AN INTERCOMPARISON AMONG FOUR MODELS OF BLOWING SNOW

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Abstract. Four one-dimensional, time-dependent blowing snow models are intercompared. These include three spectral models, PIEKTUK-T, WINDBLAST, SNOWSTORM, and the bulk version of PIEKTUK-T, PIEKTUK-B. Although the four models are based on common physical concepts, they have been developed by different research groups. The structure of the models, numerical methods, meteorological field treatment and the parameterization schemes may be different. Under an agreed standard condition, the four models generally give similar results for the thermodynamic effects of blowing snow sublimation on the atmospheric boundary layer, including an increase of relative humidity and a decrease of the ambient temperature due to blowing snow sublimation. Relative humidity predicted by SNOWSTORM is lower than the predictions of the other models, which leads to a larger sublimation rate in SNOWSTORM. All four models demonstrate that sublimation rates in a column of blowing snow have a single maximum in time, illustrating self-limitation of the sublimation process of blowing snow. However, estimation of the eddy diffusion coefficient for momentum (K_m), and thereby the diffusion coefficients for moisture (K_w) and for heat (K_h), has a significant influence on the process. Sensitivity tests with PIEKTUK-T show that the sublimation rate can be approximately constant with time after an initial phase, if K_m is a linear function with height. In order to match the model results with blowing snow observations, some parameters in the standard run, such as settling velocity of blowing snow particles in these models, may need to be changed to more practical values.

Keywords: Blowing snow, Diffusion coefficient, Intercomparison, Sublimation, Suspension.

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1. Introduction

Blowing snow is a common phenomenon in many high latitude regions. For example, over parts of the Arctic Ocean and Antarctica, blowing snow events occur on about one out of four days (Déry and Yau, 1999a; Mann, 1998). It is not only an important hydrological process, owing to its capacity for redistributing precipitated snow, but also a significant meteorological process. During blowing snow events, the sublimation of blowing snow particles can be a source of water vapour and a sink of sensible heat.

Blowing snow observations were made, for example, during the British Antarctic Survey's second Stable Antarctic Boundary Layer Experiment (STABLE2) at Halley in 1991 (Dover, 1993; Mann, 1998); on the American Great Plains at Laramie, Wyoming in 1974 (Schmidt, 1982); and on the Canadian Prairies (Pomeroy, 1988).

The physics of blowing snow saltation and suspension is complex, since processes are interactive. Accurate blowing snow observations are difficult to make. Several problems occur with instruments, especially for the observations of particle density, shape and size distribution and for relative humidity. These factors are important for correctly estimating the sublimation rate. The process is also very dependent on the surface conditions. Different observations may give different estimates of blowing snow parameters. For instance, the formula to calculate blowing snow drift density in the saltation layer given by Pomeroy et al. (1993), based on their observations over the Canadian Prairie, overestimated that observed by Schmidt (1982) in Laramie. Our understanding of the physics of blowing snow is still far from complete.

Recently, numerical models have been used as a tool to study this complex phenomenon and its possible influence on the atmospheric boundary layer. Several numerical models have been developed (Pomeroy et al., 1993; Liston, 1993; Mobbs, 1993; Mann, 1998; Déry, 1998; Bintanja, 2000a); their application can help in the design of field experiments, and in the identification of the sensitivities of the sublimation rate to parameters and the need for accurate observations of critical parameters. However, different models may give different results under given conditions. For example, the sublimation rate in a column of blowing snow calculated by one version of the Prairie Blowing Snow Model (PBSM) (Pomeroy, 1988; Pomeroy et al., 1993) is about 13 times that estimated by the steady state fetch-dependent blowing snow model PIEKTUK-F (the name PIEKTUK is from an Inuktituk word for blowing snow) in a wind speed of 20 m s^{-1} under certain conditions (Déry et al., 1998). Hence, model intercomparisons and verification become especially necessary.

In idealized circumstances, appropriate to large areas of homogeneous snow cover, blowing snow can be considered as a one-dimensional, time-dependent process. In this work, four one-dimensional, time-dependent models have been used. Three of the models are based on a size spectrum of blowing snow particles, and referred to as spectral models; they are PIEKTUK-T developed at York University in Canada (Déry et al., 1998), WINDBLAST developed at Leeds University in the United Kingdom (Mann, 1998) and SNOWSTORM developed at Utrecht University in the Netherlands (Bintanja, 2000a). The fourth is the bulk version of PIEKTUK – PIEKTUK-B, developed at McGill University in Canada (Déry and Yau, 1999b), for coupled simulations using a mesoscale compressible community (MC2) model (Benoit et al., 1997) with a simplified algorithm.

2. Model Descriptions

The three spectral models (PIEKTUK-T, WINDBLAST, SNOWSTORM) as well as the bulk model PIEKTUK-B are one-dimensional blowing snow models in which the elements are only allowed to vary in the vertical direction z and with time t. The situation is assumed to be horizontally homogeneous with an unlimited upwind snow fetch. The intercomparison is limited to pure snow surfaces with no protruding vegetation or stubble, and with no falling snow. Nocturnal situations are simulated so that solar radiation effects can be ignored. The intercomparison is limited to strong winds and near-neutral conditions.

All these models are based on a series of assumptions about the suspended particles,

(i) the blowing snow particles are spheres with the density of ice, since snow crystal structures formed during precipitation are quickly reduced by mechanical abrasion according to Schmidt (1972);

(ii) the volume concentration of blowing snow is high enough so that the snow and air two-phase flow may be regarded as a continuum, and the concentration is low enough that the interaction between particles can be neglected (Schmidt, 1972);

(iii) the particles have the same horizontal velocity as the parcel of the air in which they are suspended (Schmidt, 1982).

A brief discussion of the four models and the parameters used for the purpose of model intercomparison are given here. The detailed descriptions of each model can be found separately (Déry et al., 1998; Déry and Yau, 1999b; Mann, 1998; Bintanja, 2000a).

2.1. Particle size distribution and density in the saltation layer

The construction of the models is similar in a number of respects. All models divide the blowing snow into two layers: saltation and suspension. At the start, there are no blowing snow particles in the suspension layer, and with time, small particles are then diffused upwards and the suspension layer develops. Particle movements in the suspension layer are controlled by turbulent diffusion and settling under gravity, while sublimation leads to mass loss and size reduction of individual particles (in the spectral models).

The saltation layer acts as a lower boundary condition for the suspension layer. In the spectral models, the particle size distribution follows a normalized, two parameter gamma distribution in the saltation layer (Budd, 1966; Schmidt, 1982), with

$$f(r) = \frac{r^{\alpha-1} e^{-r/\beta}}{\beta^{\alpha} \Gamma(\alpha)} \tag{1}$$

and

$$\Gamma(\alpha) = \int_0^{\infty} r^{\alpha-1} e^{-r} \, \mathrm{d}r. \tag{2}$$

Here α is a dimensionless shape parameter, which is set to a value of 2 in the standard run for the model intercomparison, β is a scale parameter with a length dimension (m), r is the instantaneous radius of a particle (m). In the gamma distribution, $\alpha\beta$ is equal to $\overline{r}$ with $\overline{r}$ the mean radius, set to 75×10^{-6} m in the standard run. Since the saltation layer is considered as a thin, well-mixed layer, f is only a function of r in this context (i.e., not dependent on z).

In PIEKTUK-B, the size distribution is assumed to be given by the gamma function at all heights, both in the saltation layer and in the suspension layer; f is then not only a function of r but also allows a height dependence of β (α is set to a constant). Déry and Yau (1999b) constrain both α and β, estimated following a special solution of the diffusion/sedimentation equation for blowing snow particles. The blowing snow drift density in the saltation layer is set equal to a constant d = 0.2 kg m^{-3} in the model intercomparison.

In PIEKTUK-T and WINDBLAST, the particle number density is needed at the lower boundary. Because the particle size distribution in the saltation layer satisfies the two-parameter gamma distribution $f(r)$, the particle number density N_{lb} is then given by

$$N_{lb} = d/\overline{M}, \tag{3}$$

where

$$\overline{M} = \int_0^{\infty} \frac{4}{3} \pi r^3 \rho_i f(r) \, \mathrm{d}r \tag{4}$$

is a mean mass of the particles (since $\int_0^{\infty} f(r) \, \mathrm{d}r = 1$), with the density assumed to be that of pure ice ρ_i (= 900 kg m^{-3}).

In PIEKTUK-B, N_{lb} is given by Equation (3) in the saltation layer. The classical power law equation for suspended particle concentration, the so-called analytical solution, is used to determine number density at other heights as

$$N_a(z) = \int_0^{\infty} f(r) \times N_{lb} \left[\frac{z + z_0}{z_{lb} + z_0} \right]^{-w_T/\kappa u_*} \mathrm{d}r, \tag{5}$$

TABLE I
Particle size bin and bin numbers.

Models	Bin size δr (μm)	Number of bins #	Size range (μm)
PIEKTUK-T	4	192	0–768
WINDBLAST	5	80	0–400
SNOWSTORM	10	48	0–480

where z_{lb} is the lower boundary height set to 0.05 m and z_0 is the roughness length set to 0.001 m.

It is worth noting that in SNOWSTORM, the particle conservation equation is based on particle drift density. Therefore, instead of calculating the particle number density, the mass distribution is calculated. The normalized discrete distribution function for mass in a size bin (i), f_{m_i}, is given by

$$f_{m_i} = \frac{f_i m_i}{\sum(f_i m_i)}, \tag{6}$$

where m_i is the mass of one particle ($\frac{4}{3}\pi\rho_i r_i^3$), and the summation is over all size bins. In the saltation layer of SNOWSTORM, this normalized distribution is used to distribute the total mass (or density) over each size bin by

$$d_i = d \times f_{m_i}, \tag{7}$$

where d_i is the mass in each size bin.

In a test with PIEKTUK-T, with the particle size distribution covering 0–400 μm in radius, the particle number density ratio between the numerical and analytical solutions, R_n, is 99.995%. The particle mass ratio between numerical and analytic solutions, R_m is 98.11%. Table I shows particle size bin and bin number for each spectral model, where all three models cover at least 0–400 μm particles in radius, and so cover most particles very well. A sensitivity test shows that the bin size does not play an important role in the accuracy of the results.

2.2. SUBLIMATION

In the spectral models, the rate of change in mass dm/dt (kg s^{-1}) of a single particle of radius r due to sublimation is given as (Thorpe and Mason, 1966)

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{2\pi r(R_h - 1)}{\frac{L_s}{K N_{\mathrm{Nu}} T_a}\left(\frac{L_s}{R_v T_a} - 1\right) + \frac{R_v T_a}{N_{\mathrm{Sh}} D e_i}}. \tag{8}$$

In each original model, the parameter values may be slightly different. For the purpose of the model intercomparison, they all took on the same values. Here K is the thermal conductivity of air taken as 2.4×10^{-2} W m^{-1} K^{-1}, L_s the latent heat of sublimation (2.835×10^6 J kg^{-1}), R_v the gas constant for water vapour (461.5 J kg^{-1} K^{-1}), D the molecular diffusivity of water vapour in air (2.25×10^{-5} m^2 s^{-1}, although this may be a little higher at 253 K), e_i is the saturation water vapour pressure with respect to an ice surface in Pa; R_h is the relative humidity with respect to the saturation vapour pressure above an ice surface at ambient temperature T_a in K. The initial conditions for R_h and T_a will be discussed later. N_{Nu} and N_{Sh} are the Nusselt and the Sherwood numbers, noting that Lee (1975) demonstrated that in a turbulent atmosphere subject to blowing snow, the Nusselt and the Sherwood numbers are the same. For the model intercomparison, following Lee (1975) and Thorpe and Mason (1966), all models assumed

$$N_{\mathrm{Nu}} = N_{\mathrm{Sh}} = \begin{cases} 1.79 + 0.606 N_{\mathrm{Re}}^{0.5} & 0.7 < N_{\mathrm{Re}} < 10.0 \\ 1.88 + 0.580 N_{\mathrm{Re}}^{0.5} & 10.0 < N_{\mathrm{Re}} \end{cases} \tag{9}$$

with $N_{\mathrm{Re}}(= 2rV_r/\upsilon)$ being the Reynolds number, where υ is the kinematic viscosity of air (1.53×10^{-5} m^2 s^{-1}) and V_r is the ventilation velocity (m s^{-1}). Since we assume that the particles in an air parcel move with air at the same horizontal velocity, the ventilation velocity is then assumed to be equal to the settling velocity of a particle (w_f). We exceed the range given by Lee (1975) for the second formula in (9) (beyond N_{Re} =200), but only very few large particles are involved and any error should be small.

The local sublimation rate per unit volume (kg m^{-3} s^{-1}) in PIEKTUK-T and WINDBLAST is computed as

$$q_{\mathrm{subl}}(z) = \sum_i F_i \frac{\mathrm{d}m_i}{\mathrm{d}t} \delta r, \tag{10}$$

where F_i is the particle number density in each size bin predicted by the models and $\mathrm{d}m_i/\mathrm{d}t$ is computed from Equation (8) with $r = r_i$. In SNOWSTORM, q_{subl} can be obtained by summing over the equation for S_{di} in Table III.

The total vertically integrated sublimation rate over a unit horizontal land area is Q_{subl} (kg m^{-2} s^{-1}), and given by

$$Q_{\mathrm{subl}} = \int_0^\infty q_{\mathrm{subl}} \mathrm{d}z. \tag{11}$$

If the density of water is 1000 kg m^{-3}, 1 kg m^{-2} snow is equal to 1 mm water depth. Therefore, the unit can also be written as mm day^{-1} Snow Water Equivalent (SWE) after multiplication by 3600×24.

The local bulk sublimation rate S_b (kg kg^{-1} s^{-1}) for a gamma distribution of particles in PIEKTUK-B is given by the equation in Table III, where q_b is the

TABLE II
Conservation equation.

Models	Equation	Closure scheme
PIEKTUK-T WINDBLAST	$\frac{\partial F_i}{\partial t} = \frac{\partial w_f F_i}{\partial z} - \frac{\partial \overline{w' F_i'}}{\partial z} + S_{Fi}$	First order
SNOWSTORM	$\frac{\partial d_i}{\partial t} = \frac{\partial w_f d_i}{\partial z} - \frac{\partial \overline{w' d_i'}}{\partial z} + S_{di}$	E-ϵ
PIEKTUK-B	$\frac{\partial q_b}{\partial t} = \frac{\partial}{\partial z}(K_b \frac{\partial q_b}{\partial z} + v_b q_b) + S_b$	Not applicable

blowing snow mixing ratio (kg kg $^{-1}$), and the thermodynamic and diffusion terms associated with the phase change are given by the coefficients F_k and F_d (m s kg^{-1}) – see Déry and Yau (1999b) for more details.

2.3. The conservation equation of suspended blowing snow particles

Physically, all four models describe the variation in snow particle distribution based on the effects of turbulent diffusion, settling and sublimation. In PIEKTUK-T and WINDBLAST, the snow particle number density in each size bin (m^{-3} bin^{-1}) is used as the dependent variable. In SNOWSTORM, the snow drift density in each size bin (kg m^{-3} bin^{-1}) is used while in PIEKTUK-B, the blowing snow bulk mixing ratio (kg kg^{-1}) is the variable.

The particle size spectrum is shifted toward small particle sizes due to sublimation, which is described by S_{Fi}, S_{di} or S_b, as shown in Table III, where m_r is the mass of a particle with radius r, N_r is the number of particles in each size bin and the change in particle radius is given by

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\frac{\mathrm{d}m}{\mathrm{d}t}}{4\pi r^2 \rho_i}. \tag{12}$$

The turbulent diffusion of particles is represented in PIEKTUK-T and WINDBLAST by

$$\overline{w' F_i'} = -K_s \frac{\partial F_i}{\partial z}, \tag{13}$$

where $K_s(z)$ is a specified eddy diffusivity coefficient for blowing snow particles.

In SNOWSTORM, the eddy diffusivity for particles K_s is assumed equal to K_m, where K_m is obtained from an E-ϵ closure scheme used for the momentum equations (discussed below in Section 2.7).

TABLE III

Net change due to sublimation.

Models	Net change due to sublimation
PIEKTUK-T WINDBLAST	$S_{Fi} = \frac{\left[-F_{i+1}\frac{\mathrm{d}r_{i+1}}{\mathrm{d}t}+F_i\frac{\mathrm{d}r_i}{\mathrm{d}t}\right]}{\Delta r}$
SNOWSTORM	$S_{di} = -m_r\frac{\partial}{\partial r}\left(\frac{N_r}{4\pi r^2\rho_i}\left(\frac{\partial m}{\partial t}\right)_r\right)$
PIEKTUK-B	$S_b = \frac{q_b N_{\mathrm{Nu}}(R_h-1)}{2\rho_i\bar{r}^2(F_k+F_d)}$

Instead of considering the equations for a spectrum of particles, a diffusion equation for the blowing snow mixing ratio q_b (kg kg^{-1}) is solved in PIEKTUK-B, as shown in Table II, where K_b is the same as the diffusion coefficient (K_s) used in PIEKTUK-T. Also in PIEKTUK-B, v_b (m s^{-1}) is a settling velocity that characterises the particle distribution - see later discussion.

2.4. Wind profiles

In all four models, the methods to deal with wind profiles are slightly different. In PIEKTUK-B and PIEKTUK-T, we simply specify a logarithmic vertical profile with prescribed u_* and z_0 values. In SNOWSTORM, the wind speed is determined from a solution of the equations in Table IV, where $\overline{u'w'}$ is parameterized by an E-ϵ scheme including consideration of density stratification associated with the presence of snow particles.

In WINDBLAST, a full atmospheric boundary-layer model is used based on the equations in Table IV, where u and v represent the x and y velocity components respectively, and where $\overline{u'w'}$, $\overline{v'w'}$ are evaluated by a first-order closure scheme,

$$\overline{u'w'} = -K_m\frac{\partial u}{\partial z}, \tag{14}$$

$$\overline{v'w'} = -K_m\frac{\partial v}{\partial z}. \tag{15}$$

The eddy diffusivity K_m is discussed later. The geostraphic wind is $(u_g,\, v_g)$ with f_{co} the Coriolis parameter.

2.5. Heat and moisture fields

In all four models, the control equations for heat and moisture are as shown in Table V, where δq and $\delta\theta$ are local rates of change in mixing ratio and potential

TABLE IV
Wind profile in the model.

Models	Wind speed	Closure scheme
PIEKTUK	$u = (u_*/\kappa)\ln[(z+z_0)/z_0]$	Not applicable
WINDBLAST	$\frac{\partial u}{\partial t} = -\frac{\partial \overline{u'w'}}{\partial z} + f_{co}(v - v_g)$ $\frac{\partial v}{\partial t} = -\frac{\partial \overline{v'w'}}{\partial z} - f_{co}(u - u_g)$	First order
SNOWSTORM	$\frac{\partial u}{\partial t} = -\frac{\partial \overline{u'w'}}{\partial z}$	E-ϵ

TABLE V
Heat and moisture.

Models	Heat/moisture	Closure scheme	Heat source
PIEKTUK-T	$\left\{ \begin{array}{l} \frac{\partial q}{\partial t} = -\frac{\partial \overline{w'q'}}{\partial z} + \delta q \\ \frac{\partial \theta}{\partial t} = -\frac{\partial \overline{w'\theta'}}{\partial z} + \delta\theta \end{array} \right\}$	first order	Air/particle
WINDBLAST		first order	Air
SNOWSTORM		E-ϵ	Air

temperature due to sublimation. In all cases fluxes and gradients are related by eddy diffusivities, K_w or K_h.

In WINDBLAST, SNOWSTORM and PIEKTUK-B, the thermal capacity of the suspended snow is neglected, and heat is taken only from the air. In PIEKTUK-T, the heat is taken from both air and particles to maintain the sublimation, but this difference has little impact on the results (see Déry et al., 1998, for details).

2.6. Eddy Diffusion Coefficient

In the models considered here, the eddy diffusion coefficients for water vapour (K_w), heat (K_h) and blowing snow particles (K_s) are assumed proportional to the eddy viscosity (K_m), so that

$$K_w = \beta_1 K_m, \tag{16}$$

$$K_h = \beta_2 K_m, \tag{17}$$

$$K_s = \beta_3 K_m. \tag{18}$$

Here, β_1, β_2 and especially β_3 are controversial parameters and β_3 has been taken both significantly greater than and less than unity in the literature (see, for

TABLE VI
Eddy coefficients in the model.

Models	Eddy coefficient	Mixing length (m)
PIEKTUK	$K_m = u_* l$	$\frac{1}{l} = \frac{1}{\phi_{sf}\kappa(z+z_0)} + \frac{1}{\phi_{sf} l_{\max}}$
WINDBLAST	$K_m = l^2 S$	The same as PIEKTUK-T
	$S = \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]^{\frac{1}{2}}$	
SNOWSTORM	$K_m = c\frac{E^2}{\epsilon}$	Not applicable

example, Sommerfeld and Businger, 1965; Lees, 1981; Dyer and Soulsby, 1988; Mann, 1998). In our model intercomparison, we have simply taken β_1, β_2 and β_3 all equal to 1, although see the brief discussion in Section 4.2.

In PIEKTUK and WINDBLAST, near the ground, the mixing length l (see Table VI) increases with height and reaches a maximum $l_{\max}$ at upper levels. Measurements suggest that the value of $l_{\max}$ is about 40 m in a neutral boundary layer (Taylor, 1969).

In PIEKTUK, ϕ_{sf} is equal to 1.0, considering only the neutral case. In WINDBLAST, the reduction in turbulent mixing due to stable stratification is considered (see Mann (1998) for details); for the eddy coefficient in SNOWSTORM, refer to Bintanja (2000a). In the formula for K_m in SNOWSTORM in Table VI, c is a constant, E is turbulent kinetic energy and ϵ is the viscous dissipation rate, where E and ϵ are calculated from prognostic equations.

In PIEKTUK, K_m does not change with time. In WINDBLAST and SNOWSTORM, it decreases but the change with time is small, especially at lower levels where the blowing snow occurs. In PIEKTUK, K_m increases steadily with height until reaching or approaching a maximum value. In WINDBLAST, K_m increases with height, reaches a maximum value and then decreases with height. K_m in SNOWSTORM increases steadily with height and the value at upper levels is much larger than that in the other models, as shown in Figure 1. This leads to some significant difference in the model results – to be discussed later.

2.7. Settling Velocity of Blowing Snow Particles

The terminal velocity (w_T) of spheres of pure ice in still air can be calculated by analytically solving the equation of the particle motion with a balance between gravity and the drag force, if Carrier's (1953) formula for drag coefficient,

$$C_D = \frac{24}{N_{\mathrm{Re}}}(1 + 0.0806 N_{\mathrm{Re}}) \tag{19}$$

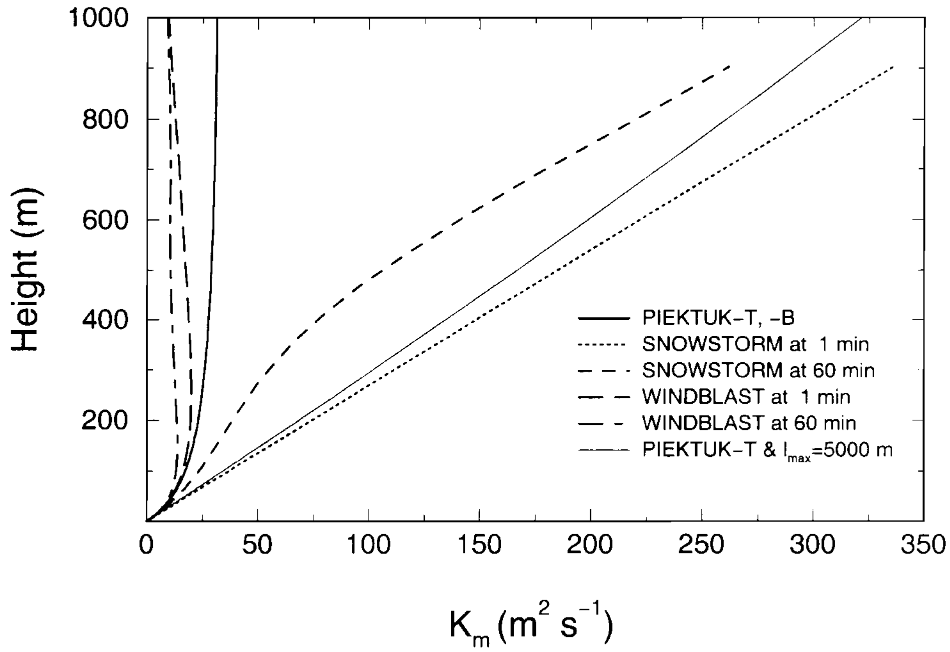


Figure 1. Profiles of eddy coefficients for momentum. The eddy coefficients for water vapour, heat and suspended blowing snow particles are assumed to be equal to that for momentum in the standard run.

is used.

However, the settling velocity (w_f) of natural blowing snow particles in a turbulent flow is generally not equal to w_T. Determination of w_f is an important and complex issue. At present, our knowledge of the settling velocity is not well established. Wang and Maxey (1993) showed that, in homogeneous turbulence the average settling velocity of a small rigid spherical particle, subject to a Stokes drag force, can increase by as much as 50% over the terminal velocity in still air while for larger particles, subject to a non-linear drag force, the net non-linear increase in average settling velocity due to turbulence is reduced. Businger (1965) pointed out that oscillations of the fluid vertical velocity may reduce the settling velocity considerably, especially for particles with Reynolds numbers large enough to be above the Stokes range. The irregularity of real blowing snow particles may reduce their settling velocity relative to the idealized spherical particles. If we let γ be the ratio between w_f and w_T ($\gamma = w_f/w_T$), γ is unlikely to be equal to 1, although, in the standard run, we set γ equal to 1. A discussion on γ and the sensitivity of sublimation rate to it will be given later.

In PIEKTUK-B, a representative terminal velocity for the particle distributions is specified by the weighted average as

$$v_b(z) = \frac{\int_0^\infty w_f r^5 F(r, z)\,\mathrm{d}r}{\int_0^\infty r^5 F(r, z)\,\mathrm{d}r}. \tag{20}$$

See Déry and Yau (1999a) for further details; the r^5 weighting is based on an empirical assessment.

2.8. Initial and Boundary Condition

The initial near-surface logarithmic wind profile is set with $u_* = 0.87$ m s^{-1} and $z_0 = 0.001$ m in all models. We assume that the initial potential temperature (θ_0) is a constant, independent of height (θ_0=253.16 K in the standard run). The absolute temperature is then

$$T_a = \theta_0 \left(\frac{p}{p_0}\right)^{\kappa_a}, \tag{21}$$

where $\kappa_a = 0.286$.

The initial pressure distribution is based on the hypsometric equation,

$$p = p_0 \exp(-\frac{zg}{R_d \theta_0}), \tag{22}$$

where p_0 is the air pressure at z_{lb} (taken as 1000 hPa) and R_d is the gas constant for dry air (287.0 J kg^{-1} K^{-1}).

The initial relative humidity, R_h, profile is set to

$$R_h(t = 0) = \begin{cases} 1 - R_s \ln(z/z_{lb}) & \text{for } z < 100 \text{ m} \\ 0.7 & \text{for } z \geq 100 \text{ m} \end{cases} \tag{23}$$

where R_h is equal to 100% at the lower boundary z_{lb} and decreases with height until z = 100 m. R_h is 70% at 100 m and above with $R_s = 0.039469$.

The model domain and vertical grid differ between the models, as shown in Table VII. In the four models, the fluxes of heat on the upper and lower boundaries are set to zero. In PIEKTUK, the particle number density is set to the value in the saltation layer at the lower boundary with no change with time and is zero at the upper boundary. In SNOWSTORM, the particle drift density is set to a constant value at the lower boundary and the flux is set to zero at the upper boundary. In WINDBLAST, Monin–Obukhov theory with Owen's adjustment to surface-layer similarity for the saltating particles is applied at the lower boundary and the particle number density is set to zero at the upper boundary. For more details, refer to Mann (1998).

TABLE VII
Model domain and vertical grid numbers.

Models	Depth (m)	Vertical grid #
PIEKTUK-T	1000	128
WINDBLAST	2500	150
SNOWSTORM	1000	76
PIEKTUK-B	1000	24

TABLE VIII
Lower boundary condition.

Models	Particles	Heat flux	Moisture
PIEKTUK-T, -B	Number density	0	R_h = 100%
WINDBLAST	Monin–Obukhov	M–O	M–O
SNOWSTORM	Drift density	0	R_h = 100%

3. Results from the Models under Standard Conditions

3.1. Drift Density

Figure 2a shows the blowing snow drift density (ρ_s, kg m^{-3}) predicted by the models at t = 1 min and 60 min. As expected, the four models show that the blowing snow drift density decreases rapidly with height. An analytical solution is also shown, obtained by neglecting sublimation and considering a steady state with a balance between turbulent diffusion and settling under gravity for uniform size particles. When $l = \kappa(z + z_0)$, the equation can be solved analytically, as shown in Equation (5). This solution, summed over the particle spectrum, is labelled as

TABLE IX
Upper boundary condition.

Models	Particles	Heat flux	Moisture
PIEKTUK-T, -B	Density = 0	0	$R_h = 70\%$
WINDBLAST	Flux = 0	0	$R_h = 70\%$
SNOWSTORM	Flux = 0	0	$R_h = 70\%$

the ‘ANALYSIS’ solution in Figure 2a. The evolution of drift density profiles from the four models shows that the profiles approach the analytic solution with time at higher levels. Note however that in practice the balance between gravitational setting and turbulent diffusion may never be reached for small particles (Kind, 1992) at all height.

Since sublimation is neglected in the analytic solution, the numerical predictions from all models with sublimation and the same u_* and l should have smaller drift densities (at the same height and at all times) than those computed from the analytic solution. However, near the surface (well below 10 m) where the density is largest, the numerical solutions are close to the analytic one. Above this level, the speed of approach to the analytic solution is different for different models. The drift density in SNOWSTORM soon reaches a steady state and the profile of drift density at 5 min (not shown) is very close to that at 60 min. This result is probably related to the larger K_s, see later discussion of the sensitivity tests.

Solutions for ρ_s tend to zero with increasing heights; with time increasing, the heights at which $\rho_s \rightarrow 0$ increase. This illustrates the development of the suspended blowing snow layer.

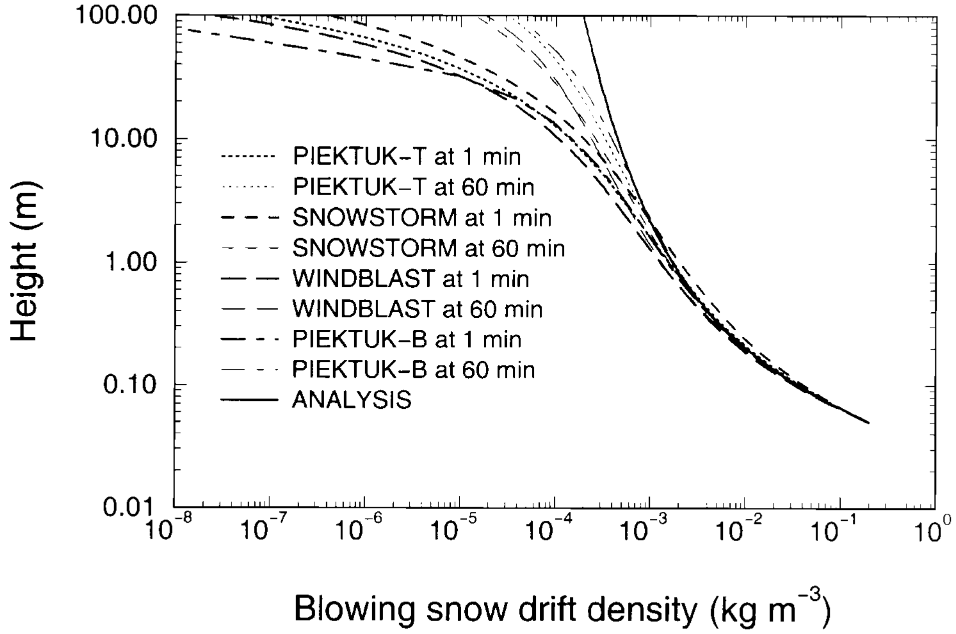
Figure 2b shows the profiles of particle number densities from each model. These correspond to the drift densities in Figure 2a, but generally show a slower (though still large) variation with height since there are smaller particles at the higher levels. Among the spectral models, SNOWSTORM predicts the lowest particle numbers above 1 m after 60 min; the curve labelled as ‘ANALYSIS’ is also shown in the figure.

PIEKTUK-B is anomalous in that it assumes a number density profile matched to the analytic power-law solution. The profile differs slightly from the similar ‘ANALYSIS’ solution because of differences in the particle bin size used for these computations (16 μm in PIEKTUK-B). Within PIEKTUK-B, this number density profile is assumed to be independent of time and the increases in snow drift density with time shown in Figure 2a, apart from diffusion within this bulk model, are caused by increases in particle mean radius – see also discussion of Figure 3 below.

3.2. Mean Radius and Size Distributions

In all models, the particle mean radius decreases with height (Figure 3) since only smaller particles are diffused to higher levels. For a given height, SNOWSTORM predicts the largest mean particle radius, followed by PIEKTUK-T and WINDBLAST, while PIEKTUK-B predicts a substantially smaller mean radius than those from the spectral models, even smaller than the analytic solution (labelled as ‘ANALYSIS’ in Figure 3), especially at upper levels.

The mean radius at fixed height reduces with time in the spectral models. Sensitivity tests with PIEKTUK-T without sublimation also show that the mean radius decreases with time at fixed z and tends to the steady-state analytic solution. Diffusion is assumed equally effective for all size particles while settling is



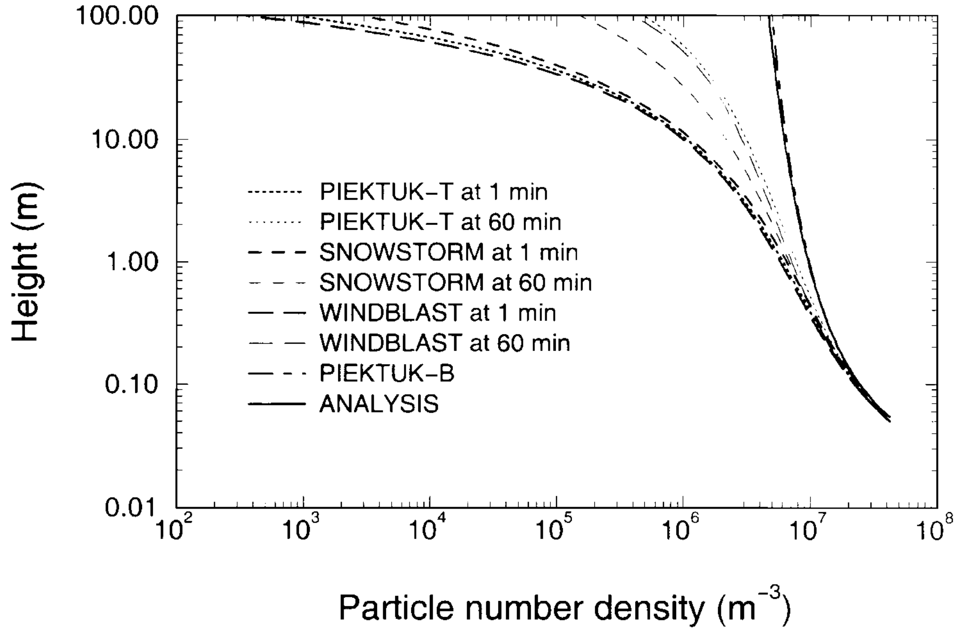


Figure 2. Profiles of blowing snow drift (a, upper) and number (b, lower) density predicted by the models after 1 and 60 min. The analytical solutions without sublimation, labelled 'ANALYSIS', are also given.

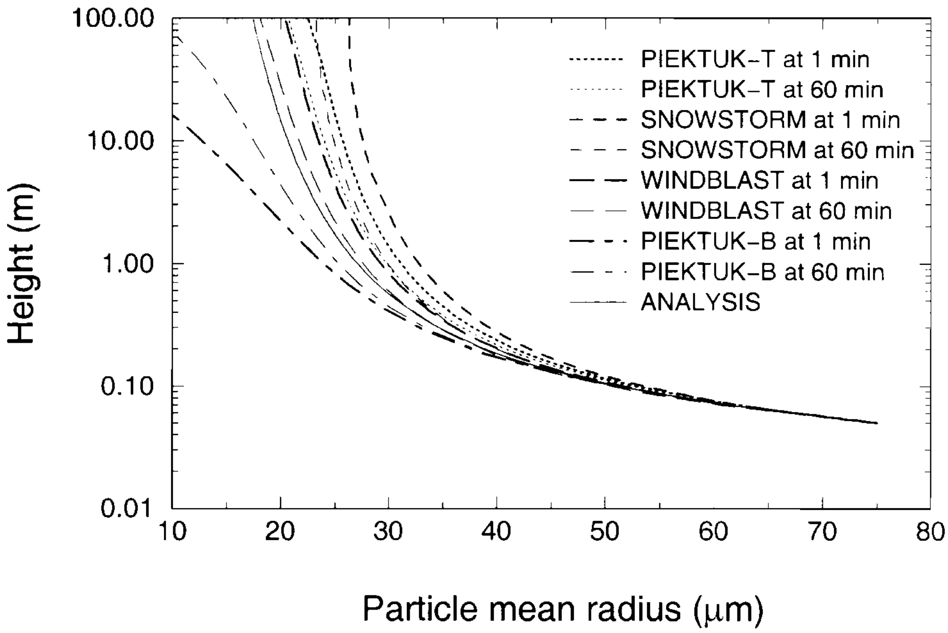


Figure 3. Profiles of particle mean radii forecast by the models after 1 and 60 min. The analytical solution, labelled 'ANALYSIS' is also shown.

stronger for larger particles; initially, diffusion is greater than settling. With time, larger particles settle faster than smaller ones, reducing mean sizes even though the concentration increases. Meanwhile, sublimation of snow particles makes the size distribution move towards even smaller sizes. However, the particle mean radius given by PIEKTUK-B increases with time at a fixed z.

As mentioned before, we assumed that the size distribution in the saltation layer follows the gamma distribution in the spectral models. The measurements by Schmidt (1982) suggest that the size distribution in the suspension layer also approximately follows a gamma distribution. The results from the three spectral models also predict approximate gamma distributions in the suspension layer. The three models all show that the α parameters in these gamma distributions generally increase with height and time (Figure 4), which indicates that the distributions become narrower and more symmetric. In PIEKTUK-T, α increases with height above 0.1 m more rapidly than in SNOWSTORM and WINDBLAST. The reason for the change of α with time and height may partly be because the smaller particles become smaller with time due to sublimation and because the relative radius change is larger for smaller particles. Recall that in PIEKTUK-B, $\alpha = 2$ and is constant throughout the vertical domain, explaining in part the lower values for the profiles of mean radius predicted by this model.

Figure 5 shows the particle size distributions and their time evolution for the four models at 1 m high. At this level, most of the particles are smaller than 50

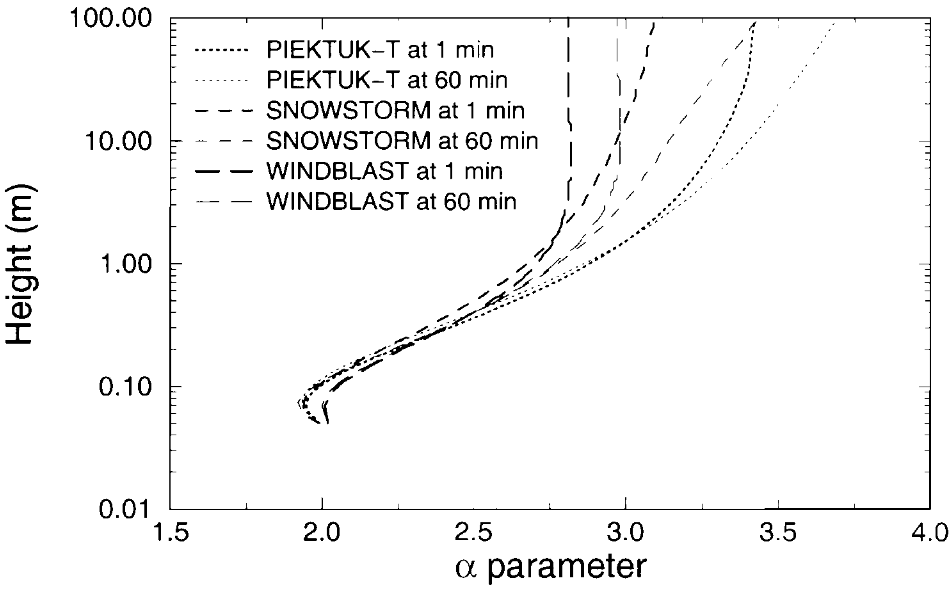


Figure 4. Profiles of the α parameter from the models after 1 and 60 min; note that the α parameter is a constant (= 2) in PIEKTUK-B.

μm, and with time, the particle number densities increase. However, the increase is different in the different models. The particle number density in SNOWSTORM is smallest at 60 min, corresponding to the drift density results in Figure 2a or the number density in Figure 2b. The results from the three spectral models are, however, generally similar.

3.3. Relative humidity, temperature and local sublimation rates

WINDBLAST and PIEKTUK-T generally predict similar profiles of relative humidity as shown in Figure 6 and the relative humidity after 60 min is above 95% for $z < 100$ m. SNOWSTORM predicts smaller relative humidity values after 60 min, and has results less than 70% for large z. Though the region we are mainly concerned with is the near surface layer where the blowing snow occurs, the sensitivity test shown later explains that the lower relative humidity values aloft are the results of assuming K_w to be linear with height.

Figure 7 shows vertical profiles of the local sublimation rate. The sublimation rate decreases with time since the relative humidity increases and the temperature decreases due to the sublimation itself. The lower boundary condition of relative humidity has been fixed at 100% and the sublimation rate is therefore zero at z_{lb}. The local sublimation rate reaches a maximum value just above z_{lb} where the particle number density is relatively large, even though the relative humidity is high. Above this level, the particle number density becomes smaller and the local sublimation rate decreases with height. Since the relative humidity in

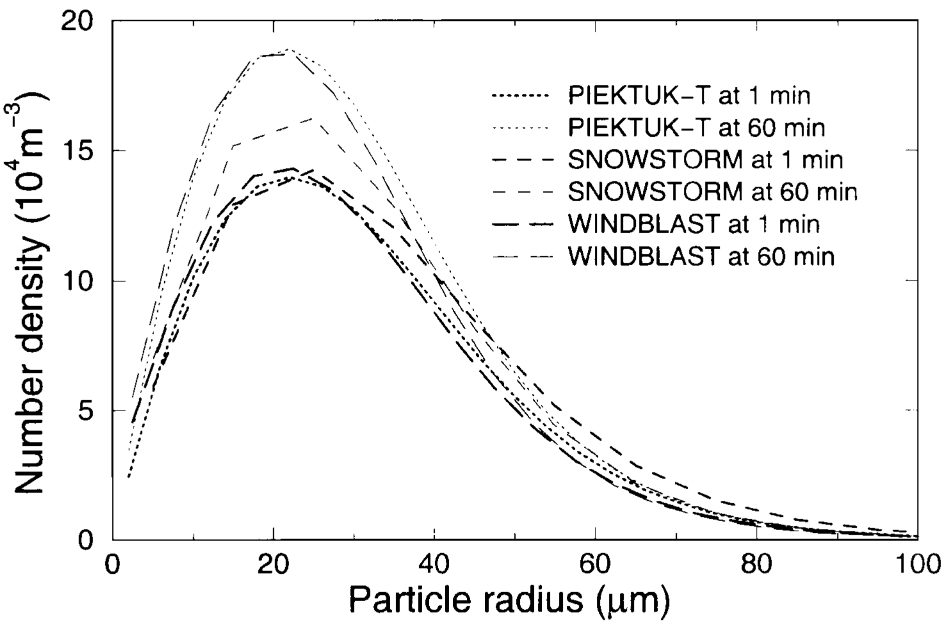


Figure 5. Particle number density distribution at $z = 1$ m predicted by the spectral models after 1 and 60 min.

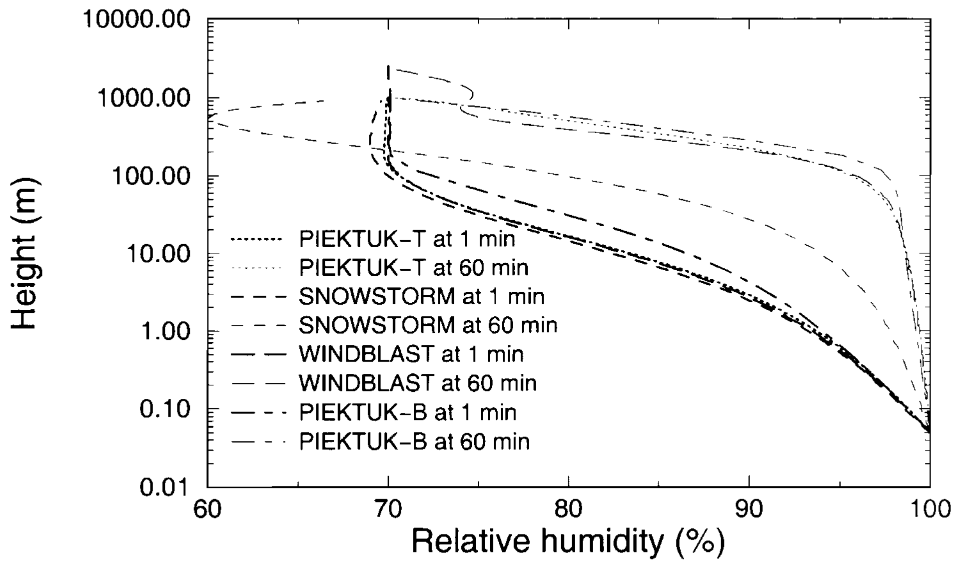


Figure 6. Profiles of relative humidity predicted by the models after 1 and 60 min; 100% relative humidity is set at the lower boundary.

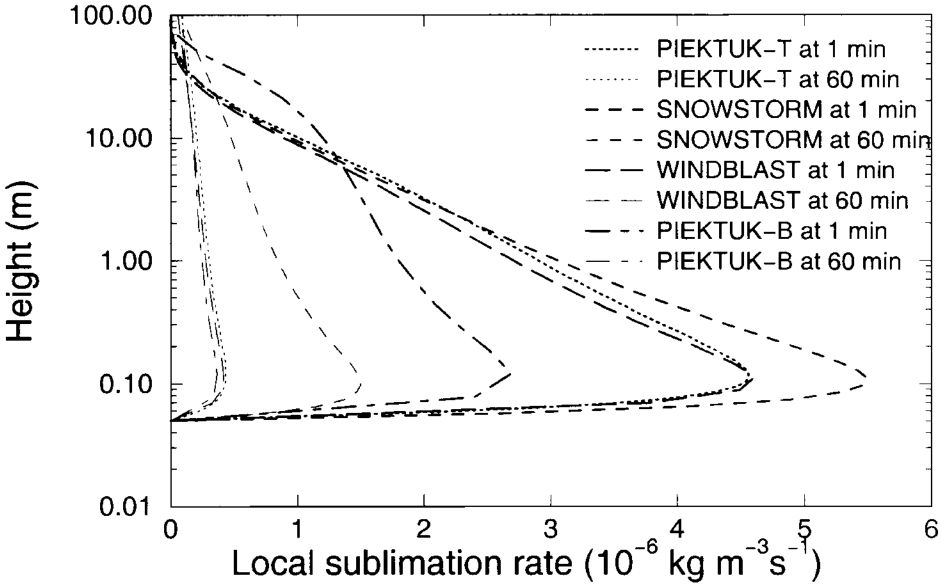


Figure 7. Profiles of local sublimation rate for the suspension layer predicted by the models after 1 and 60 min.

SNOWSTORM is lower than in the other models at t = 60 min, the sublimation rates in SNOWSTORM are noticeably larger.

PIEKTUK-B predicts less sublimation than that predicted by the spectral models at lower levels ($z < 10$ m for t = 1 min) but this is balanced by more sublimation at higher levels. We attribute this primarily to the differences in number density profiles (Figure 2b).

As a result of the sublimation of blowing snow particles, the near-surface potential temperature decreases (Figure 8), and at 1 min, the four models give very similar temperature profiles. As the process develops, at 60 min, SNOWSTORM predicts the lowest near-surface temperatures, because the sublimation rate in SNOWSTORM is the largest. The temperature predicted by WINDBLAST is the highest near the surface.

3.4. Temporal Evolution of the Vertically Integrated Sublimation and Transport Rates

For many applications, the vertically integrated rates are the most important predictions of the models. Figure 9 shows the time evolution of the vertically integrated sublimation rate in a column of blowing snow. As before, the sublimation rate predicted by SNOWSTORM is the largest among the four models on account of the lower relative humidity. We notice that the higher sublimation rate is not the result of particle density; the particle density in SNOWSTORM (Figure 2a) is no larger than that in other models because of its linear K_s. The sublimation rate in

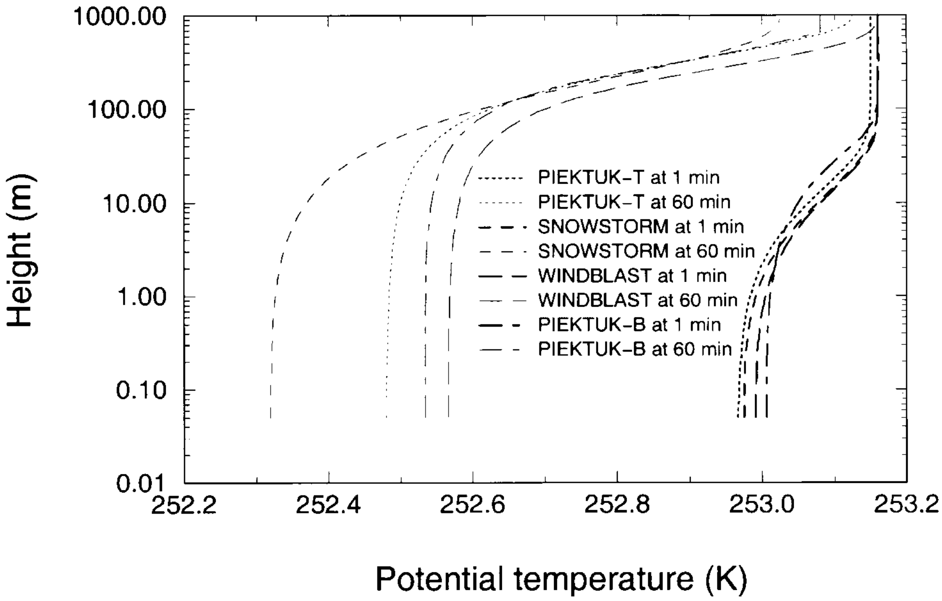


Figure 8. Profiles of potential temperature predicted by the models at 1 and 60 min.

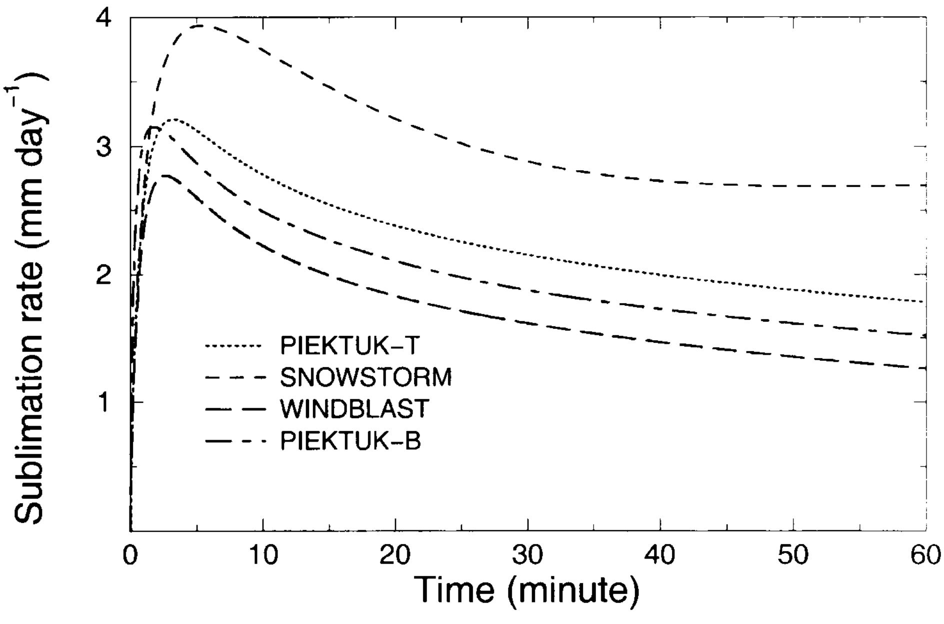


Figure 9. Temporal evolution of the vertically integrated sublimation rate in the suspension layer of a column of blowing snow.

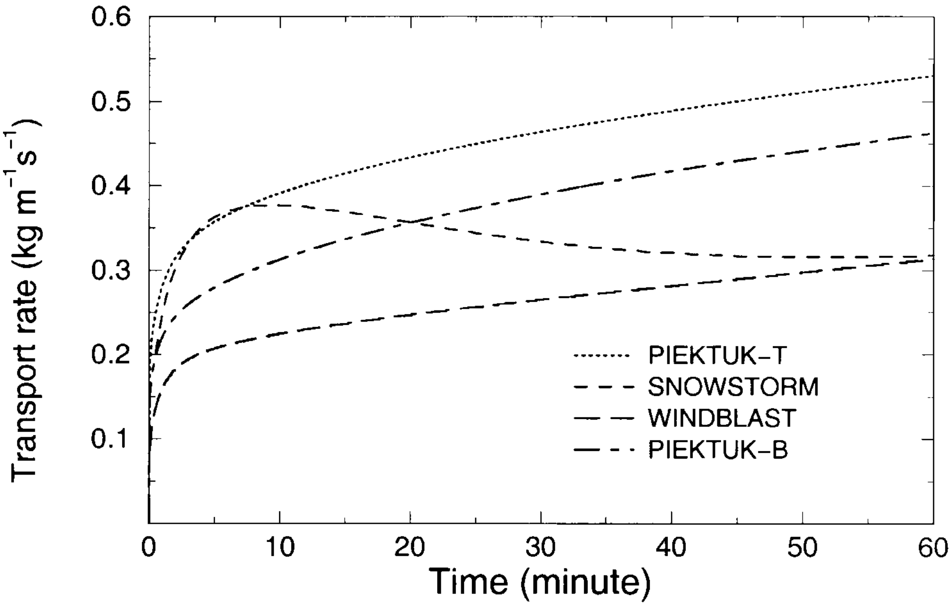


Figure 10. Temporal evolution of the vertically integrated transport rate.

WINDBLAST is smallest among the models. PIEKTUK-B predicts a peak sublimation rate about 2 min, while in SNOWSTORM, the maximum occurs later (about 6 min). The maximum values predicted by PIEKTUK-T and WINDBLAST occur at the same time at about 3 min. Under the conditions set for the intercomparison, the sublimation rates range from about 2–4 mm day^{-1}, corresponding to a relatively large (64–128 W m^{-2}) latent heat flux.

Figure 10 gives the temporal evolution of the vertically integrated transport rate $\int_{z_{lb}}^{\infty} U\rho_s \,\mathrm{d}z$, with those predicted by PIEKTUK and WINDBLAST increasing with time. The transport rate in SNOWSTORM increases, and reaches a maximum value then decreases slowly. This temporal change tendency of the transport rate in SNOWSTORM may be related to its K_s, which increases with height linearly and decreases with time, as analysed in the next section. The transport rate predicted by PIEKTUK-T is the largest, but all models predict comparable results.

4. Sensitivity Tests

4.1. Sensitivity to $l_{\max}$

The results presented above suggested a number of sensitivity tests, and these tests have been performed with the PIEKTUK-T model. One sensitivity test has been designed to test the effect of the eddy diffusion coefficient on the model predictions. Notice that we have assumed that the eddy coefficients for water vapour, mixing ratio and blowing snow particles are all equal to that for momentum. Therefore, the change on K_m is also applied to K_s, K_w and K_h. Within PIEKTUK-T, we change the maximum mixing length $l_{\max}$ (see Table VI) to 5000 m instead of 40 m as in the standard run. This effectively corresponds to K_m varying linearly with z, as in SNOWSTORM.

Drift density profiles show that when $l_{\max}$ is equal to 5000 m, the density profiles are very similar to those predicted by SNOWSTORM in Figures 2a and 2b. The vertical diffusion of particles with $l_{\max}$ = 5000 m fails to lead to higher density, however, since most particles are restricted to the lower levels where the change in $l_{\max}$ has little impact on K_m, as can be seen in Figure 1.

As in the profiles predicted by SNOWSTORM in Figure 6, the relative humidity profile simulated by PIEKTUK-T with $l_{\max}$= 5000 m at 60 min also gives values less than 70% at higher levels and the relative humidity is smaller than that with $l_{\max}$= 40 m at lower levels. Since the initial temperature decreases with height, the initial saturation mixing ratio decreases with height. As a result, the initial mixing ratio as a function of relative humidity and saturation mixing ratio also decreases with height. In other words, there exists a gradient of mixing ratio with height. In the model, it is the mixing ratio that is diffused and the large $l_{\max}$ allows ‘drier’ air (R_h = 70%, but with lower temperature and lower mixing ratio) to be mixed down from above. The performance of PIEKTUK-T with $l_{\max}$ = 5000 m is very similar to that of SNOWSTORM in this respect.

The transport rate with $l_{\max}$ = 5000 m in Figure 11a shows that the mass flux reaches an approximately stable value around 10 min instead of increasing with time. However, this is different from the prediction with SNOWSTORM, in which the transport rate (Figure 10) reaches a maximum value and then decrease slowly. The reason may be that the K_s in PIEKTUK-T is fixed while it becomes smaller with time in SNOWSTORM. In PIEKTUK-T with $l_{\max}$= 5000 m, since the relative humidity is far from saturation due to increased vertical diffusion, the self-limitation of sublimation is lessened. The vertically integrated sublimation rate predicted by PIEKTUK-T with $l_{\max}$= 5000 m appears to reach a steady state of about 4 mm day $^{-1}$ (Figure 11b). The sublimation rate in SNOWSTORM reaches a maximum value with time and then decreases, as for the transport rate previously analysed. It is clear that the value of the sublimation rate can be strongly influenced by assumptions concerning the eddy diffusion coefficient for moisture above the blowing snow. If we do not consider the effect of the feedback of sublimation in blowing snow, the model estimates an upper bound for sublimation rate. The curves labelled ‘FTW’ (Fixed Temperature and Mixing ratio) in Figure 11b show these upper bounds. The change in FTW due to the different $l_{\max}$ is not as great as the change in the standard runs due to $l_{\max}$ because FTW has disabled the diffusion on mixing ratio and the difference is caused only by the different K_s.

It has been concluded in Déry et al. (1998) that the increase in sublimation rate due to the increase in $l_{\max}$ is not significant. That is because the initial temperature and mixing ratio were set to constant values with height by them. Under this

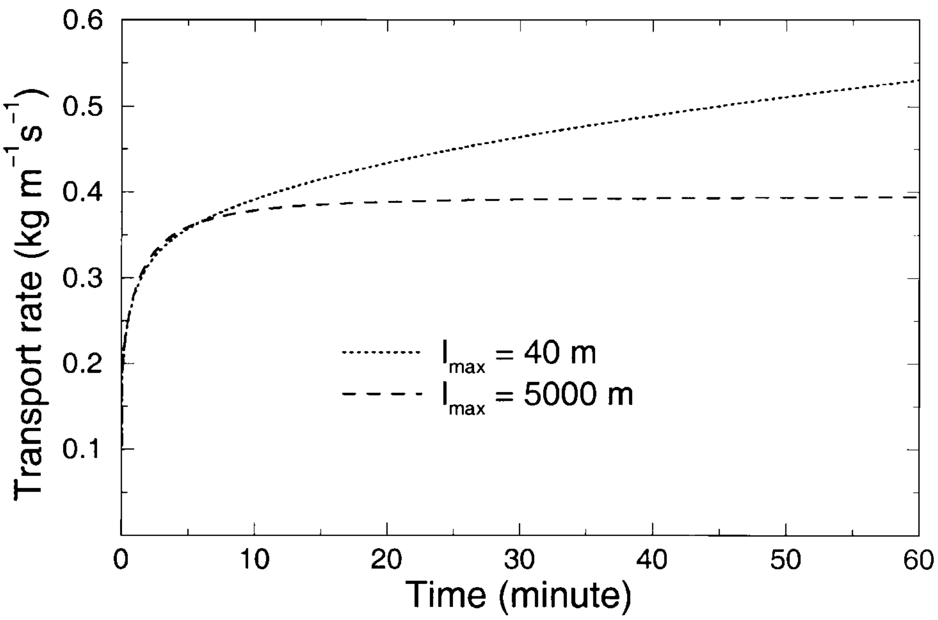


Figure 11a. Temporal evolution of the transport rates predicted by PIEKTUK-T with l_{max} = 40 m and 5000 m. When l_{max} is equal to 5000 m, K_m increases approximately linearly with height.

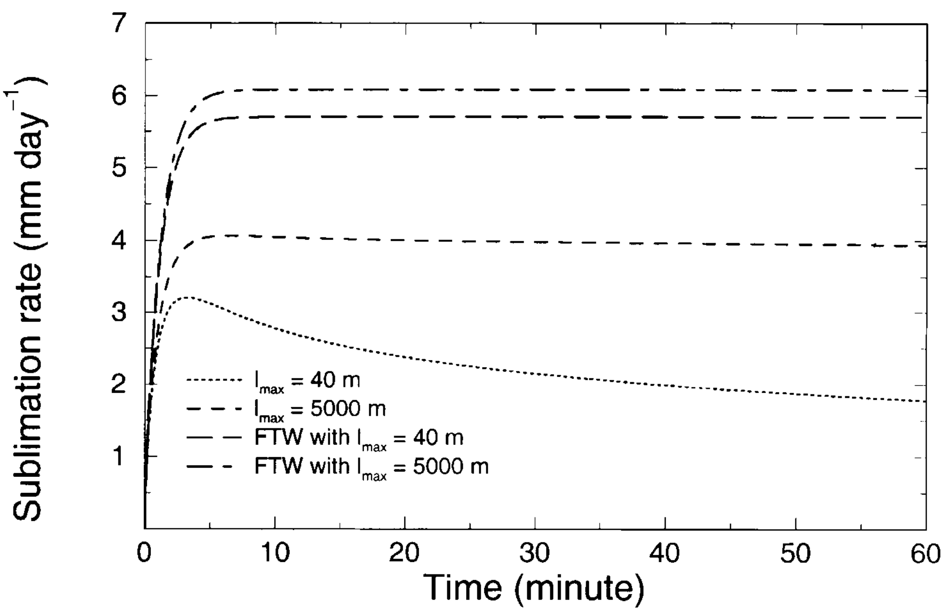


Figure 11b. Temporal evolution of the vertically integrated sublimation rate in the suspension layer of a column of blowing snow under the same condition of Figure 11a. Curves labelled 'FTW' are also plotted for l_{max} = 40 m and 5000 m, respectively.

condition, the change of mixing ratio was only due to the sublimation in blowing snow; there was no gradient of mixing ratio at higher levels. The larger l_{max}, and therefore larger K_w, made little difference to the mixing ratio or relative humidity. Consequently, the sublimation rate had no significant change with l_{max}.

4.2. Sensitivity to Ratios γ and β_3

When these numerical models are run to match observed particle drift densities, it is generally assumed that the settling velocity of particles w_f is equal to the terminal velocity of particles in still air (γ = 1.0), as in the model intercomparison. However, this assumption is open to challenge. The larger drag force on non-spherical snow particles than on spherical particles, and the smaller density of snow particles compared to pure ice, may significantly reduce the terminal velocity of the particles. For the larger blowing snow particles, where the Reynolds number is well above the Stokes range, following Sommerfeld and Businger (1965), the particle settling velocity in turbulent flow is reduced. As a result, the models generally predicted a smaller drift density than those observed. When γ is set to 0.2, a relatively large number of particles remains in the suspension layer. Model results can then be greatly improved in comparison with observations and a reduction in the settling velocity may be sufficient to explain the snow density profiles.

However, instead of reducing γ, many researchers have increased β_3 to adjust the model performance. In Mann (1998), β_3 has been taken as 5 in order to match observations from STABLE2. When β_3 is equal to 5, the particle drift density profile is shown in Figure 12. As we can see from this figure, increasing β_3 or decreasing γ by a factor of 5 results in similar changes to particle drift density. However, the sublimation rates in a column are quite different. When γ = 0.2 and β_3 = 1, the maximum sublimation rate is around 7 mm day^{-1}, while the maximum value can reach 25 mm day^{-1} with γ = 1 and β_3 = 5 as shown in Figure 13. This is because reducing the settling velocity also means reducing the ventilation velocity. As a result, the predicted sublimation rate with γ = 0.2 should be smaller than that for the same particle density under β_3 = 5, as seen in Figure 13, although the particle drift density is almost the same.

Though other work also showed that β_3 could be larger than 1, as in Lees (1981) and Sommerfeld and Businger (1965), this is still very questionable because the conclusion may be based upon γ = 1. Meanwhile, Bintanja (2000b) demonstrated that the particle concentration gradient reduces the eddy coefficients, even though there was no difference between K_m and K_s. Rouault et al. (1991) argued that the 'counter-diffusion' of particles always reduces the eddy coefficient for particles and gave β_3 as a function of particle settling velocity and friction velocity, which is always less than 1.

We realised that either β_3 or γ should probably be a function of the particle size and z, and further investigation of this is in process.

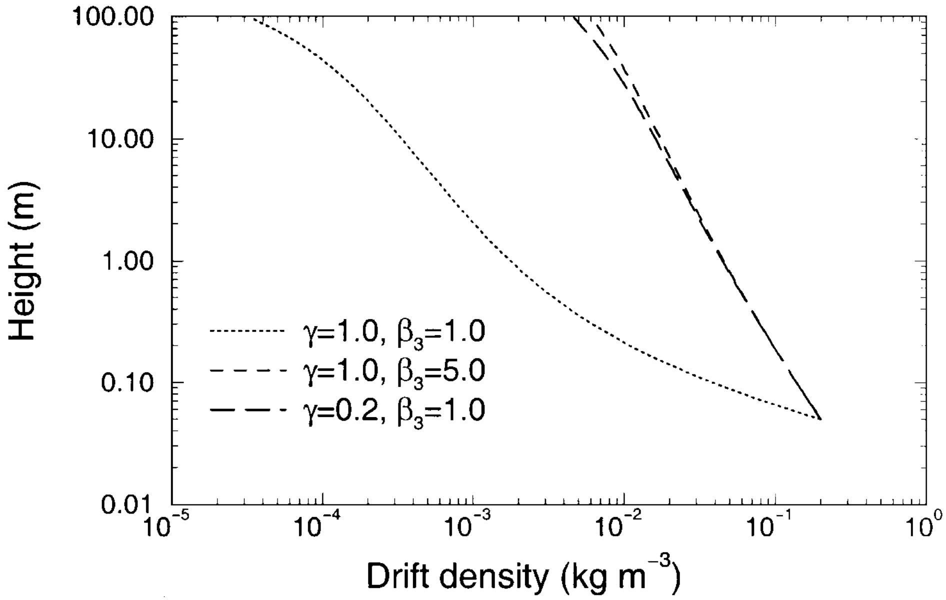


Figure 12. Profiles of suspended blowing snow drift density predicted by PIEKTUK-T with different K_s ($\beta_3 = K_s/K_m$) or settling velocity w_f ($\gamma = w_f/w_T$) from the standard run.

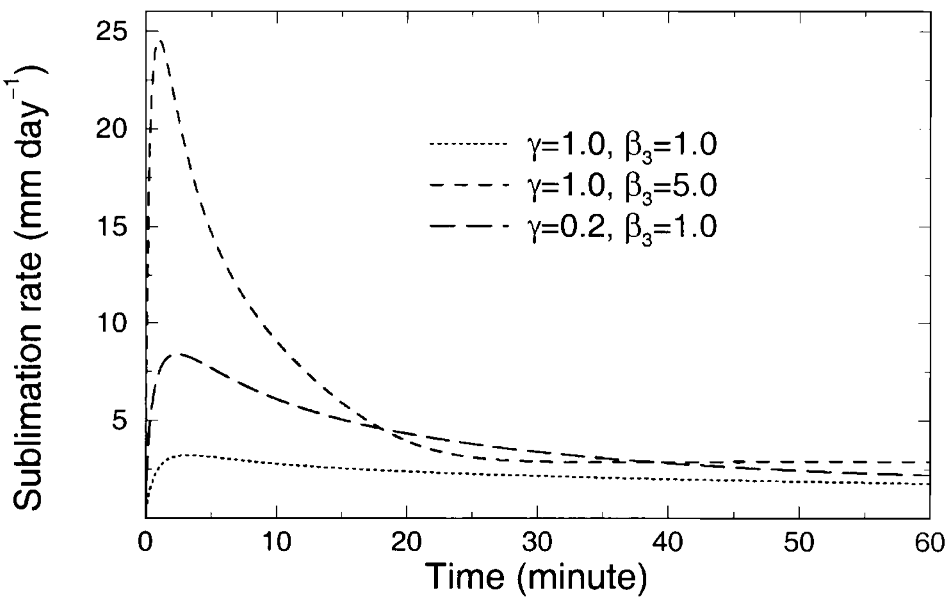


Figure 13. Vertically integrated sublimation rate in the suspension layer of a column of blowing snow predicted by PIEKTUK-T with different K_s ($\beta_3 = K_s/K_m$) or settling velocity w_f ($\gamma = w_f/w_T$).

5. Conclusion

Four one-dimensional, time-dependent, blowing snow models are intercompared. The models generally give similar results for most variables but some details of the bulk model predictions differ from those of the spectral models. The models give similar tendencies for most results. The models are sensitive to ambient factors such as initial temperature, wind speed, relative humidity and the particle size distribution in the saltation layer. Sensitivity tests for these were performed in Déry et al. (1998). In addition to these factors, parameters such as eddy viscosity (K_m) are very important. A linear K_m for all height may not be appropriate for these models.

All the model results presented were obtained under idealized conditions in order to intercompare the numerical models themselves, rather than with observations. However, we realize that the more important work lies in the comparison of the model results with field observations. To achieve satisfactory agreement, some parameters, such as settling velocity of blowing snow particles, may have to be adjusted. Decreasing the settling velocity by a factor $\gamma = 0.2$ or increasing the K_s with $\beta_3 = 5$ will increase the particle drift density in order to better match the observed data. However, the predicted sublimation rates are different. Further study of these parameters is necessary for the correct prediction of the sublimation rate in blowing snow.

Acknowledgements

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