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SOME ASPECTS OF THE INTERACTION OF BLOWING SNOW WITH THE ATMOSPHERIC BOUNDARY LAYER

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ABSTRACT

Several possible effects of blowing snow on the atmospheric boundary layer are investigated, mostly within the general framework of the Prairie Blowing Snow Model (PBSM). The processes of snow saltation and suspension are first described. Variations to the drift density profile are tested and the effects of stratification and density variation calculations are evaluated. Despite high density gradients of blowing snow, stratification effects on turbulence and the velocity profiles can generally be neglected. However, with saltating or suspended snow in a constant shear stress layer, part of the shear stress is carried by the particles. A highly simplified, single-phase approach, based on the density variation of the air–snow mixture coupled to a simple turbulent stress–strain relationship, is used to illustrate this. Sublimation rates in a column of blowing snow are calculated using the PBSM and results are compared with those obtained with a modified formulation which incorporates a spectrum of sublimating particles of varying sizes at each height in a steady-state surface boundary layer and different specifications of the ventilation velocity.

KEY WORDS blowing snow; saltation; suspension; sublimation

INTRODUCTION

Across most of Canada, snow is a common overlying surface during winter-time. Snow is a loose surface, which allows the processes of snow suspension and saltation above its surface in high winds. This, in turn, can have a significant effect on the structure of the atmosphere near the snow surface owing to modified density gradients and the sublimation of airborne snow.

Bagnold (1941) provided one of the first thorough examinations of suspension and saltation of sediments in a fluid, with an emphasis on the movement of sand in air. He also noted that the log-law wind profile became inaccurate near the surface in situations where sediments were airborne. Subsequent studies of snow saltation and suspension exhibited similar deviations in the log-law wind profile (Maeno *et al.*, 1979; Kikuchi, 1981; Schmidt, 1982a). Several authors have therefore proposed, with some success, velocity profiles that take into account the presence of a sediment load. For instance, Taylor and Dyer (1977) use Monin–Obukhov stability parameters to represent density stratification effects and determine density and fluid velocity profiles with suspended sediments, while several authors, including Kind (1990), McEwan (1993) and McKenna-Neuman and Nickling (1994) describe the role played by sediment in momentum transfer within the saltation layer.

During snow transport by saltation and suspension, sublimation of blowing snow particles can also occur. Sublimation can be a significant source of water vapour and sink of sensible heat in the air, and will act to reduce particle sizes and, consequently, terminal velocities.

Several numerical models describing snow transport and snow drift formation have recently been developed by Uematsu *et al.* (1989), Liston *et al.* (1993) and Uematsu (1993), among others. In this paper, we will make use of the Prairie Blowing Snow Model (PBSM) developed by Pomeroy (1988) and Pomeroy *et al.* (1993). This combines physically based algorithms to simulate snow transport due to saltation and suspension, while taking into account the sublimation of blowing snow. We propose to study interactions

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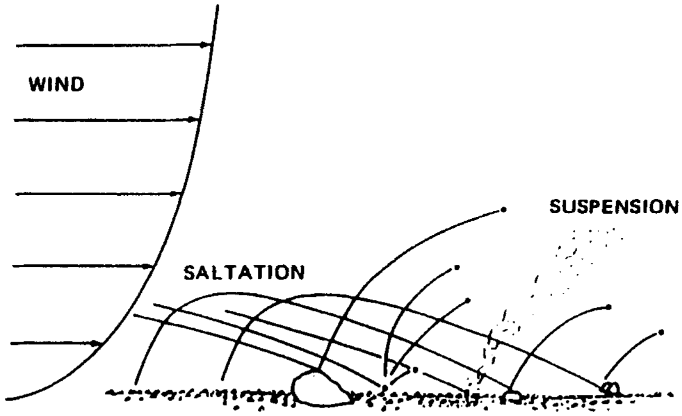


Figure 1. Diagram showing the processes of saltation and suspension of sediments into the atmosphere (adapted from Greeley and Iversen, 1985)

between blowing snow and the atmospheric boundary layer (ABL) with a modified version of the PBSM that takes into account a full distribution of particles at each height. Some initial steps in this model development are described.

SALTATION AND SUSPENSION

The occurrence of high winds over a loose surface can induce particles into several types of aeolian movement: saltation, suspension and, in some instances, surface 'creep' (Kind, 1990). Saltation is the process by which particles are transported horizontally by bouncing along the surface (Bagnold, 1941). The saltation of a particle is usually initiated by the collision of one particle into another (Figure 1). The distance travelled by a saltating particle is determined in part by the mean flow of the fluid, the particle size and surface conditions (Pomeroy and Gray, 1995).

The vertically integrated saltation transport rate per unit cross-wind distance, Q_{salt} (kg m^{-1}s^{-1}), used in the PBSM is assumed by Pomeroy and Gray (1990) to be of the form

$$Q_{salt} = (C_{salt}\rho_a u_{*_t}/g)(u_*^2 - u_{*_n}^2 - u_{*_t}^2) \tag{1}$$

where ρ_a is the air density, $\tau = \rho_a u_*^2$ is the total atmospheric shear stress, $\rho_a u_{*_n}^2$ is the portion applied to non-erodible elements (e.g. rocks or bushes protruding through the snow) and $\rho_a u_{*_t}^2$ is the threshold shear stress on erodible elements (the value at the cessation of saltation). The dimensionless coefficient C_{salt} is set equal to $0{\cdot}68/u_*$, where u_* is in m s^{-1}. The threshold 10-m wind, U_t(10 m) is typically set at 5 m s^{-1} and u_{*_t} computed assuming a log profile with $z_0 = 0{\cdot}0002$ m, giving $u_{*_t} = 0{\cdot}03697\ U_t(10\,\mathrm{m}) = 0{\cdot}185$ m s^{-1}.

The friction velocity, u_*, corresponding to the total shear stress, is dependent on the wind speed, at, for example, 10 m, and the surface roughness. With a fixed surface, near-neutral stratification and no material being transported by the wind then the log-law velocity profile, $U(z)$, is well established both theoretically and experimentally as

$$U(z) = (u_*/\kappa)\ln[(z + z_0)/z_0)] \tag{2}$$

where $\kappa (= 0{\cdot}4)$ is von Kármán's constant and z_0 is the aerodynamic roughness length for momentum (m). This form has $U = 0$ at $z = 0$, and corresponds to a 'constant stress' layer in which τ is independent of z. If we know z_0 and have a wind measurement at, for example, 10 m, we could then determine u_*. With sand it is found (see, for example, McEwan, 1993) that, close to the surface, this profile is modified since some of the shear stress is carried by the saltating or suspended particles. The apparent roughness length is also modified. We will discuss some details of this below but for the moment can remark that, for blowing snow conditions in the PBSM, u_* is related to the (nominal) 10-m wind speed, U_{10}, by the empirical formula proposed by Pomeroy (1988),

$$u_* = 0{\cdot}02264\ U_{10}^{1{\cdot}295} \tag{3}$$

where u_* and U_{10} are both in units of m s^{-1}.

Pomeroy and Gray (1990) make use of a 'mean saltating mass flux', $\bar{q}_{\text{salt}} = Q_{\text{salt}}/h_s$ (kg m^{-2}s^{-1}), where h_s is the 'mean saltation trajectory height'. The mean 'saltation drift density' is then,

$$\bar{\rho}_{\text{salt}} = \bar{q}_{\text{salt}}/u_{\text{p}}$$

where u_{p} is the 'saltation velocity' [$= cu_{*_t}$, where we will use $c = 2{\cdot}3$ — the value used in Pomeroy (1988), although Pomeroy and Gray (1990) gave 2·8]. P. R. Owen (cited by Pomeroy and Gray, 1990) and Greeley and Iversen (1985) propose $h_s = (a/2)u_*^2/g$ and choose $a = 1{\cdot}6$. This then also corresponds to the $h_* (= u_*^2/12{\cdot}25)$ used by Pomeroy and Gray (1995, Equations 39, 40) as the 'thickness of the saltation layer'.

In the PBSM, q_{salt} and ρ_{salt} are, in effect, set equal to the mean values defined above and assumed not to vary with height through the depth of the saltation layer. We have also considered an exponential variation of ρ_{salt} with z, following Kawamura (1948) and Takeuchi (1980), see also Pomeroy and Gray (1995, p. 61). With $\rho_{\text{salt}} \propto \exp(-z/\pi h_s)$, and assuming the same mean concentration within the saltation layer, this modification to the PBSM had little effect except when it was used to modify the lower boundary condition for the suspension layer.

Suspension of snow can occur only when particles initially in the saltation mode have a terminal velocity that is balanced by upward motions in the fluid resulting from turbulence. The suspended snow transport rate Q_{susp} (kg m^{-1}s^{-1}) developed by Pomeroy and Male (1992) is given by

$$Q_{\text{susp}} = (u_*/\kappa)\int_{z_{\text{lb}}}^{z_{\text{ub}}} \rho_{\text{susp}}(z)\ln(z/z_0)\text{d}z \tag{4a}$$

where z_{lb} and z_{ub} are the lower and upper boundaries for the suspension layer (m). Within the PBSM the roughness length and friction velocity are replaced by effective values, as discussed below. In the PBSM, the upper boundary height is assumed to be dependent on the fetch over uniform, mobile snow, although in the PBSM code an upper limit of 5 m is often imposed for snow transport calculations, as suggested by Pomeroy *et al.* (1993) 'for surface hydrology purposes'. The (non-linear) equation used for z_{ub} (m) at fetch x (>300 m) is

$$z_{\text{ub}} = 0{\cdot}3 + \kappa^2(x - 300)[\ln(0{\cdot}3/z_0^{\text{eff}})\ln(z_{\text{ub}}/z_0^{\text{eff}})]^{-0{\cdot}5} \tag{4b}$$

Iterative solutions to this equation are shown in Figure 2a for three values of U_{10}, the wind speed dependence is through the effective roughness length, z_0^{eff}, to be discussed below [Equation (12)]. We can see that the growth is almost linear, since the logarithm term in Equation (4b) will vary only slowly with x.

For most of the results to be presented in this paper we have set the fetch distance to 500 m, and impose the 5-m upper limit on z_{ub} for transport calculations, but not for sublimation calculations. Values of z_{ub} from Equation (4b) are typically between 5 and 10 m, depending on wind speed. This fetch was chosen

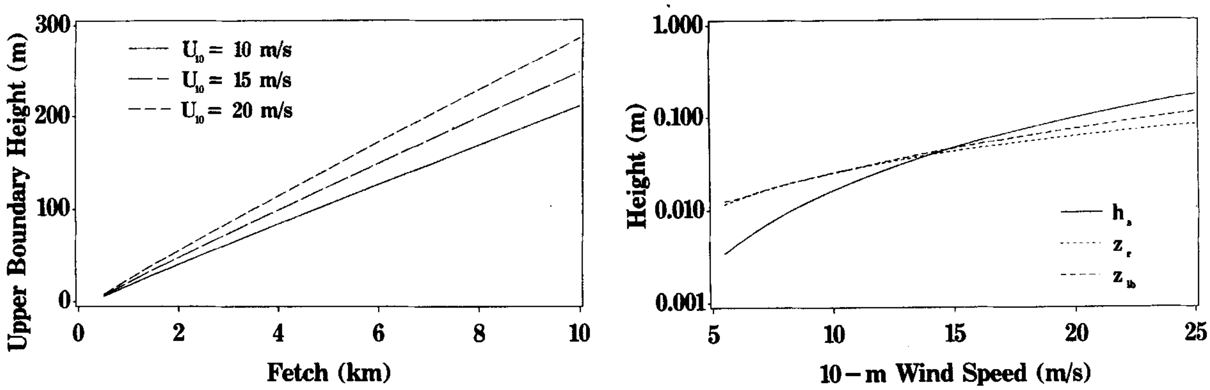
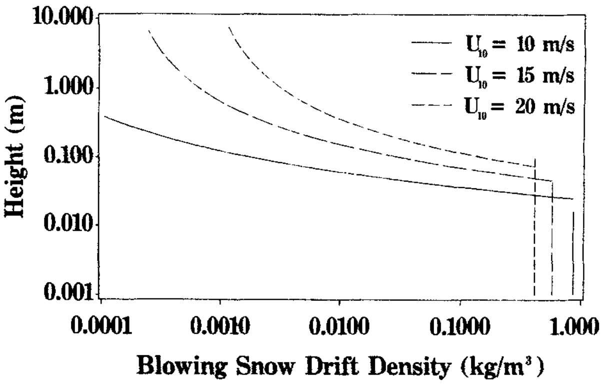


Figure 2. (a) Upper boundary height, z_{ub}, as a function of fetch for three wind speeds. (b) Variation of h_s, z_r and z_{lb} with wind speed in the PBSM model



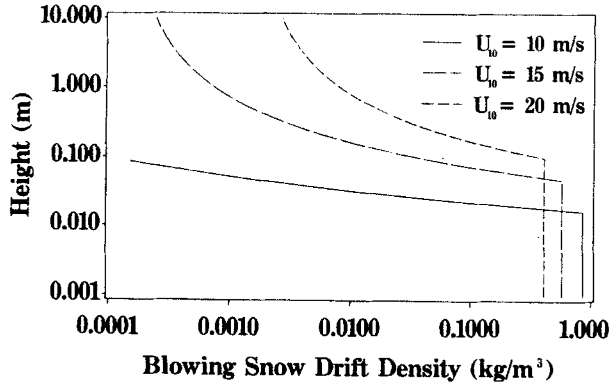


Figure 3. (a) The snow drift density profiles used in the PBSM for U_{10} values of 10, 15 and 20 m s^{-1}. Vertical lines indicate snow drift densities in the saltation layer, while the lines extend just to h_s. (b) As in (a), but for the snow drift density profiles used in a modified version of the PBSM, with $\rho_{susp} = \rho_{salt}$ at $z = h_s$

as typical and for comparison with the results presented by Pomeroy *et al.* (1993, Figure 3). Calculations for longer fetches show significant increases in vertically integrated sublimation rates, despite relatively low suspended snow concentrations at upper levels. The suspension snow drift density, $\rho_{susp}(z)$ (kg m^{-3}), for drifting snow with a spectrum of particle sizes, is given (Pomeroy and Male, 1992) by the finite difference recursion formula,

$$\rho_{susp}(z) = \rho_{susp}(z - \mathrm{d}z)[z/(z - \mathrm{d}z)]^{w_*} \tag{5a}$$

equivalent, in differential form to,

$$\mathrm{d}\rho_{susp}/\mathrm{d}z = \rho_{susp} w_*/z. \tag{5b}$$

Here, w_* is a dimensionless vertical velocity, proportional to a mean settling velocity divided by u_*, and was found to vary with height as

$$w_*(z) = -0{\cdot}8412 z^{-0{\cdot}544} \tag{6}$$

for data collected in the Canadian Prairies. With this form for w_* we can solve Equation (5b) to give

$$\rho_{susp}(z) = \rho_r \exp[-1{\cdot}55(z_r^{-0{\cdot}544} - z^{-0{\cdot}544})] \tag{5c}$$

The lower boundary condition on ρ_{susp} for the PBSM was formulated by Pomeroy *et al.* (1993) in terms of a height, z_r, at which ρ_{susp} is equal to a standard value, ρ_r, of 0·8 kg m^{-3}. This height, z_r, is given by the relationship,

$$z_r = 0{\cdot}05628 u_* \tag{7}$$

where z_r is in m and u_* is in m s^{-1} (Pomeroy and Male, 1992, Equation 33). [Note that the coefficient is 0·05628s, not s$^{-0.544}$, as indicated in Pomeroy *et al.* (1993, p. 169).] An increase in u_* will cause z_r to increase and thereby increase ρ_{susp} at fixed values of z. Pomeroy (1988) uses Equation (5a), equivalent, in effect, to Equation (5c), to fix the lower boundary of the suspension layer. This is placed at the level, z_{lb}, where $\rho_{susp} = \rho_{salt}$. The three levels h_s, z_r, and z_{lb} are shown in Figure 2b for a range of wind speeds. The three levels are similar for $U_{10} = 15$ m s^{-1} but are different in general. When $z_{lb} < h_s$ (high wind speeds) there will be an overlap of the saltation and suspension layers, while for low wind speeds there will be a gap between them. These features clearly have an influence on the mass flux and sublimation calculations. The rational for this arrangement (J. W. Pomeroy, personal communication) lies in the fact that there will, in practice, be a transition region between the lower, primarily saltation, layer and the upper, suspension, layer and that h_s is a representative height for the saltation layer rather than a well-defined upper boundary. Snow density profiles, $\rho_s(z)$, with the PBSM assumptions, are shown for three wind speeds

Table I. Original PBSM and variable density model results for a range of wind speeds*

PBSM results							
U_{10} (m/s)	U(10 m) (m/s)	u_* (m/s)	ρ_{salt} (kg/m^3)	Q_{salt} (g/ms)	Q_{susp} (g/ms)	z_0^{eff} (m)	h_s (m)
5·0	4·92	0·18	0·000	0·00	0·000	0·0002	0·000
10·0	10·06	0·45	0·856	5·95	15·03	0·0012	0·016
15·0	15·02	0·76	0·575	11·41	75·83	0·0035	0·047
20·0	19·75	1·10	0·409	17·11	268·7	0·0074	0·098
25·0	24·24	1·46	0·310	23·13	697·1	0·0131	0·175

Variable density model results					
U_{10} (m/s)	U(10 m) (m/s)	u_p (m/s)	Q_{salt} (g/ms)	Q_{susp} (g/ms)	z_0^{eff} (m)
5·0	5·00	0·000	0·00	0·000	0·0002
10·0	10·29	0·425	5·92	22·70	0·0027
15·0	17·74	0·425	11·36	116·2	0·0023
20·0	26·10	0·425	17·03	464·5	0·0020
25·0	35·17	0·425	23·03	1332·4	0·0018

* Nominal U_{10} values are used to determine the u_* values used in both models from Equation (3). U(10 m) values are the actual 10-m wind speeds from the models

in Figure 3a. The depths of the saltation layers are indicated by the extent of the vertical lines with $\rho_s = \rho_{salt}$, while the suspension layer profiles stop at the point where $\rho_{susp} = \rho_{salt}$. Scaling apart, these profiles only differ from those presented by Pomeroy and Gray (1995, Figure 36) in that we have not attempted to blend the saltation and suspension layer segments. Vertically integrated transport rates, Q_{salt} and Q_{susp}, together with other parameters are listed in Table I for a range of wind speeds.

One problem with this formulation for ρ_{susp} is that as $z \to \infty$, $\rho_{susp} \to [\rho_r \exp(-1{\cdot}55 z_r^{-0{\cdot}544})]$ which, though small, is non-zero. The upper limit for the suspension layer, z_{ub}, is thus an important parameter in determining Q_{susp} and in limiting the sublimation rates predicted by the model.

As a possible modification of the PBSM we tried using $\rho_{susp} = \rho_{salt}$ at $z = h_s$ as the lower boundary condition for Equation (5b). Snow drift density profiles for this case are shown in Figure 3b. Compared with the profiles in Figure 3a, these show higher concentrations at high wind speeds ($20\,\mathrm{m\,s^{-1}}$) and lower concentrations at low speeds ($10\,\mathrm{m\,s^{-1}}$) in the suspension layer. For U_{10} of the order of $10\,\mathrm{m\,s^{-1}}$, compared with the observations presented by Pomeroy and Male (1992, Figure 2), these modified profiles appear to predict ρ_{susp} values that are too low and we do not recommend this alternative.

STRATIFICATION OWING TO SNOW SALTATION AND SUSPENSION

In the absence of blowing or precipitating snow, the horizontal velocity of air in the neutrally stratified surface boundary layer (say the lowest 10–50 m) is well approximated by the log-law profile discussed above [Equation (2)]. Taylor and Dyer (1977), following Barenblatt (1953, 1955), have shown, however, that the presence of suspended sediment load in water flows can lead to density stratifications and modified velocity profiles. Furthermore, it has been shown that, when snow transport is occurring, the log-law velocity profile becomes inaccurate (Bagnold, 1941; Owen, 1964; Schmidt, 1982a), particularly in the saltation layer where drift densities of the order of $1\,\mathrm{kg\,m^{-3}}$ are not uncommon (Mellor and Fellers, 1986; Pomeroy and Gray, 1990). This suggests that we need to consider possible effects of density stratification in blowing snow.

To represent modified velocity and concentration profiles taking into account stratification effects, Taylor and Dyer (1977) develop profile equations for the suspension of sand in water using Monin–Obukhov similarity theory (Monin and Obukhov, 1954). They assume that the mixture of water plus suspended particles behaves as if it were a single-phase fluid and ignore any two-phase effects. In steady, equilibrium flow there is a balance between upward diffusion and downward settling. Ignoring stratification effects and assuming that the suspended material consists of particles of the same size with a single settling velocity, w_s, leads to the classic solution for suspended sediment density,

$$\rho_s(z) = \rho_r[(z + z_0)/(z_r + z_0)]^{-b} \tag{8}$$

where $b = w_s/\kappa u_*$. This is generally applied for z, $z_r \gg z_0$ for which the z_0 terms can be neglected. If suspension were the dominant process it may be appropriate in some models to take $z_r = 0$ and the z_0 terms must be retained.

The effects of density stratification on the eddy diffusivity are assumed to be given by Monin–Obukhov similarity theory, applied with local, rather than surface, values of the Obukhov length L, which is given, for example by Taylor and Dyer (1977), as,

$$L = -u_*^2\rho_a/[\kappa^2 g(z + z_0)\partial\rho_s/\partial z] \tag{9}$$

In the present case it may be appropriate to define L in terms of u_*^{eff}(defined below) rather than u_*, but differences will generally be small.

Applying this equation for blowing snow in air we find that typical values of z/L are generally very small ($<0{\cdot}02$) except in a thin layer near z_{lb} where values are of the order of 0·1, as illustrated in Figure 4 for the PBSM blowing snow density profiles represented in Figure 3a. Note that L will be infinite in the saltation layer under the assumption that ρ_{salt} is independent of z. Thus, these stratification effects can generally be ignored for the suspension and saltation of snow in air, confirming the conjecture made by Pomeroy and Male (1987). The corresponding velocity profiles deviate only marginally from the log-law velocity profile, unlike the cases of suspended sediments in water presented by Taylor and Dyer (1977). In addition this implies that Equation (8) could be applied independently for each size group in suspended material with a range of particle sizes, as in Anderson and Hallett (1986), and provided of course that no sublimation or other processes causing modifications to individual particle sizes were active.

DENSITY AND STRESS PARTITION EFFECTS ON THE CONSTANT STRESS LAYER VELOCITY PROFILE

A possible effect of the presence of suspended material such as blowing snow lies in the potential modification caused to a constant stress layer velocity profile due directly to density effects. The Boussinesq approximation, in which density variations are retained in buoyancy terms but neglected in inertia terms (see, for example, Baines, 1995, p. 7), is normally invoked in a flow with small relative density variations. However, with snow densities of the order of 1 kg m^{-3} in air of similar density, this may no longer be appropriate.

In saltation conditions it is clear that particle velocities are significantly different from the local air velocity and a two-phase treatment of the flow is normally required. One way to approach this, as described by

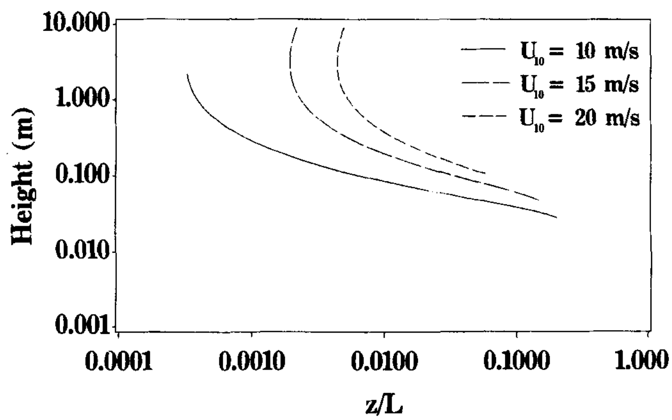


Figure 4. The profiles of height (z) divided by the Obukhov length (L) for the blowing snow profiles depicted in Figure 3a

Kind (1990) or McEwan (1993) for example, is to separate the momentum transfers associated with the air and with the saltating material. Their sum will be independent of height. Horizontal acceleration of the saltating particles over the ascent, and possibly part or most of the descent portions of their trajectories, extracts momentum from the air. Some momentum may be returned to the air as the particle descends close to the surface through slow moving air, but most will be lost when the particle crashes into the surface. Both mechanisms represent a downward transfer of momentum across horizontal surfaces — a shear stress contribution associated with the saltating particles. McEwan (1993) offers two alternative empirical approximations to the stress profiles and explains how the resulting air velocity profile can depart from the basic logarithmic form [Equation (2)]. Once material is in suspension a single-phase treatment becomes more acceptable but there may still be an effect from differences between instantaneous, turbulent fluid and particle velocities.

As a rather crude but simple illustration of density effects, we treat the air–snow mixture (in both saltation and suspension situations) as a single-phase fluid with total density $\rho(z) = \rho_a + \rho_s(z)$. We consider a steady, horizontally homogeneous flow and assume that the shear stress (τ) is constant with height. Then, a simple mixing length formulation of the relationship between the shear stress and the velocity gradient leads to

$$\tau = \rho(z)[\kappa(z + z_0)\mathrm{d}U/\mathrm{d}z]^2 = \rho_a u_*^2 = \text{constant} \tag{10}$$

where u_* is the friction velocity required to give the same stress in clear air. This then gives,

$$\kappa(z + z_0)\mathrm{d}U/\mathrm{d}z = u_*^{\mathrm{eff}} \tag{11a}$$

where

$$u_*^{\mathrm{eff}}(z) = [\rho_a/(\rho_a + \rho_s)]^{0.5} u_* \tag{11b}$$

Equation (11a) can be integrated numerically from the initial condition, $U = 0$ and $z = 0$, given the vertical distribution of ρ_s, and assuming that the implied simple mixing length closure still applies. In principle it could alternatively be integrated upwards from z_{lb}, if one knew the velocity at that level. The quantity u_*^{eff} is equivalent to the effective friction velocity discussed by Anderson and Haff (1991) or McEwan (1993) or the 'two-phase' and 'local' friction velocities defined by Pomeroy and Male (1987) to account for flow density effects in the saltation and suspension layers respectively.

Figure 5a depicts u_*^{eff}/u_* profiles for the PBSM blowing snow density distribution of Figure 3a, for nominal 10-m wind speeds of 10, 15 and 20 m s^{-1}. We say 'nominal' because these were the wind speeds used in

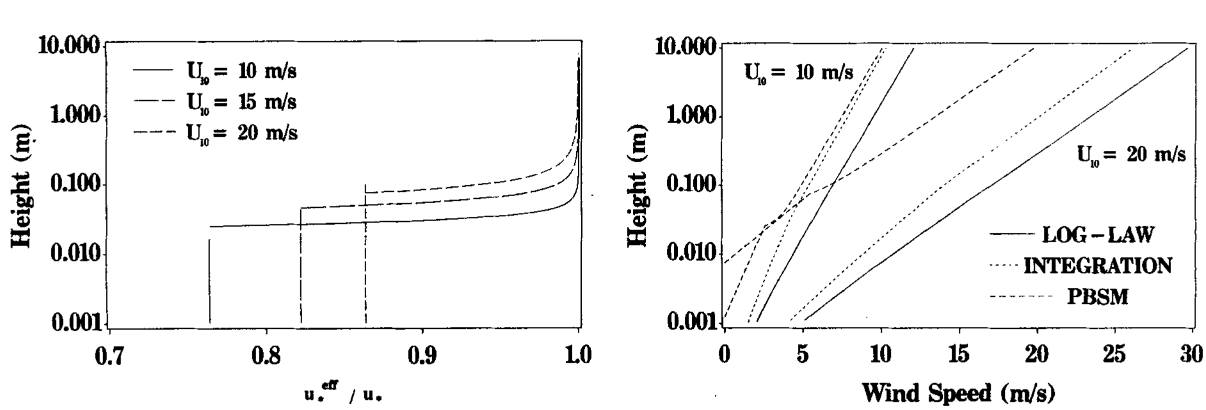


Figure 5. (a) Profiles of effective friction velocity (u_*^{eff}) divided by the reference friction velocity (u_*) for the blowing snow profiles depicted in Figure 3a. (b) Velocity profiles (labelled 'INTEGRATION') integrated from the surface using Equation (11a) and the u_*^{eff} shown in Figure 5a for $U_{10} = 10$ and 20 m s^{-1}. Results are compared to log-law wind profiles (labelled 'LOG-LAW') with the basic u_* and z_0 and with log-law profiles (labelled 'PBSM') based on u_*^{eff} and z_0^{eff}, as used by the PBSM in the suspension layer, but here extended into the saltation layer

Equation (3) to calculate u_* and the subsequent snow densities, but integration of Equation (11a) above may give different values. We see in Figure 5a, for example, that u_*^{eff} is reduced by approximately 25% relative to u_* near the surface for $U_{10} = 10\,\text{m s}^{-1}$, in order to maintain a constant shear stress layer as the density increases, while for $U_{10} = 20\,\text{m s}^{-1}$ there is about a 14% reduction at a depth of 0·1 m (the saltation layer). Note that for $u_* > \sqrt{3}u_{*_t}$ ($U_{10} > 7{\cdot}7\,\text{m s}^{-1}$), ρ_{salt} decreases with increasing wind speed. Figure 5b illustrates the variable density model wind profiles for the 10 and $20\,\text{m s}^{-1}$ cases, as well as the log-law profile in the absence of blowing snow [with $z_0 = 0{\cdot}0002$ m but the same u_* from Equation (3)] and the modified log-law profiles assumed in the PBSM (with $z_0 = z_0^{\text{eff}}$ and $u_*(z) = u_*^{\text{eff}}$). The assumption within the PBSM that u_*^{eff} is reduced when part of the stress is carried by the particles is sound. The model's use of the log-law profile [Equation (2)] with u_*^{eff} is, however, not justified and leads to a failure to satisfy Equation (11a) because of an additional term involving $\partial u_*^{\text{eff}}/\partial z$. This anomaly means that a reduced friction velocity in the lower part of the suspension layer occurs in conjunction with an increased velocity shear $\partial U/\partial(\ln z)$, as can be seen in Figure 5b. We consider this to be unrealistic. However, these PBSM profiles have been extended below z_{lb} here since we will use them later in determining ventilation velocities for the saltation layer. The PBSM values for z_0^{eff} are based on the result from Pomeroy and Gray (1990),

$$z_0^{\text{eff}} = 0{\cdot}1203u_*^2/2g \tag{12}$$

For large z (say 10 m), when $u_*^{\text{eff}} \approx u_*$ (see Figure 5a), Equation (3) for u_* and Equation (12) for z_0^{eff}, as used in the PBSM, can lead to a wind speed U(10 m) for the logarithmic profile that may differ slightly from U_{10}. J. W. Pomeroy (personal communication) remarks that U_{10} is a nominal value intended to correspond to data from a surface synoptic station and need not exactly match the actual wind speed at the site with blowing snow.

In both the basic PBSM and our illustrative variable density model, as a consequence of the presence of saltating and suspended snow, velocity profiles are modified and the effective roughness lengths, z_0^{eff}, are higher than the original z_0. There is reasonable agreement between our variable density model result and the profile obtained from the PBSM for $U_{10} = 10\,\text{m s}^{-1}$ but not for $U_{10} = 20\,\text{m s}^{-1}$. For profiles based on integration of Equation (11a) we can extrapolate down from the upper portions of the profile to obtain variable density model z_0^{eff} values. Values for a range of wind speeds and with blowing snow density profiles based on the PBSM model are given in the lower half of Table I. Adjusted values of U(10 m), h_s, $\bar{\rho}_{\text{salt}}$ and the transport rates Q_{salt} and Q_{susp} are also listed.

The increase during saltation in z_0^{eff} from its value under non-transport conditions (0·0002 m here) has been widely reported in the literature (Bagnold, 1941; Owen, 1964; Kind, 1976; Iversen, 1980; Pomeroy and Gray, 1990) and has been shown to be a function of u_*^2 as in Equation (12). It is clear that, with the assumptions made about ρ_s in our variable density model, the increase in z_0^{eff} owing to the presence of suspended material is not as large as expected for the higher wind speed cases and does not match the quadratic dependence on u_* postulated above. It also leads to much higher values of U(10 m) for a given u_* and, consequently, to higher estimates of Q_{susp}. Overall it would appear that the simple, single-phase approach to accounting for the dynamic effects of blowing snow through density increases fails to predict as large an effect on the velocity profiles as observed or as included in the original PBSM model, and in the next section we will revert to the basic PBSM formulation. Although the abrupt discontinuities between the saltation and suspension layers may seem a little artificial, it appears that this may be the most practical modelling approach at this time.

BLOWING SNOW SUBLIMATION

During snow transport by saltation or suspension, sublimation can play an important role. In addition to acting as a significant source of water vapour and sink of sensible heat in the air, sublimation may lead to a decrease in radius and mass of the snow or ice particles and hence will reduce their drag coefficient. This will mean that, strictly speaking, one cannot treat the blowing snow 'problem' by considering particles of a uniform size (and terminal velocity) and then adding or integrating over the spectrum of sizes present as a final step in determining snow mass distributions and fluxes as a function of z. As a result of sublimation, there will be a continuous transfer through the drop-size spectrum towards smaller particle radii, and it will be necessary to treat the advection, diffusion and sublimation of particles with a spectrum of sizes.

In the Canadian Prairies, Pomeroy and Gray (1994, 1995) have argued that up to 75% of the annual snowfall over a one-kilometre fallow field is eroded by wind, and that, typically, half of this amount is sublimated before deposition. In the PBSM, Pomeroy (1988) and Pomeroy *et al.* (1993) had anticipated the significance of snow transport by wind and its sublimation in the prairies of Canada. The model determines the sublimation rate for a column of blowing snow from the particle of mean size from a certain distribution of particles in a steady-state atmosphere. The essential details will be described below.

The distribution of particles in blowing snow is shown by Budd (1966) and Schmidt (1982b, 1986) to be well represented by a two-parameter gamma distribution given by

$$f(r,z) = \frac{r^{(\alpha-1)}e^{(-r/\beta)}}{\beta^{\alpha}\Gamma(\alpha)} \tag{13}$$

where $f(r,z)$ is the particle frequency, α and β are shape and scale parameters respectively, and Γ is the gamma function. Schmidt (1982b) found that in the suspension layer, α varies with height z (m) as

$$\alpha(z) = 4{\cdot}08 + 12{\cdot}6z \tag{14}$$

The scale parameter is then given by $\beta = \bar{r}/\alpha(z)$ where $\bar{r}$ is the mean particle radius (m). The mean particle radius also varies with height, z (m), as, (Pomeroy, 1988)

$$\bar{r}(z) = 4{\cdot}6 \times 10^{-5} z^{-0{\cdot}258} \tag{15}$$

In the saltation layer, however, values of $\bar{r} = 100\,\mu$m and $\alpha = 5$ are used to determine $f(r,z)$.

Given the size of any particle, assuming sphericity, its mass can then be found by

$$m = (4/3)\pi\rho_i r^3 \tag{16}$$

where m is the mass of a single snow particle (kg) and ρ_i is the density of ice (900 kg m^{-3}). Blowing snow is assumed to have the density of ice, since snow that has precipitated to the surface is usually transformed into ice very quickly by the effects of the wind (Pomeroy, 1988).

Ideas on the sublimation of blowing snow have been developed in part by Dyunin (1959), Schmidt (1972, 1991), Lee (1975), Male (1980) and Pomeroy (1988). The process is controlled by the rate at which water vapour is being removed from the snow particle, as well as the amount of energy being delivered to the same particle. The final equation (Pomeroy and Gray, 1995, Equation 47) for the rate at which snow mass is lost by sublimation is,

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{2\pi r\sigma - \frac{Q_s}{\lambda_T T_a Nu}\left[\frac{L_s M}{RT_a} - 1\right]}{\frac{L_s}{\lambda_T T_a Nu}\left[\frac{L_s M}{RT_a} - 1\right] + \frac{1}{D\rho_{sat} Sh}} \tag{17}$$

where σ (dimensionless and negative) is the water vapour deficit with respect to ice $(e - e_i)/e_i$, where e_i is the saturation vapour pressure (svp) over ice, and approximately equal to relative humidity -1; Q_s is the radiant energy received by the particle (W m^{-2}); λ_T is the thermal conductivity of air (W m^{-1} K^{-1}); T_a the ambient air temperature (K); M the molecular weight of water (18·01 kg kmol^{-1}); L_s the latent heat of sublimation (2·838 $\times$ 10^6 J kg^{-1}); R the universal gas constant (8313 J kmol^{-1} K^{-1}); D the diffusivity of water vapour (m^2 s^{-1}); ρ_{sat} the saturation density of water vapour (kg m^{-3}); and Nu and Sh the Nusselt and Sherwood numbers, respectively. Note that the equation for λ_T given in papers describing the PBSM (e.g. Pomeroy *et al.* 1993) appears to be in error by a factor 10. We have corrected this in the PBSM calculations included in the present paper.

In essence, the sublimation calculations of the PBSM and our modified spectral version are quite similar. However, the inclusion of a spectrum of particles at each height in our version of the model leads to some modifications of the PBSM equations. For instance, the Nusselt and Sherwood numbers of Equation (17) are dependent on the Reynolds number $Re(r,z)$ as

$$Nu(z) = Sh(z) = 1{\cdot}79 + 0{\cdot}606Re^{0{\cdot}5} \tag{18}$$

and

$$Re = 2rV_r/\nu \tag{19}$$

where V_r is the ventilation velocity of the particles (m s^{-1}) and ν is the molecular kinematic viscosity. For simplicity, one could neglect the effects of turbulence on V_r, as does Schmidt (1982b), such that, in the suspension layer, V_r is equal to the terminal velocity (w_s) of the particles (Pomeroy and Male, 1986) described by

$$V_r = w_s(r) = 1{\cdot}1 \times 10^7 r^{1{\cdot}8} \tag{20}$$

One set of calculations has been made with this assumption while another set includes the effects of turbulence, following the model described by Lee (1975).

For the saltation layer, the PBSM (Pomeroy and Gray, 1995, Equation 59) sets

$$V_r = 0{\cdot}68u_* + 2{\cdot}3u_{*_t}$$

on the basis that this is a sum of a vertical component ($0{\cdot}68u_*$, or $0{\cdot}63u_*$ in Pomeroy, 1988) and a horizontal component which equals the saltation velocity, u_p. Pomeroy (1988) argues that, in terms of order of magnitude, 'the speed of the saltating particles relative to the wind speed is equal to the speed of the saltating particles relative to the surface'. Although we see little justification for this, it may well be as good an estimate as any other!

If we assume that the saltating particles are generally moving with lower horizontal velocity (u_p) than the air $U(z)$, and that this difference will provide the main component of the ventilation velocity, we arrive at

$$V_r(z) = |U(z) - u_p| \tag{21}$$

which we use for our calculations with the spectral model. Here, $U(z)$ is determined from Equation (2) with $z_0 = z_0^{\mathrm{eff}}$ and $u_* = u_*^{\mathrm{eff}}(z)$, as in the PBSM profiles of Figure 5b.

From Equation (17), we can determine a sublimation loss rate coefficient $V_s(r)$(s^{-1}) given by

$$V_s(r) = (\mathrm{d}m/\mathrm{d}t)/m \tag{22}$$

The sublimation rate (negative) per unit area of snowcover, Q_{subl} (kg m^{-2} s^{-1}), can then be found by integrating the sublimation loss rate coefficient, multiplied by the frequency of a certain particle size and its mass, over the particle spectrum and over height. This gives

$$Q_{\mathrm{subl}} = \int\int N(z)V_s(r)f(r,z)m(r)\,\mathrm{d}r\,\mathrm{d}z \tag{23}$$

where dr is the particle size increment (m) and N the total number of particles per unit volume.

We now compare results of the regular version of the PBSM with the modified version, including the sublimation of a spectrum of snow particles with, for example, the snow drift density profiles of Figure 3a. Particle size bins of 4 μm are used and calculations are performed for the particle of mean size in each bin varying from 2 to 254 μm. We assume that $T_a = -10$°C throughout the blowing snow column and set the relative humidity $RH = 70\%$ at $z = 2$ m. Pomeroy *et al.* (1993, Equation 19) assume a profile for the undersaturation, $\sigma(= RH - 1)$ given by

$$\sigma(z) = \sigma_2(1{\cdot}02 - 0{\cdot}027 \ln z) \tag{24}$$

where σ_2 is the undersaturation at $z = 2$ m, stating that 'prairies consistently showed a decrease in relative humidity with increasing height during blowing snow'. Unfortunately Equation (24) has the opposite tendency. A profile with the opposite sign for the vertical gradient would be given by

$$\sigma(z) = \sigma_2(0{\cdot}98 + 0{\cdot}027 \ln z) \tag{25}$$

This will be used for most of our sublimation calculations and is apparently what Pomeroy *et al.* had intended (J. W. Pomeroy, personal communication). The difference between the two profiles has most effect in the saltation layer.

Sublimation rate results using Equation (25) for σ are depicted in Figure 6 for a fetch of 500 m and with the total incident radiation, $Q_* = 120$ W m^{-2}. These show that Q_{subl} increases rapidly with wind speed (note

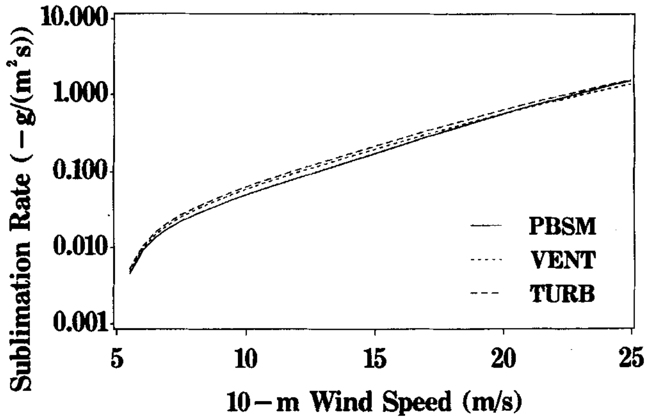


Figure 6. The cumulative sublimation rates Q_{subl} ($-\mathrm{g\,m^{-2}\,s^{-1}}$) for columns of blowing snow over a unit area of snow surface extending to the top of the surface boundary layer for U_{10} varying from 5·5 to 25·0 m s^{-1}. The transport threshold wind speed at $z = 10$ m is 5 m s^{-1}, the relative humidity at $z = 2$ m is 70%, the ambient air temperature is −10°C and the unobstructed fetch is 500 m. The line labelled 'PBSM' is based on original PBSM results, the line labelled 'VENT' is for our spectral version of the PBSM with no turbulence effects on ventilation velocity, as opposed to the line labelled 'TURB' in which turbulence effects are included

the logarithmic scale). Our spectral version of the PBSM shows slightly higher values of Q_{subl} than the original version, when $U_{10} \leqslant 20\,\mathrm{m\,s^{-1}}$. The differences are a result both of the changes in assumptions concerning ventilation velocities and of the use of a spectrum of particles at each height. Table II provides additional details of the sublimation rates per unit area for selected wind speeds, and also compares PBSM values with the two different undersaturation profiles discussed above. We can see that there are some differences between sublimation rates calculated with the mean particle PBSM and the spectral version in both the suspension and saltation layers. The largest relative changes are in the saltation layer. The

Table II. Vertically integrated sublimation rates from various versions of the PBSM

U_{10} (m/s)	Model	SUB_{salt} ($-$g/m^{2}s)	SUB_{susp} ($-$g/m^{2}s)	SUB_{tot} ($-$g/m^{2}s)	SUB_{tot} ($-$g/m^{2}s)	SUB_{tot} ($-$g/m^{2}s)
10	$PBSM_{25}$	0·0369	0·0127	0·0496		
	VENT	0·0450	0·0137	0·0587		
	TURB	0·0450	0·0182	0·0632		
15	$PBSM_{25}$	0·0774	0·0951	0·1725		
	VENT	0·1109	0·0854	0·1963		
	TURB	0·1109	0·1042	0·2151		
20	$PBSM_{25}$	0·1255	0·4343	0·5598		
	VENT	0·1994	0·3656	0·5649		
	TURB	0·1994	0·4320	0·6313		
25	$PBSM_{25}$	0·1815	1·3203	1·5019		
	VENT	0·3103	1·0516	1·3618		
	TURB	0·3103	1·2300	1·5403		
					$x = 1$ km	$x = 10$ km
10	$PBSM_{24}$	0·0477	0·0150	0·0627	0·068	0·085
15	$PBSM_{24}$	0·0947	0·1000	0·1947	0·301	1·597
20	$PBSM_{24}$	0·1474	0·4366	0·5840	1·199	8·817
25	$PBSM_{24}$	0·2068	1·2994	1·5062	3·498	28·720

Fetch, $x = 500$ m except where indicated. Values labelled 'PBSM' are based on original PBSM results, the subscript (24 or 25) relates to those equation numbers and identifies the assumed subsaturation profile. Values labelled 'VENT' are for our spectral version of the PBSM with no turbulence effects on ventilation velocity, as opposed to those labelled 'TURB' in which turbulence effects are included. VENT and TURB results use Equation (25) for subsaturation

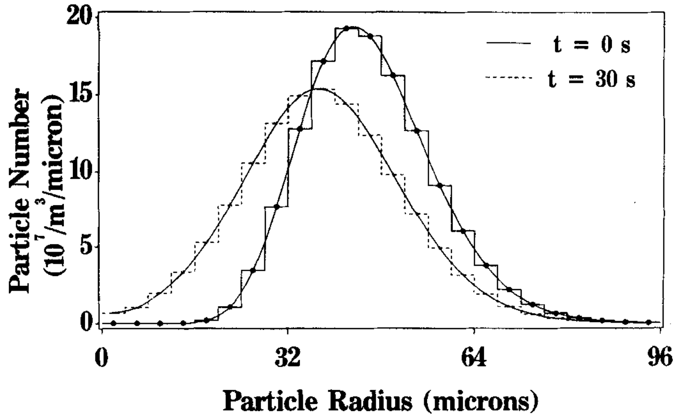


Figure 7. The initial particle distribution and the particle distribution after 30 s of sublimation at $z \approx 1{\cdot}0$ m. Environmental conditions are as in Figure 6 for a 10-m wind speed of $15\,\mathrm{m\,s^{-1}}$. No turbulence effects are considered here. Particle size bins are of 4 μm and remain constant, but the number of particles in each will vary in time due to sublimation

change in the assumed ventilation velocities and the relatively large (negative) undersaturations assumed in the model calculations within the saltation layer, even after the profile changes described above, will be factors in this. In practice, and for long fetches with blowing snow, values of σ in the range $-0{\cdot}1$ to 0 may be more likely than values of the order of $-0{\cdot}25$ (cf. Schmidt, 1982b). For the contributions to sublimation rates from the suspension layer we do see some effect of the switch from a mean particle size calculation in the original PBSM to the spectral model. The VENT model gives lower values of Q_{subl} in the suspension layer at moderate and high wind speeds ($>15\,\mathrm{m\,s^{-1}}$), but TURB predictions appear closer to the original PBSM values. It is significant that the spectral model results presented here suggest that the contributions to sublimation from the saltation layer are at least as large as from suspended snow for $U_{10} \leqslant 15\,\mathrm{m\,s^{-1}}$. There would appear to be a definite need for further modelling and measurements to test this prediction and the assumptions behind it.

Sublimation calculations for longer fetches (1 km, 10 km) show large increases in the vertically integrated sublimation rates, as indicated by the sample results shown in Table II. This was anticipated by Pomeroy (1988, p. 161), and arises simply from the increased range of z over which Equation (23) is integrated. As noted earlier, we can see from Equation (5c) that $\rho_{\mathrm{susp}}(z)$ has $\rho_{\mathrm{susp}} \rightarrow$ constant $[\rho_{\mathrm{r}} \exp(-1{\cdot}55 z_{\mathrm{r}}^{-0{\cdot}544})]$ as $z \rightarrow \infty$. Unless V_{s} decays to 0 for increasing z, Q_{subl} will increase as z_{ub} increases, and for infinite fetch we would have an infinite sublimation rate, given the assumed suspended snow distribution, ventilation velocities, etc.

If we consider the impact of sublimation on the particle spectrum, under the same environmental conditions as before, we can determine the evolution of the spectrum with time. Figure 7 represents the initial gamma distribution of particles at $z \approx 1{\cdot}0$ m (for $x = 500$ m) found with Equations (13)–(15) and the resulting distribution of particles after 30 s of sublimation. Here, the humidity and temperature profiles remain fixed, and the diffusion or advection of particles is not considered. As expected, the snow particles are decreasing in size such that the whole distribution is moving towards the left and a continuous transfer through the drop-size spectrum is occurring.

CONCLUDING REMARKS

This paper has briefly examined some of the effects of blowing snow in air. We based these investigations on the PBSM but our aim in future studies will be to develop a 'self-contained' model which will include the processes giving rise to the spectrum of particles, the temperature and humidity values and their variation with height. As shown in this study, blowing snow can be a significant process for water budget considerations and should be included when modelling atmospheric boundary layers in cold environments.

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