Corporate Finance Review, Nov/Dec,7,3,13-21, 2002

The Jackknife Estimator for Estimating Volatility of Volatility of a Stock

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Abstract

In this paper we demonstrate application of a statistical technique to estimate the volatility of volatility of a stock, based on re-sampling method. The jackknife technique is easy to implement, useful in case of small sample data and does not place a heavy burden on data requirements. The paper describes the jackknife procedure and illustrates how it can be used to estimate the volatility of volatility. To demonstrate its practical use the pricing bias is analyzed using the stochastic volatility estimate as input in Hull and White (1987) model. Finally, confidence intervals are constructed for selecting among different weighting schemes as summarized in Mayhew (1995). The proposed technique is ideal for small data sets when implementing stochastic option pricing models.

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Introduction

The Black-Scholes option-pricing model for valuing a European option on a dividend-protected stock depends upon five parameters, the stock price (*S*), exercise price (*K*), volatility of the stock (σ), time to maturity (*T*) and the risk-free rate (*r*). The value of a call option (*C*) can be written as

$$C = S\mathrm{N}(d_1) - Ke^{-rT}\mathrm{N}(d_2)$$

$$\text{where} \quad d_1 = \frac{\log(S/T) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T} \ .$$

and N(*d*) is the cumulative normal distribution value evaluated at *d*.

Except for the volatility of the stock, all of the other parameters of the model are relatively easy to observe. The historic volatility of the stock can be estimated by computing the standard deviation of continuous price return of a series of recent stock prices. Along with the other parameters, the historic volatility can be plugged into the Black-Scholes formula to derive a price for the option. As an alternative, the stock's volatility can be inferred from the market price of a stock by inverting the Black-Scholes formula. This procedure involves substituting the option's market price along with values for the exercise price, the time to maturity, risk free rate and the initial stock price in the model and solving for the volatility parameter. The volatility parameter based on the market price of an option is called the implied volatility.

The Black and Scholes (BS) formula depends on 10 unrealistic assumptions but the formula works well than any other formula in a wide range of circumstances (Black 1993). Among others, a critical assumption of the BS model is that a stock's volatility is known and doesn't change over the life of the option. Contrary to this assumption, empirical evidence on stock prices and their derivatives strongly suggest that the volatility of a stock is not constant. The asset price volatility is indeed stochastic. Consequently, changes in the volatility of a stock may have a major impact on the value of an option, especially if the option is far out-of-the money.

Since the development of the path breaking BS (1973) model, many researchers have tried to relax its most stringent assumptions. Researchers have introduced stochastic volatility models that relax the constant volatility assumption. When asset prices do not exhibit continuous processes, and researchers have introduced jump diffusion models Merton (1976), Cox and Ross (1976). Many researchers including Scott (1987, 1991), Hull and White (1987, 1988), Heston (1993a and 1993b) have generalized the BS model to incorporate stochastic volatility. In order to implement stochastic option pricing models, an input parameter, the volatility of volatility of the asset has to be estimated. Ball and Roma (1994), state that estimation problems in implementing stochastic volatility models are a promising area for future research.

The following quote from Black's (1993) article and above observation by Ball and Roma (1994) motivated our research: " Since the volatility can change, we should really include ways in which it can change the formula. The option value will depend on the entire future path that we expect the volatility to take, and on the uncertainty about what the volatility will be at each point in the future. One measure of that volatility is the volatility of volatility". The basic question that leads from the above is: how can one measure a stock's volatility of volatility? Also, if the volatility of an underlying asset itself is stochastic, as assumed in stochastic volatility models, from an implementation point of view, it is important to estimate the volatility of volatility.

Hull and White (1987) suggest two alternative methods to estimate the volatility of volatility, which is a parameter in their stochastic option model. They compare each of the two methods. First, the volatility of volatility is estimated by examining the changes in volatility implied by option prices. Using the implied volatility is an indirect procedure and the results they argue can be contaminated since the changes in implied volatility to some extent can be attributed to pricing errors in the options. Alternatively, they suggest that one could use the changes in estimates of the actual variances to estimate the volatility of volatility. This however, would require very large amounts of data.

In this paper, we propose yet another statistical method to estimate the volatility of volatility based on re-sampling techniques. The jackknife re-sampling technique is easy to implement and does not place a heavy burden on data requirements. The jackknife is a versatile statistical procedure based on the principle of replicability for estimating the standard error of a statistic non-parametrically. We demonstrate the usefulness of the jackknife procedure in estimating the volatility of volatility of a stock. Although the procedure is simple to apply there is a general lack of its use in option pricing applications. The proposed technique is ideal for small data sets when implementing stochastic option pricing models.

The paper is organized as follows. First, we describe the jackknife procedure in detail. Second, we illustrate the jackknife procedure using an example from Gemmill (1992). Weighted-average techniques for computing the implied volatility have received quite a bit of attention. However, there are mixed results as to whether implied volatility is better at forecasting future volatility than estimators based on historic data. Next, we use the jackknife estimate of stochastic volatility as input in Hull and White (1987) stochastic volatility model to illustrate the pricing bias. Finally, in order to select volatility estimates based on statistical comparison of different weighting schemes, we use the jackknife volatility of volatility estimates to construct confidence intervals for different weighting schemes as summarized in Mayhew (1995).

Jackknife procedure

The jackknife method is computationally intensive but is easy to use especially if the sample size is small. Efron and Gong (1983) discuss properties of the jackknife and compare it with the bootstrap method. Buzas (1997) provides an approach that is faster for estimating the jackknife standard error. The jackknife procedure has several advantages over other methods. First, it is appropriate for small samples such as the weekly two-year stock price data typically used for estimating the historic volatility of a stock. Second, the procedure is sensitive enough to detect the influence of outliers on the analysis. Third, it uses all the data while eliminating potential bias related to the inclusion of atypical data. Finally, as we will demonstrate it can easily be implemented on a spreadsheet and does not need special statistical packages or computer programs as required for the bootstrap method.

Let R_1, R_2, ……,R_n be the percent returns for a stock over the n trading periods calculated from the sequence of the successive closing prices for that stock, i.e., the percent return

$$R_t = 100\left[\log \frac{price_t + dividends_t}{price_{t-1}}\right]$$

The standard deviation of the returns σ is a commonly used measure of the volatility of a stock. In order calculate the price on a European call using the Black and Scholes model the estimate of the historic volatility $\sigma = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(R_t - \overline{R})^2}$ can be used where the mean return $\overline{R} = \frac{1}{n}\sum_{t=1}^{n} R_t$ is based on the n returns.

In order to implement stochastic option pricing models, we need to measure the volatility of the volatility of a stock, say psi (ξ). We argue that the jackknife technique can be efficiently employed to estimate ξ, using the same returns data used to find a stock's volatility σ. The procedure is as follows. A given sample n is partitioned into m sub-samples all of which must be the same size. The value of m can range between one and the largest multiplicative factor of n. We first calculate a pseudovalue $\theta_{n-1,i}$, which is the standard deviation of returns after each observation is omitted. We repeat computation of pseudovalues based on sub-samples to obtain the jackknife estimate of the volatility of the volatility. Notice that the jackknife statistic is based on the (n – 1) returns excluding R_i for the ith sub-sample. For example, if there are 19 observations for stock returns in the sample, to generate the first pseudovalue, the first observation is omitted and the statistic computed from the remaining 18 observations: to generate the second sub-sample the second observation is omitted from the given sample of 19 observations. This procedure is repeated until 19 pseudovalues are computed. Then following Tukey (1958), we estimate the volatility of volatility ξ, by

$$\hat{\xi} = \sqrt{\frac{n-1}{n}\sum_{i=1}^{n}(\theta_{n-1,i} - \bar{\theta}_{n-1})^2}$$

where $\bar{\theta}_{n-1} = \sum_{i=1}^{n} \theta_{n-1,i} / n$.

An Example

We illustrate the jackknife procedure using an example from Gemmill (1992). We selected the same data that Gemmill (1986) used to analyze different volatility weighting schemes and implied volatility. In Table 1, 20 end of week prices and dividends for the British oil company, BP for the period August 23 to January 3, 1992 are presented. The return for week 2 in Table 1 is computed as R_2 = 100[ln (352.5/347)] = 1.5726. The average return of the stock price is -0.8452 per week and standard deviation is (σ_w = 3.0363%) per week. The annual historic volatility of BP stock is calculated as 22%. The historic volatility of 22% could then be used in the Black and Scholes model as a forecast of future volatility.

Next, by applying the jackknife procedure we estimate the volatility of volatility of BP stock as ξ = 3.26%. The Microsoft Excel formulas for computing the average returns, volatility, pseudovalues and the volatility of volatility are presented in Table 1. The jackknife procedure is computationally efficient, since the same 20 end of week prices that are used to estimate the historic volatility can be used to estimate the volatility of volatility. It requires less data unlike the estimation procedures of Hull and White that uses changes in estimates of the actual variances. Furthermore, it is also easy to implement unlike bootstrap procedures. These advantages make it a good choice as an estimation procedure for implementing stochastic volatility models. In the next section, we use the stochastic volatility parameter that we estimated using the jackknife technique to illustrate the pricing bias of a BP call option.

[Insert Table 1 Here]

Pricing Bias of BP Calls Caused by Stochastic Volatility

In order to illustrate the pricing bias due to stochastic volatility, we use BP call option data from Gemmill (1992). A series of BP calls with an initial stock price S = 291, volatility σ = 22%, days to maturity (T) = 19 for January call, 110 for April call and 201 for July has the market prices as shown in Table 2. The risk free rate is r = 11% or (0.1044 compounded continuously). The BS prices based upon the historic volatility of 22% are given in brackets in Table 2.

[Insert Table 2 Here]

In order to demonstrate the pricing bias we use the stochastic volatility parameter ξ = 3.26% estimated for BP stock using the jackknife procedure as an input in the Hull and White (1987) stochastic volatility model. Hull and White (1987) developed a model to price a European call on an asset with stochastic volatility. The option price is determined in series form for the case where the stochastic volatility is independent of its underlying stock price. The series form of Hull and White model with the first three terms of the series solution is given below.

$$f(S,\sigma^2) = S\mathrm{N}(d_1) - Ke^{-rT}\mathrm{N}(d_2) + \frac{1}{2}\frac{S\sqrt{T}\mathrm{N}'(d_1)(d_1 d_2 - 1)}{4\sigma^3}\left[\frac{2\sigma^4(e^k - k - 1)}{k^2} - \sigma^4\right]$$

$$+\frac{1}{6}\frac{S\sqrt{T}\mathrm{N}'(d_1)(d_1d_2-3)(d_1d_2-1)-(d_1^2+d_2^2)}{8\sigma^5}$$

$$\times\sigma^6\left[\frac{e^{3k}-(9+18k)e^k+(8+24k+18k^2+6k^3)}{3k^3}\right]+.....,$$

where, $k = \xi^2 T$, $d_1=\frac{\log(S/T)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}$ and $d_2=d_1-\sigma\sqrt{T}$.

In order to compare the pricing bias, of the Hull and White and BS models we use the at-the-money April 300 call which has just over three months to maturity. The April 300 call is treated as the marker since it is nearest to the money. The implied volatility is 16% since, by setting σ = 16% we obtain a BS price of 10 which is exactly equal to the market price. The call option parameters to determine the pricing bias are σ = 16%, ξ = 3.26%, T = 110 days, and r = 10.44% respectively. In Figure 1, we compare the Hull and White call prices with the BS model. In order to illustrate the pricing bias, we have exaggerated the bias 25,000 times since the actual pricing error is quite small. When the volatility is un-correlated with the stock we find that the Hull and White option price is lower compared to the BS price for near the money options. The BS price is too low for deep in the money and deep out of the money and high at the money. These observations are consistent with the Hull and White (1987) findings.

[Insert Figure 1 Here]

Confidence Intervals for Weighted Average Implied Volatility

Weighted-average techniques for computing the implied volatility have received quite a bit of attention. For prices of multiple options with varying strike prices and maturity written on the same underlying asset implied volatility are not the same. For instance, for the six BP call options in Table 1 the implied volatility are found to be Jan-280 (15%), Apr-280 (8%), July-280 (1%), Jan –300 (22%), Apr-300 (16%) and July-300 (14%). Consequently, researches have developed various weighting schemes to derive a single implied volatility estimate that can be used to price options. Mayhew (1995) provide an excellent literature review on option–implied volatility. In his article, he describes four implied volatility-weighting schemes that range from simple equal weights to more complex elasticity weighting.

An important issue in volatility is its predictive ability. Among others, early literature such as, Latene and Rendleman (1976), Schmalensee and Trippi (1978) and Beckers (1981) found that implied volatility is better than historic volatility at predicting actual volatility. Gemmill (1986) compared historic volatility and six different implied volatility schemes- equal weights, elasticity weights, minimized squared pricing errors, at-the-money implied volatility, out-of-the money implied volatility, in the money implied volatility to ascertain the predictive ability of actual volatility. By regressing the predictors on the actual volatility, Gemmill found that at-the-money implied volatility is the best predictor of future volatility. Subsequent literatures also support this claim but results have been mixed Mayhew (1995).

In this section, we use the volatility of volatility estimate obtained using the jackknife technique to construct confidence intervals for historic volatility (*Historic*) and four volatility schemes described in Mayhew (1995) to identify the best predictor of actual volatility. The four weighted average implied volatility measures are:

(1) Equal weights used by Schmalensee and Trippi (1978) - (*Equal weighting*);

(2) Black-Scholes Vega weighting scheme of Latene and Rendleman (1976) –(*Vega Weighting*);

(3) Volatility elasticity used by Chiras and Manaster (1978) – (*Elasticity Weighting*);

(4) Beckers (1981) Implied Standard Deviation – (*Beckers ISD*).

The different weighting schemes along with the parameter description and the volatility estimates are summarized in Table 3. The volatility based on the four weighting schemes range from 8.3% to 18.5%. Of the four weighting schemes, Beckers ISD provide the volatility estimate closest to the implied volatility (16%) of the April 300-marker call option.

[Insert Table 3 Here]

Next, using the volatility of volatility estimates obtained from the Jackknife procedure we develop the 95% and 99% confidence intervals for each of the weighted average implied volatility scheme. The confidence intervals for historic and the four weighted average volatility are presented in Figure 2. In Table 4, we present the estimated call values for the six BP call options. The call prices indicate that for near-the-money options (300-Jan, 300-April and 300-July) historic volatility, elasticity weighted and Beckers ISD perform better than the other weighting schemes. Vega weighting of Chiras and Manaster (1978) price the in-the-money options better than the other methods. The shaded regions in Table 4 show the best weighting schemes for near-the-money and in-the-money options.

The confidence intervals for the call prices using the jackknife-based volatility of volatility parameter are given in Table 5. Based on a 95% confidence interval for implied volatility, we find that Vega weight is more appropriate compared to the other weighting schemes for in-the money options. In case of pricing near-the-money options the results are mixed. However, Beckers ISD is found to be consistently performing better for all three near-the money options followed by historic volatility and elasticity weighted scheme as shown by the shaded regions in Table 5.

[Insert Figure 2 Here]

[Insert Table 4 Here]

[Insert Table 5 Here]

Next, we study the pricing bias of Hull and White stochastic volatility model and the BS model when different weighted volatility values are used. According to Hull and White (1987) the principle result of increasing volatility is to make the bias more positive for out-of-the options and more negative for in-the money options. Consistent with their findings, the pricing bias is minimum when Hull and White model with historic volatility of 22% is compared to the BS with implied volatility of 16%. The pricing bias of BS is maximum when Vega weighting of 8.3% is used in the Hull and White stochastic volatility model. As shown in Figure 3 the graphs indicate that the bias is inversely related with the volatility of the stock.

[Insert Figure 3 Here]

Conclusions

In this paper we introduce a relatively easy and efficient statistical approach to estimating the volatility of volatility of an asset. A key contribution of this paper is the Jackknife procedure to empirically estimate the volatility of volatility parameter, which is an input to stochastic volatility models. The jackknife method is easy to use especially if the sample size is small such as the weekly two-year stock price data typically used for estimating the volatility of a stock. It can easily be implemented on a spreadsheet and does not need special statistical packages or computer programs. It provides more conservative and less biased volatility of volatility estimates for implementing stochastic option pricing models. Further since the purpose of deriving prediction models is for prediction with future samples, and if a model does not predict well with future samples, the purpose for which model is designed is lost. Thus, when an external replication is not feasible, the jackknife statistic is the most appropriate technique to determine result stability [Ang(1998)].

Next, we demonstrated how one could use volatility of volatility estimates to develop confidence intervals for selecting among different weighting schemes. As future extension of this research, we plan to perform a comparative study of different replication techniques to estimate the volatility of volatility of a stock.

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TABLE 1

Weekly Prices Returns and Jack-knife Estimates for BP Stock (August – January 1992)

Week	Price	Return (r_i)	Standard Deviation of r_i (θ_{18})
Cell	A	B	C
1	347		
2	352.5	1.5726	3.0657
3	346	-1.8612	3.1140
4	337	-2.6356	3.0923
5	331	-1.7965	3.1153
6	336.5	1.6480	3.0619
7	339	0.7402	3.0992
8	341	0.5882	3.1038
9	352	3.1749	2.9594
10	331	-6.1513	2.8307
11	328	-0.9105	3.1243
12	332.5	1.3626	3.0755
13	324	-0.8760	3.1243
14	311	-4.0951	3.0175
15	302	-2.9366	3.0805
16	291	-3.7104	3.0416
17	297.5	2.2091	3.0302
18	279	-6.4202	2.7985
19	277	-0.7194	3.1241
20	290.5	4.7586	2.7949

$$R_t = 100\left[\ln \frac{352.5}{347}\right]$$

$$\theta_{(18,1)} = \text{STDEV(B3 : B20)}$$

$$\theta_{(18,2)} = \text{STDEV(B2, B4 : B20)}$$

$$\overline{R}_w = \text{AVERAGE(B2 : B20)} = -0.8452$$

$$\sigma_w = \text{STDEV(B2 : B20)} = 3.0363$$

$$\overline{\theta_{18}} = \frac{\text{SUM(C2 : C20)}}{19} = 3.0344$$

TABLE 2

Market and BS Model Prices for BP on 3 January 1992

Expiry price	**January**		**April**		**July**	
	Market price	BS price	Market price	BS price	Market price	BS price
280	13	(14)	20	(26)	26	(34)
300	3	(3)	10	(14)	16	(23)

TABLE 3

Weighted Average Volatility

Description	**Model**	**Estimated Volatility**
Historic	-	22%
Schmalensee and Trippi (1978) – (*Equal weighting*)	$\hat{\sigma} = \frac{1}{N}\sum_{i=1}^{N}\sigma_i$ σ_i: implied volatility of ith call	12.67%
Latene and Rendleman (1976) (*Vega Weighting*)	$\hat{\sigma} = \frac{1}{\sum_{i=1}^{N} w_i}\sqrt{\sum_{i=1}^{N} w_i^2\sigma_i^2}$ w_i: Black-Scholes vegas of option *i*	8.30%
Chiras and Manaster (1978) (*Elasticity Weighting*)	$\hat{\sigma} = \frac{\sum_{i=1}^{N}\sigma_i \frac{\delta C_i}{\delta\sigma_i}\frac{\sigma_i}{C_i}}{\sum_{i=1}^{N}\frac{\delta C_i}{\delta\sigma_i}\frac{\sigma_i}{C_i}}$ C_i: Market price of option *i*	18.5%
Beckers (1981) (*Beckers ISD*)	Minimize $\sum_{i=1}^{N} w_i\left[C_i - BS_i(\hat{\sigma})\right]^2$ BS_i: Black-Scholes price of option *i*	16%

TABLE 4

Call Prices Based on Weighting Schemes

Call Option	Market Price	Historic	Equal Weighting	Vega Weighting	Elasticity Weighting	Beckers ISD
280-Jan	13	14	13	13	13	13
280-April	20	26	21	20	24	23
280-July	26	34	29	27	32	31
300-Jan	3	3	1	0	2	2
300-April	10	14	8	5	12	10
300-July	23	23	15	12	20	18

TABLE 5

Estimates of Call Prices Based on Confidence Intervals (CI) for Implied Volatility

Call Option	Market Price	Historic				Equal Weighting				Vega Weighting				Elasticity Weighting				Beckers ISD			
		95% CI		99%CI		95% CI		99%CI		95% CI		99%CI		95% CI		99%CI		95% CI		99%CI	
280-Jan	13	13.0	15.2	12.7	15.9	12.5	13.5	12.5	14.1	12.5	13.0	0.0	13.4	12.7	14.6	12.5	15.2	12.6	14.1	12.5	14.7
280-April	20	22.4	29.0	21.1	30.9	19.7	24.2	19.7	25.8	19.7	22.1	0.0	23.6	21.0	27.2	20.1	29.0	20.3	25.9	19.7	27.6
280-July	26	30.2	39.0	28.5	41.4	26.7	32.5	26.6	34.7	26.6	29.8	0.0	31.8	28.4	36.6	27.2	38.9	27.4	34.8	26.7	37.1
300-Jan	3	1.4	4.5	0.8	5.3	0.1	2.3	0.0	3.0	0.0	1.3	0.0	2.0	0.8	3.7	0.3	4.5	0.4	3.1	0.1	3.8
300-April	10	9.9	18.2	7.9	20.3	4.1	12.4	2.0	14.4	1.3	9.6	0.0	11.6	7.8	16.1	5.7	18.2	6.2	14.5	4.1	16.6
300-July	23	17.3	28.2	14.6	30.9	10.0	20.5	8.1	23.2	7.8	16.8	0.0	19.5	14.6	25.4	12.0	28.1	12.5	23.2	10.1	26.0

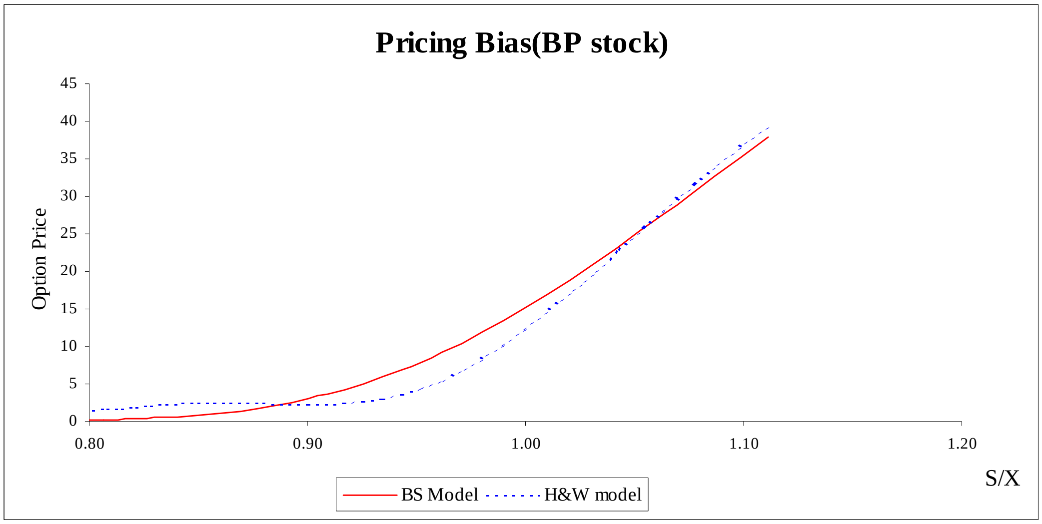
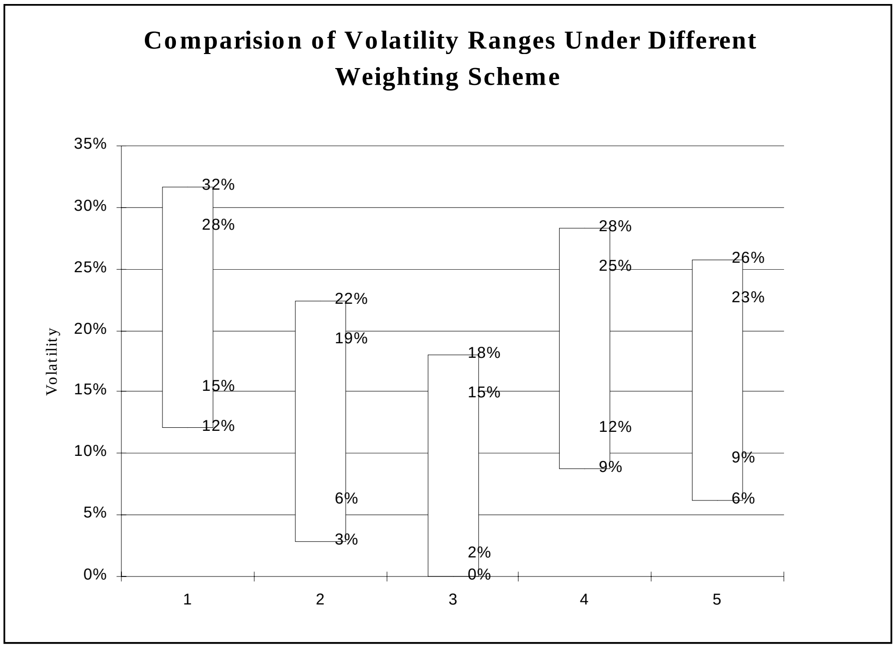


Figure 1: Pricing Bias when σ = 16%, ξ = 3.26%, T = 110 days, r = 10.44



1 Historic, 2 Equal Weighting, 3 Vega Weighting, 4 Elasticity Weighting, 5 Beckers ISD

Figure 2: Confidence Intervals for Historic and Weighted Average Volatility Schemes.

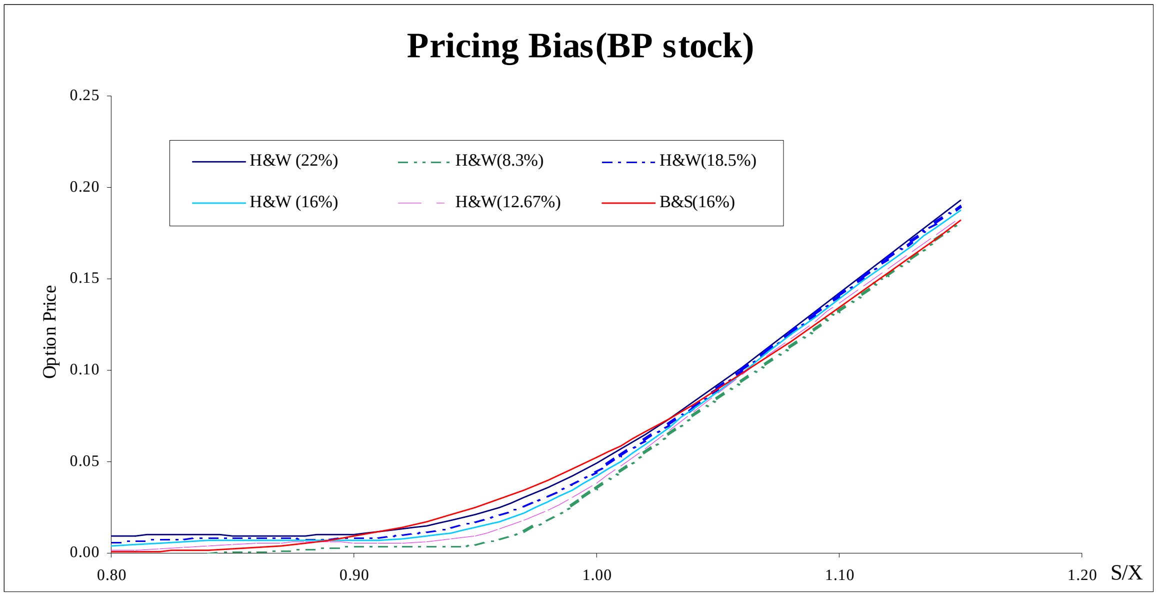


Figure 3: Effect of Pricing Bias with Varying Volatility