply using Equation (1) to compute the loss of mass, dm, in time dt and then noting the corresponding reduction in radius, dr, of a particle of size  $r_i$ . It is assumed that the dt or dx step is sufficiently small that  $dr < \Delta r$ . This can cause problems for the smallest size bin (i = 1) where consideration of Equation (1) shows that rcan go to zero in a finite time with a singularity in dr/dt at that point. These very small particles do not contribute significantly to the budgets and so we simply set  $dr = \Delta r$  with complete removal of all particles from that size bin if  $dr_1 > \Delta r$ .

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For eddy diffusivity, we follow Rouault et al. (1991) and set

$$K_s = \frac{K_{s0}}{\left(1 + \frac{c_2\omega^2}{1.56u_*^2}\right)},$$
(11a)

to calculate the particle diffusion coefficient, where  $K_{s0}$  is the gas diffusion coefficient,

$$K_{s0} = u_* l(z) \tag{11b}$$

and the coefficient 1.56 arises from an assumed ratio between the vertical velocity variance  $\overline{\omega'^2}$  and  $u_*^2$ . Note that  $u_*$  is the friction velocity and the mixing length l(m) is specified through the equation

$$\frac{1}{l} = \frac{1}{\kappa(z+z_0)} + \frac{1}{l_{\max}}.$$
(11c)

In Equation (11c) above,  $\kappa$  (= 0.4) is the von Kármán constant,  $z_0$  the roughness length (or effective roughness length), and  $l_{max}$  a constant usually set equal to 40 m. Values of  $u_*$  and  $z_0$  are discussed in Section 3. In Equation (11a),  $c_2$  is the 'counter diffusion' parameter, and when it is equal to zero, then  $K_s = K_{s0}$ . With an increase in  $c_2$ ,  $K_s$  becomes smaller and, through the settling velocity  $\omega$ , is a function of particle size. In the PIEKTUK calculations presented here,  $c_2$  is taken as unity. This is based on tests with values between 0 and 10. We found that there are only small changes between results with  $c_2 = 4$  and 10, and with  $c_2 = 1$  we obtain values about midway between those with values of 0 and 4. In their spray droplet study, Rouault et al. (1991) selected  $c_2 = 5$  on the basis of comparisons with droplet volume spectra but other factors are involved and we felt that a value of 1.0 was representative. In this case Figure 1a shows the profile of  $K_s$  for particle radii of 50 and 100  $\mu$ m. Figure 1b shows that the larger the particles, the smaller the K<sub>s</sub> value. We note as well that Sommerfeld and Businger (1965) argued that particle eddy diffusivities could be an order of magnitude *larger* than the eddy viscosity (which would lead to  $c_2 < 0$ ; however their conclusion is based on limited data and has sometimes been considered incorrect (Radok, 1968) but see Dyer and Soulsby (1988). Other studies (e.g., Mobbs and Dover, 1993; King et al., 1996), indicate  $K_{s} = K_{s0}$ .

For settling velocity, we initially used Pomeroy's (1988) formula,

$$\omega(r) = 1.1 \times 10^7 r^{1.8},\tag{12a}$$

to describe the dependence of  $\omega$  on particle size, where *r* is in metres and  $\omega$  in m s<sup>-1</sup>. However, this was formulated for mean settling velocity and mean particle size and predicts significantly higher settling velocities for large (>100  $\mu$ m radius) particles than those obtained from the drag formulae proposed by Carrier (1953)



*Figure 1.* (a)  $K_s$  profiles for the standard case and different particles sizes, following Equation (11a). (b)  $K_s$  as a function of particle radius at z = 1 m, following Equation (11a).



*Figure 2*. The variation of terminal velocity with particle radius using the Carrier (1953) and Pomeroy (1988) formulae.

and others. In the results to be presented here we determine settling velocities by assuming the Carrier formula for the drag coefficient,

$$C_d = \frac{24}{N_{\rm Re}} \left(1 + 0.0806 N_{\rm Re}\right). \tag{12b}$$

Settling velocities from the Carrier and Pomeroy formulae are shown in Figure 2.

Equations (9)–(12), plus the specification of the velocity profile, essentially describe our model for snow particles of radius r in suspension. We solve the diffusion equation for each particle size bin individually and then sum the particle frequencies to obtain the snow drift density and sublimation rate at each height. The excess of diffusion of snow particles from the saltation/suspension layer interface to higher levels over settling back to the surface will replenish the concentration of snow particles that are being sublimated in the suspension layer.

## 2.4. VELOCITY PROFILE AND DIFFUSION OF THERMODYNAMIC QUANTITIES

The diffusion process is not limited to the snow particles and advection and diffusion of potential temperature and moisture will also occur. We take the eddy diffusivity coefficients for temperature and moisture to be the same as the gas diffusivity (see Equation (11b)), and we also make no distinction between temperature

and potential temperature in the snow microphysics equations above. Following sublimation, heat and moisture will be redistributed within a column of blowing snow. Cooling and moistening of the air will extend to higher levels than the suspended snow since the snow particles are also subject to settling.

For near-neutral atmospheric conditions over homogeneous terrain, U(z) is determined by the log-law velocity profile, as in the Prairie Blowing Snow Model or PBSM, described by Pomeroy et al. (1993) (hereafter P93). We set:

$$U(z) = \frac{u_*^{\text{eff}}}{\kappa} \ln\left[\frac{z+z_0}{z_0}\right],\tag{13a}$$

where  $u_*^{\text{eff}}$  is an effective friction velocity (m s<sup>-1</sup>) that takes into account flow density effects in the suspension layer (Pomeroy and Male, 1987; Pomeroy, 1988). Thus,

$$u_*^{\text{eff}}(z) = u_* \left(\frac{\rho_a}{\rho_a + \rho_s}\right)^{0.5}.$$
 (13b)

This is not entirely consistent with a constant stress layer or Equation (11c), but the logarithmic profile is consistent with observations and differences will be small for most heights. The assumed constant shear stress throughout the layer has stress equal to  $\rho_a u_*^2$ . In this context note that a test with  $l_{\text{max}} = 200$  m and  $\infty$  will be discussed below.

In the present model we assume that there are no changes to the horizontal velocity, or to the turbulence as a result of either the presence of suspended snow or of thermal stratification since, in high winds, we anticipate that density stratification and related effects will be small (see Déry and Taylor, 1996).

### 2.5. INITIAL AND BOUNDARY CONDITIONS

Values of the initial (x or t = 0) and boundary conditions (z) must be assigned for the calculations in the suspension layer. In the present work we set the top boundary,  $z_{ub}$ , to be equal to 1000 m, well above the heights that we expect to be influenced over the times or fetches considered here. At that level temperature and humidity gradients ( $\partial/\partial z$ ) and the particle frequencies are assumed zero.

Lower boundary conditions are imposed at a height  $z_{lb}$  (m), and we follow the method adopted in the PBSM (P93) by assuming a lower boundary for the suspension layer to be at:

$$z_{lb} = \left[ z_r^{-0.544} + \left( \ln \frac{\rho_{\text{salt}}}{\rho_r} \right) / 1.55 \right]^{-1.838}, \tag{14}$$

where  $\rho_r$  is the snow drift density at a reference height  $z_r$  (= 0.05628 $u_*$ ) and we take  $\rho_r = 0.8$  kg m<sup>-3</sup> as in P93;  $\rho_{salt}$  is the mean saltation snow drift density

(kg m<sup>-3</sup>). Note that  $z_{lb}$  does not necessarily coincide with  $h_s$ , the saltation layer height (see Déry and Taylor, 1996). Modelling the saltation layer as well as the suspension layer might be a preferable approach but the processes within that layer are extremely complex (e.g., Pomeroy and Gray, 1990; Anderson and Haff, 1991). The empirical Equation (14) used above, and the particle size distribution assumed for that level, are based on an examination of observational data by Pomeroy (1988).

We take the initial particle frequencies and the snow drift density in the suspension layer to be zero except at  $z_{lb}$  where they are assumed to equal the values in the saltation layer. This differs from the approach adopted in the PBSM, which assumes a specified vertical distribution of suspended particles at a fetch of 300 m as the starting point for downwind application of the model. In our saltation layer the value of the total snowdrift density,  $\rho_{salt}$  (kg m<sup>-3</sup>), is dependent on the current and threshold surface shear stresses such that (Pomeroy, 1988):

$$\rho(z_{lb}) = \rho_{\text{salt}} = 0.4615 \left[ 1 - \frac{u_{*t}^2}{u_*^2} \right] / u_*, \tag{15}$$

where  $u_{*t}$  (m s<sup>-1</sup>) is the threshold friction velocity for wind transport of snow. The saltation layer is assumed to be fully-developed at *x* or t = 0 and invariable, such that it provides a continual source of particles for the suspension layer. The size distribution of blowing snow particles has been observed to follow the 2-parameter gamma distribution (Budd, 1966; Schmidt, 1982b). Thus, at  $z_{lb}$  we assign the normalized number density, f, to be:

$$f(r, z_{lb}) = \frac{r^{\alpha - 1} \exp(-r/\beta)\beta^{-\alpha}}{\Gamma(\alpha)},$$
(16)

where  $\alpha$  (dimensionless) and  $\beta$  (m) are the shape and scale parameters of the gamma distribution, and  $\Gamma$  denotes the gamma function. In the saltation layer, it is common to assume that  $\alpha = 5$  and that  $r_m$ , the mean particle radius, is 100  $\mu$ m (Pomeroy, 1988) although variation is these parameters can be expected with changing environmental conditions. Recent analyses of Antarctic data by the University of Leeds group have suggested, however, values of  $\alpha = 2$  and  $r_m = 75 \ \mu$ m (Mobbs, personal communication). Tests with both values will be described. Since the gamma distribution also has the property that  $r_m = \alpha\beta$ , then  $\beta$  and subsequently the particle size distribution at  $z_{lb}$ , are easily found.

The gamma distribution in Equation (16) is normalized such that its integration over the particle spectrum is unity. Since the particle frequency must satisfy the following relationship (Pomeroy and Male, 1988):

$$\rho_{\text{salt}} = \frac{4}{3} \pi \rho_i N(z_{lb}) \int_0^\infty f(r, z_{lb}) r^3 \,\mathrm{d}r,\tag{17}$$

in which we take the density of ice  $\rho_i = 900$  kg m<sup>-3</sup>, we can then obtain the total number of particles per unit volume,  $N(z_{lb})$  and hence  $F_i(z_{lb})$ . By taking  $\alpha = 5$ 

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and a nominal 10-m wind speed  $U_{10} = 15 \text{ m s}^{-1}$  with threshold for transport  $U_{10t} = 5 \text{ m s}^{-1}$ , we obtain, following P93, values of  $u_* = 0.75 \text{ m s}^{-1}$  and  $u_{*t} = 0.18 \text{ m s}^{-1}$ , and then find that  $N(z_{lb}) = 9.09 \times 10^7 \text{ m}^{-3}$  while with  $\alpha = 2$  we have  $N(z_{lb}) = 5.09 \times 10^7 \text{ m}^{-3}$ .

At  $z_{lb}$  we assume that the surface is insulating and the temperature gradient,  $\partial T_a/\partial z = 0$ . However, there will certainly be situations where the surface sensible heat flux is different from zero and other conditions are appropriate. The relative humidity is fixed at 100%.

## 2.6. NUMERICAL ASPECTS

Once the background environmental conditions are assigned, PIEKTUK first determines the blowing snow transport rate for the steady-state saltation layer and the boundary conditions on suspended snow particles for the lowest level of the suspension layer (as described above). The model calculations proceed by marching forward from t or x = 0, computing profiles of the concentrations in each size bin plus temperature and absolute humidity at each step. These are obtained by an implicit (in x or t) finite difference scheme. A transformation from z to  $\zeta = \ln [(z + z_0)/z_0]$  is used for the vertical coordinate and central differences are used in  $\zeta$ . This allows for higher accuracy near the lower boundary of the suspension layer. Convergence tests were made to verify independence of results relative to  $\Delta \zeta$  and  $\Delta x$  or  $\Delta t$  and satisfactory agreement was obtained with analytic solutions for cases with no sublimation and  $l_{max} = \infty$ , i.e.,  $l(z) = \kappa(z + z_0)$ .

### 2.7. OTHER PROCESSES

The diffusion, suspension and sublimation of blowing snow, as we have seen, can be rather complex to describe. Nonetheless, other processes that may affect windblown snow such as the precipitation of new snow, or the collision and fractionation of snow particles, have not been included so far. They do, however remain important processes that may need to be considered for the precise modelling of blowing snow. The precipitation of snow will seriously complicate the situation by adding snow particles or snowflakes with different properties to the suspended, blowing snow, at the top boundary of the model. The collision of particles while in transport may also produce a higher number of relatively small particles than would otherwise be expected.

The representation of turbulent transport by a diffusion process is always open to criticism as an over-simplification of an extremely complex process and we recognize this limitation of the model. The model results to be presented below are critically dependent on the lower boundary conditions specified, especially those relating to the particle size distribution, F(r, z) and the height at which it is applied,  $z_{lb}$ . The empirical approach used here was adopted more from necessity than conviction and we consider this as the weakest link in the model's construction.

Despite these limitations we believe that the present model can provide some insight into the thermodynamic impacts of blowing snow in the ABL and we present some sample results in the following sections. Our initial application of the model will be for simple, 1D, step changes from non-blowing to blowing snow over flat terrain. In nature many situations will involve continuous, 2D, changes in terrain, topography, and the nature of the snowcover. All of these factors may need to be considered at some point in the future.

## 3. Results

We present results for the steady-state, fetch dependent model (PIEKTUK-F). Results from PIEKTUK-T are quite similar, after allowing for the transposition of time and fetch. PIEKTUK-T results are relevant to the onset of high winds with blowing snow over vast snow-covered plains (e.g., in Antarctica), while PIEKTUK-F results are more applicable to limited areas of blowing snow.

## 3.1. BASIC MODEL RESULTS

We use the following environmental conditions as the basic state for our 'standard' model run:  $T_a = -10^{\circ}$ C with initial RH<sub>a</sub>, or RH<sub>a0</sub> = 70% except at  $z_{lb}$  and below where RH<sub>a</sub> remains fixed at 100%, with  $Q_* = 120$  W m<sup>-2</sup> and  $\alpha_p = 0.1$ . Following Pomeroy (1988), a nominal 10-m wind speed is also selected from which quantities such as  $u_* (= 0.02264U_{10}^{1.295})$  and  $z_0 (= 0.06u_*^2/g)$  are derived. As in Section 2.5, we choose  $U_{10} = 15$  m s<sup>-1</sup>, giving  $u_* = 0.75$  m s<sup>-1</sup> and  $z_0 = 0.0035$  m, while the initial particle concentration is zero at all heights except at the saltation/suspension layer interface. The usual particle bin size  $\Delta r$  is 4  $\mu$ m and the sublimation calculations are performed for particles of mean radius for each bin from 2 to 254  $\mu$ m.

The evolution of the snow drift density ( $\rho_s$ ) profile is first examined in Figure 3. Note the log scales and the ranges of both axes. This figure shows that a steady state between the processes of diffusion, settling and sublimation of blowing snow particles is soon ( $x \approx 100$  m) reached near the surface (z < 1 m). However, snow particles require a longer fetch to diffuse to higher levels where  $\rho_s \rightarrow 0$  and a steady-state profile is unlikely to be attained even after a fetch of 10 km for blowing snow. For comparison, the limiting case in which settling balances the diffusion of snow particles, i.e. excluding sublimation, is determined analytically and included in Figure 3. Note that this analytical solution, based on the summation of the classic power law solution for single size particles (Anderson and Hallet, 1986), is based on  $l(z) \approx \kappa (z + z_0)$ , rather than Equation (11c). However, the analytic solution corresponds well with our modelled results for z < 10 m where the sublimation process is still effective, for the fetches considered here. Note that the inclusion of



*Figure 3.* The profiles of blowing snow drift density forecast by PIEKTUK at x = 0.1, 1.0 and 10.0 km for our standard case with  $U_{10} = 15 \text{ m s}^{-1}$ ,  $T_a = -10^{\circ}\text{C}$  and  $\text{RH}_{a0} = 70\%$ . The analytical profile without sublimation ('ANALYSIS') is also shown.

a sublimation term in Equation (9a) may also imply that, in an equilibrium or near equilibrium situation with ongoing sublimation of wind-blown snow, a net vertical flux of snow particles is to be expected (Kind, 1992).

Figure 4 depicts the downwind development of ambient air temperature profiles after the onset of blowing snow at x = 0. The profiles show, as expected, that near the surface  $T_a$  decreases with fetch under a weak thermal inversion before reverting back to -10 °C aloft. At x = 10 km, the temperature near the surface has decreased by about 0.55 °C. ABL observations of temperature also suggest that thermal inversions are common in blowing snow events (Budd et al., 1966; Schmidt, 1982b). Note that these temperature decreases represent the additional cooling and the evolution of increased temperature gradients as the result of sublimation. In practice there may also be a flux of heat at the ground leading to modified cooling rates and temperature gradients.

Changes in the moisture profiles are significant, with the mixing ratio w and RH<sub>a</sub> profiles increasing with fetch. Figure 5a shows that, within the blowing snow,  $w_s$  decreases with fetch as a result of lower air temperatures; on the other hand, w increases as ice particles are converted into water vapour. At x = 10 km, we see



*Figure 4*. The profiles of temperature forecast by PIEKTUK at x = 0.5, 1.0, 5.0 and 10.0 km, for the standard case.

from Figure 5b that the near-surface air (z < 2 m) is nearly saturated (RH<sub>a</sub> > 95%) while RH<sub>a</sub> is > 90% for z < 10 m. Figure 6 shows the predicted evolution of temperature and relative humidity (with respect to ice) with x at  $z \approx 1$  m and 10 m. One can see that near the surface (z = 1 m) RH<sub>a</sub> increases rapidly from 70% to  $\approx$  90% in the first few hundred metres and then slowly climbs towards saturation. At z = 10 m the humidity has risen to approximately 90% by x = 10 km. These results are in contrast to the relative humidity assumptions made within the PBSM and, in that sense, are one of the more controversial predictions.

In Figure 7 we can see the particle distributions assumed at  $z = z_{lb}$  and predicted at  $z \approx 1.0$  m and x = 1 km, with and without sublimation. In addition the analytical distributions for infinite fetch with no sublimation are shown for comparison (analysis). First note the change in particle distributions with height, with fewer ice particles as z increases and particle distributions tending towards smaller sizes. For  $z \approx 1.0$  m, we see that the particle distribution affected by sublimation is shifted towards the left as sublimation leads to an increased number of relatively small particles and a decreased number of relatively large particles (note the change in radius and number scales in the figures). This result was anticipated by Déry and Taylor (1996) who have shown the effects of sublimation alone on the evolution of a blowing snow particle distribution. Mean particle radius ( $r_m = N^{-1} \int r F(r) dr$ )



*Figure 5.* The profiles of (a) mixing ratio and saturation mixing ratio, and (b) relative humidity with respect to ice forecast by PIEKTUK at x = 1.0, 5.0 and 10.0 km, for the standard case.



*Figure 6.* The evolution with fetch of ambient air temperature  $(T_a)$  and relative humidity  $(RH_a)$  at  $z \approx 1$  m and  $z \approx 10$  m above the blowing snow surface, for the standard case.

is plotted as a function of z at a fetch of 1 km in Figure 8. Also plotted are results from our computations with a version of the PBSM and the mean radii for the 'analysis' solution with infinite fetch and no sublimation. The PBSM uses results from Pomeroy and Male (1992), who developed a power law relationship based on Schmidt's (1982b) field measurements of blowing snow, given by:

$$\bar{r} = 4.6 \times 10^{-5} z^{-0.258},\tag{18}$$

where  $\bar{r}$  and z are in metres. This gives larger mean particle sizes compared to the modelled and analytical solutions in the lowest metre or so of the ABL, and smaller mean sizes above (although there are very few particles there and the lower section is more significant). The 'analysis' solution has larger mean particle radii than PIEKTUK for z > 1 m, presumably because sublimation reduces the size of all particles at these levels.

For a fetch of 1 km and varying 10-m nominal wind speeds, we now present in Figure 9 the vertically integrated horizontal transport rate of blowing snow  $QT_{susp}$  (kg m<sup>-1</sup> s<sup>-1</sup>) in the suspension layer determined from

$$QT_{\text{susp}} = \int_{z_{lb}}^{z_{ub}} U(z)\rho_s(z)\,\mathrm{d}z,\tag{19}$$

together with snow sublimation rates in the suspension layer, calculated as in Equation (3a). As expected there is a very strong, non-linear dependence on wind speed.



*Figure 7.* The particle distribution assigned at  $z_{lb}$  as well as the distribution of snow particles determined by PIEKTUK at x = 1.0 km, with or without sublimation, for  $z \approx 1.0$  m for the standard case.



*Figure 8.* The profiles of mean particle radii in the PBSM and calculated by PIEKTUK at x = 1 km, and the analytical solution without sublimation, labelled 'ANALYSIS', for the standard case.

Table I provides some of the values for the transport and sublimation rates in the suspension layer for  $U_{10} = 10$ , 15, 20 and 25 m s<sup>-1</sup> from both PIEKTUK and from our version of the PBSM. Note the significantly lower predictions of the sublimation rate from the PIEKTUK model for strong winds. The PBSM and PIEKTUK forecasts of transport rates in the saltation layer are identical as we assume the same constant snow drift density and particle velocities within this layer. Transport rates in the suspension layer differ as a result of differences in predicted or assumed snow drift density between the two models.

We now consider the effect of the predicted changes in temperature and humidity on the vertically integrated sublimation rate as a function of downwind fetch (Figure 10) for our standard case. The curve labelled 'FTW' (fixed temperature and mixing ratio) shows the PIEKTUK forecasts of  $Q_{subl}$  when the thermodynamic quantities are kept fixed at their assigned initial (x = 0) values. Results show that the inclusion of the effects of sublimation on the relative humidity distribution in our blowing snow model leads to much reduced sublimation rates in the suspension layer. In the FTW run, and in the PBSM, the temperature and moisture profiles remained unchanged with increasing x from their initial values. In this situation, our model predicts that  $Q_{subl}$  approaches a constant of about 0.4 mm h<sup>-1</sup> snow water equivalent (swe). For large x, our FTW model should have an asymptotic



*Figure 9.* The vertically integrated transport and sublimation rates for the suspension layer of a column of blowing snow with varying wind speed at x = 1 km, as determined by PIEKTUK with RH<sub>a0</sub> = 70% and  $T_a = -10^{\circ}$ C.

#### TABLE I

The transport rates for the saltation layer  $(QT_{salt})$  and the suspension layer  $(QT_{susp})$  as well as the vertically integrated sublimation rates  $(Q_{subl})$  predicted by PIEKTUK and by our version of the PBSM, for varying nominal 10-m wind speeds  $(U_{10})$  at a fetch of 1 km

Model	$U_{10}$ (m s <sup>-1</sup> )	$QT_{salt}$ (g m <sup>-1</sup> s <sup>-1</sup> )	$\begin{array}{l} QT_{\rm susp} \\ ({\rm g}\;{\rm m}^{-1}\;{\rm s}^{-1}) \end{array}$	$Q_{subl}$ (mm h <sup>-1</sup> swe)
PBSM	10.0	5.92	16.69	0.0693
PIEKTUK			41.59	0.0324
PBSM	15.0	11.21	128.5	0.7552
PIEKTUK			175.8	0.1277
PBSM	20.0	16.80	672.7	3.905
PIEKTUK			538.8	0.2938
PBSM	25.0	22.71	2329.0	12.32
PIEKTUK			1378.1	0.5213

<i>u</i> *	<i>z</i> <sub>0</sub>	$\rho_{\rm salt}$	$z_{lb}$
$(m s^{-1})$	(m)	$(\text{kg m}^{-3})$	(m)
0.45	0.0012	0.86	0.0248
0.75	0.0035	0.58	0.0456
1.10	0.0070	0.41	0.0741
1.46	0.0130	0.31	0.1127
	$ \begin{array}{c}  u_{*} \\  (m  \mathrm{s}^{-1}) \\  0.45 \\  0.75 \\  1.10 \\  1.46 \end{array} $	$\begin{array}{ccc} u_{*} & z_{0} \\ (m \ s^{-1}) & (m) \\ \hline 0.45 & 0.0012 \\ 0.75 & 0.0035 \\ 1.10 & 0.0070 \\ 1.46 & 0.0130 \\ \end{array}$	$\begin{array}{cccc} u_{*} & z_{0} & \rho_{salt} \\ (m \ s^{-1}) & (m) & (kg \ m^{-3}) \end{array}$ 0.45 0.0012 0.86 0.75 0.0035 0.58 1.10 0.0070 0.41 1.46 0.0130 0.31

TABLE II

Assumed variation of  $u_*$ ,  $z_0$ ,  $\rho_{salt}$ , and  $z_{lb}$  with wind speed

steady state with a local balance between upward diffusion, downward settling and sublimation effects ( $S_i$ ), and a constant vertically integrated sublimation rate. This would not be the case with the PBSM model, which has a continually deepening suspension layer and a sublimation rate that continues to increase with x.

When the temperature and humidity are allowed to vary, we have just seen that, with increasing x, the near-surface air tends to become saturated and sublimation is constrained. We refer to this as 'self-limitation'. In the basic PIEKTUK prediction, beyond  $x \approx 1$  km,  $Q_{subl}$  starts decreasing with fetch and is about one third of the value predicted with fixed  $T_a$  and w profiles at x = 10 km. This would indicate that both our FTW calculations and the PBSM, in which the feedback from sublimating blowing snow on the temperature and humidity fields is not considered, may significantly over-estimate blowing snow sublimation rates. This will be especially so at long fetches, but will also apply at the short fetches (of order 1 km) for which the PBSM was developed and tested.

The differences between PIEKTUK and PBSM results for  $Q_{subl}$  listed in Table I arise partly from the humidity change factor and partly from the differences between our predicted variations of particle size distribution with height and those assumed, based on observations, in the PBSM. They are included to indicate the magnitude of the differences, even at a fetch as short as 1 km.

In the PIEKTUK model with sublimation affecting the temperature and humidity fields, we assume that the asymptotic situation far downstream would eventually depend upon the upper boundary conditions imposed on temperature and humidity, but could have 100% relative humidity and minimal sublimation (not zero because of the radiation term in Equation (1)). The boundary layer would, however, have exceeded our constant flux layer assumption and we should not place too much emphasis on this asymptotic behaviour.

Model predictions of local sublimation rate,  $q_{subl}$ , are shown for  $z > z_{lb}$  in Figure 11 to vary considerably with height and fetch. Near the point of blowing snow initiation, relative humidities are still low near  $z_{lb}$ , allowing for relatively high rates



*Figure 10.* The vertically integrated sublimation rates for the suspension layer of a column of blowing snow with varying fetch for  $U_{10} = 15 \text{ m s}^{-1}$ , as determined by PIEKTUK. See text for details on the curve labelled 'FTW'.

of  $q_{subl}$ . However, the moisture content increases with fetch and this corresponds to lower values of  $q_{subl}$  near the surface. For larger z we require more time or fetch before significant numbers of particles diffuse upwards. The competing processes are illustrated at z = 10 m where  $q_{subl}$  is higher for a fetch of 1 km than for either 0.1 km (few particles present at that level) or 10 km (humidity at 10 m increased to reduce sublimation). Nonetheless we note that in all three cases, the maximum in  $q_{subl}$  is associated with the significant snow drift densities near the saltation layer. Note that, even at  $z_{lb}$  where the air is assumed to be saturated, the sublimation rate can be above zero as a result of the radiation term in Equation (1). In general, however, with unsaturated air, the radiation term will be small.

The perturbations in sensible  $(Q_H)$  and latent  $(Q_E)$  heat flux profiles caused by the sublimation of blowing snow are shown in Figure 12 at x = 1 and 10 km. PIEKTUK assumes, at the point of blowing snow initiation, no sensible or latent heat flux (except at  $z_{lb}$ ). As expected, the heat fluxes are of opposite signs. There is a maximum in  $Q_E$  of about 90 W m<sup>-2</sup> and a minimum  $Q_H$  of approximately -40 W m<sup>-2</sup> at x = 1 km and  $z \approx 10$  m, while the  $Q_E$ ,  $Q_H$  extrema are approximately 74 W m<sup>-2</sup> and -45 W m<sup>-2</sup> at x = 10 km and  $z \approx 40$  m. The contribution of blowing snow sublimation to the heat fluxes may therefore be significant, partic-



*Figure 11.* The local sublimation rates  $(q_{subl})$  for the suspension layer of a column of blowing snow at x = 0.1, 1 and 10 km, for the standard case.

ularly for cold Arctic or sub-Arctic environments where in winter-time, nights are long and daytime incoming solar radiation at the surface may not reach 100 W m<sup>-2</sup>. Energy to sublimate the snow is essentially being drawn from the air above the blowing snow while latent heat is being added there. A simple picture is of sensible heat diffusing down from above into the suspended snow region, sublimating snow and being converted to latent heat then diffusing upwards, out of the blowing snow 'processor'. There is a net upward flux of total heat ( $Q_H + Q_E > 0$ ) contributed by latent heat flux from the lower boundary, the cooling of air in the suspension layer plus any radiation absorbed by the blowing snow.

Note that the surface boundary conditions (Section 2.5) assume no sensible heat flux but there may be a latent heat flux. The latent heat flux at the lower boundary of the model can be thought of as emanating from the saltation layer. Without saltation the effective roughness length and  $u_*$  values would be smaller and these fluxes would be reduced.

## 3.2. SENSITIVITY EXPERIMENTS I – INITIAL AND BOUNDARY CONDITIONS

In developing a numerical model such as PIEKTUK, it is essential to examine the sensitivity of the output to the assigned background, initial and boundary condi-



*Figure 12.* The perturbations in sensible  $(Q_H)$  and latent  $(Q_E)$  heat fluxes as a result of blowing snow sublimation forecast by PIEKTUK in a column of blowing snow, 1 and 10 km downwind from blowing snow initiation, for the standard case.

tions and to the model assumptions. In this section, we test the model by varying the background, initial and lower boundary conditions on temperature, relative humidity, wind speed and incoming solar radiation. Variation in the top boundary conditions do not affect the results presented here. In Section 3.3 we address the sensitivity to some of the model's assumptions. Parameters other than those being adjusted are the same as in our standard case described in Section 3.1.

Figure 13 presents the vertically integrated sublimation rates in the suspension layer of blowing snow with increasing fetch and varying initial temperature (constant with height at x = 0) but with all other parameters as before. As shown by Pomeroy and Gray (1995), sublimation rates are reduced with decreasing ambient air temperatures due to reductions in  $e_i(T_a)$ . In our results there are variations in  $Q_{\text{subl}}$  by a factor of about five between the -1 °C and the -30 °C curves.

The model is also quite sensitive to the relative humidity assigned to the ABL. Figure 14 shows the results when the initial relative humidity with respect to ice  $(RH_{a0})$  is set to a constant value throughout the whole blowing snow column but varies for  $z > z_{lb}$  and x > 0. At  $z_{lb}$  we assume  $RH_a = 100\%$  as before. As expected sublimation rates decrease with increasing relative humidity but the reduction is not simply proportional to  $(100 - RH_{a0})$ , as it would be if relative humidities were



*Figure 13.* The vertically integrated sublimation rates forecast by PIEKTUK in the suspension layer of a column of blowing snow for varying fetch. The environmental parameters are as before ( $U_{10} = 15 \text{ m s}^{-1}$ , RH<sub>a0</sub> = 70%), except for the ambient air temperature (-1, -10, -20 and -30°C) assigned to the ABL at x = 0.

invariant with x and z. Even with  $RH_{a0}$  as low as 30% we are unable to increase sublimation rates to those obtained from the FTW calculations and a  $RH_a$  of 70% (Figure 10).

By controlling the snow drift density profile, the wind speed profile is a key determining factor in predicting the sublimation rates. Model results are shown in Figure 15 for 10 m s<sup>-1</sup>  $\leq U_{10} \leq 25$  m s<sup>-1</sup>. Higher wind speeds promote the turbulent diffusion of snow particles to higher elevations than at low wind speeds, thus increasing  $Q_{\text{subl}}$ . There is an impact through the lower boundary conditions, since the snow drift density at the lower boundary (Equation (15)) reduces with  $u_*$ , and thus wind speed, for sufficiently high values ( $u_* > \sqrt{3}u_{*t} = 0.31$  m s<sup>-1</sup>,  $U_{10} > 7.54$  m s<sup>-1</sup>). There is, however, an added complication due to the variation in the height of  $z_{lb}$  with  $\rho_{\text{salt}}$ .

Further tests examined the variation in sublimation rates with the particle albedo and incoming solar radiation. We found that increasing  $Q_*$  by factor of three, from 120 W m<sup>-2</sup> to 400 W m<sup>-2</sup> could increase  $Q_{subl}$  by about 20% with zero albedo, while reducing  $Q_*$  to zero (or setting the albedo to unity) lowered  $Q_{subl}$  by



*Figure 14.* As in Figure 13 with  $T_a = -10^{\circ}$ C,  $U_{10} = 15 \text{ m s}^{-1}$  and with RH<sub>a0</sub> values of 30, 50, 70 and 90% at x = 0 for  $z > z_{lb}$ .

about 8%. These changes are not usually as important as changes in temperature, humidity or wind speed, as noted by Pomeroy and Gray (1995).

# 3.3. SENSITIVITY EXPERIMENTS II - MODEL ASSUMPTIONS

The choice of settling velocity can lead to substantially different sublimation rates. With a lower settling velocity, a particle may stay longer in the suspension layer and the sublimation rate may increase. To counter this, a lower settling velocity, and ventilation velocity,  $V_r$ , leads to less sublimation. When the particle radius is 120  $\mu$ m, the settling velocity is 0.94 m s<sup>-1</sup> in Pomeroy's formula (Equation (12a)) and 0.78 m s<sup>-1</sup> in Carrier's (Equation (12b)), with corresponding  $N_{\text{Re}}$  values of 14.66 and 12.17 respectively. The rate of loss of mass can be 0.032  $\mu$ g s<sup>-1</sup> for the Pomeroy case and 0.029  $\mu$ g s<sup>-1</sup> for Carrier. The results predicted in PIEKTUK in Figure 16 show that the sublimation rate of a column of blowing snow using Pomeroy's settling velocity formula is larger than that with the Carrier specification.

In a previous version of the model, the particle diffusion coefficient was assumed to be equal to the gas diffusion coefficient  $K_{s0}$ , neglecting the particle



*Figure 15.* As in Figure 13 with  $T_a = -10^{\circ}$ C, RH<sub>a0</sub> = 70% and values of 10, 15, 20 and 25 m s<sup>-1</sup> for the 10-m nominal wind speed,  $U_{10}$ ,

counter diffusion effect. With the larger  $K_{s0}$  more particles are in suspension and this leads to a higher sublimation rate for a column of blowing snow (Figure 17).

In the PIEKTUK standard run, the  $\alpha$  parameter in the gamma function, describing the particle size distribution at  $z_{lb}$ , is taken as 5 and the mean  $r_m$  in the saltation layer is taken to be 100  $\mu$ m. As mentioned before, analysis of Antarctic data suggests that  $\alpha$  may be closer to 2 and  $r_m$  to 75  $\mu$ m. When  $\alpha = 2$  more large particles are present in the distribution of particle sizes. The modelled size range should be enlarged to allow for this. Figure 18 shows the sublimation rate when  $\alpha$  is 5 and 2, and  $r_m$  is 100 and 75  $\mu$ m. Decreasing  $\alpha$  results in slightly reduced sublimation rates. The mean radius in the saltation layer is, however, the more important parameter. When the particles are smaller, more particles enter the suspension layer through turbulent mixing, reach higher levels and remain in suspension longer. Therefore, the sublimation rate is larger than that when larger particles exist in the saltation layer.

For completeness in the model, the particle temperature is assumed to be at the ice bulb temperature instead of the ambient temperature. Results have shown, however, that the difference due to this change is negligible and that for practical purposes the particle can be assumed to be at the ambient temperature.



*Figure 16.* The vertically integrated sublimation rates forecast by PIEKTUK in the suspension layer of a column of blowing snow for varying fetch. The environmental parameters are for the standard case but two different assumptions for terminal velocity are contrasted.

The maximum value of the mixing length,  $l_{\text{max}}$ , is taken as 40 m in the standard PIEKTUK runs but values of  $l_{\text{max}}$  of 200 m and  $\infty$  have been tested. The increase in sublimation rate due to the increase in  $l_{\text{max}}$  is not significant. The peak value of sublimation rate is 0.1278 mm h<sup>-1</sup> with  $l_{\text{max}} = 40$  m, and 0.1335 mm h<sup>-1</sup> and 0.1351 mm h<sup>-1</sup> with  $l_{\text{max}} = 200$  m and  $\infty$ , respectively.

# 4. Discussion

It is evident from the results presented in the previous section that the inclusion of thermodynamic variables in our blowing snow model leads to decreased temperatures and increased humidities. This subsequently yields reductions of suspended snow sublimation compared to predictions with either our own model with the thermodynamic effects suppressed, or from other models, notably the PBSM, which exclude these effects. The forecast sublimation rates of  $1-12 \text{ mm d}^{-1}$  swe shown in Table I agree generally with the findings of Schmidt (1982b), Dyunin et al. (1991), Mobbs and Dover (1993) and Bintanja (1998), and suggest that the PBSM



Figure 17. As in Figure 16 except for different assumptions concerning  $K_s$ .

of Pomeroy et al. (1993) may over-estimate the contribution of sublimation to the fluxes of snow, especially at long fetches.

A determining factor in applying our model to estimate sublimation rates for northern regions such as the Mackenzie River Basin may well be the fetch available for the development and maintenance of a blowing snow boundary layer. In the Canadian Prairies, impediments to blowing snow occur on average every 0.5 to 1 km (Pomeroy, personal communication). For Arctic tundra conditions, we might expect the relatively unobstructed fetch for blowing snow to increase, perhaps to a few kilometres, so that sublimation rates could be strongly modified by negative thermodynamic feedbacks.

The results presented in this paper indicate that the blowing snow sublimation process is particularly sensitive to the environmental conditions of temperature, humidity, and wind speed in addition to the particle distributions. In order to facilitate tests of PIEKTUK against experimental data, future field experiments on blowing snow should include these critical measurements. The integration of PIEKTUK, or of calculations based on it, into other models would also allow the examination of more complex and realistic situations, including blowing snow over gentle topography, the presence of snowfall and the possible dynamical interactions between the ABL and the blowing snow boundary layer. This would then provide the necessary thermodynamic inputs to, or blowing snow parameterizations for, numerical mod-



Figure 18. As in Figure 16 except for variation of particle size distribution assumed at  $z_{lb}$ .

els such as MC2, the Canadian mesoscale compressible community model (Benoit et al., 1997) or MM5, the Pennsylvania State University/NCAR mesoscale model (Grell et al., 1993), and allow an assessment of the importance of blowing snow fluxes on the moisture budgets of high-latitude regions.

# 5. Conclusions

We have presented in this paper a blowing snow model which includes processes that modify the ambient particle, humidity and temperature distributions found in an atmospheric boundary layer with blowing snow. In comparison to a previous model, the Prairie Blowing Snow Model or PBSM of Pomeroy (1988) and Pomeroy et al. (1993), our PIEKTUK model predicts that the blowing snow sublimation process is self-limiting, yielding substantially smaller sublimation rates (as much as two-thirds less) in comparison to situations when the thermodynamic feedbacks are neglected. Although ambient air temperature changes are small (of the order of  $-0.5^{\circ}$ C), the relative humidity can increase significantly near the snow surface and may approach saturation with respect to ice when sublimation is occurring. The humidity assumption, as stated by King et al. (1996), becomes the critical parameter in evaluating sublimation rates in blowing snow. Future field experiments

examining wind-blown snow should include detailed observations of the spatially evolving humidity field so that modelled results may be tested.

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