Chapter 14

Nuclear Physics Applications. Home Work Solutions

14.1 Problem 14.9

When the nuclear reaction represented by:

$$Q = (K_Y + K_b) - K_a = (M_x + M_a - M_Y - M_b)c^2$$

is endothermic, the disintegration energy Q is negative. In order for the reaction to proceed, the incoming particle must have a minimum kinetic energy, called *threshold energy*, K_{th}.

(a) show that

$$K_{th} = -Q\left(1 + \frac{M_a}{M_X}\right)$$

by using the fact that $K_{th} = -Q$ in the CM frame and by transforming this result for the CM frame back to the laboratory frame. Note that in the CM frame, a and X have equal and opposite momenta, $p = M_a v = M_X V$. In the lab frame, a has momentum $p_{lab} = M_a(v + V)$ and X is at rest.

(b) Calculate the threshold energy of the incident particle in the reaction

$$^4_2He +^{14}_7 N \longrightarrow {}^{17}_8O +^1_1 H$$

Solution

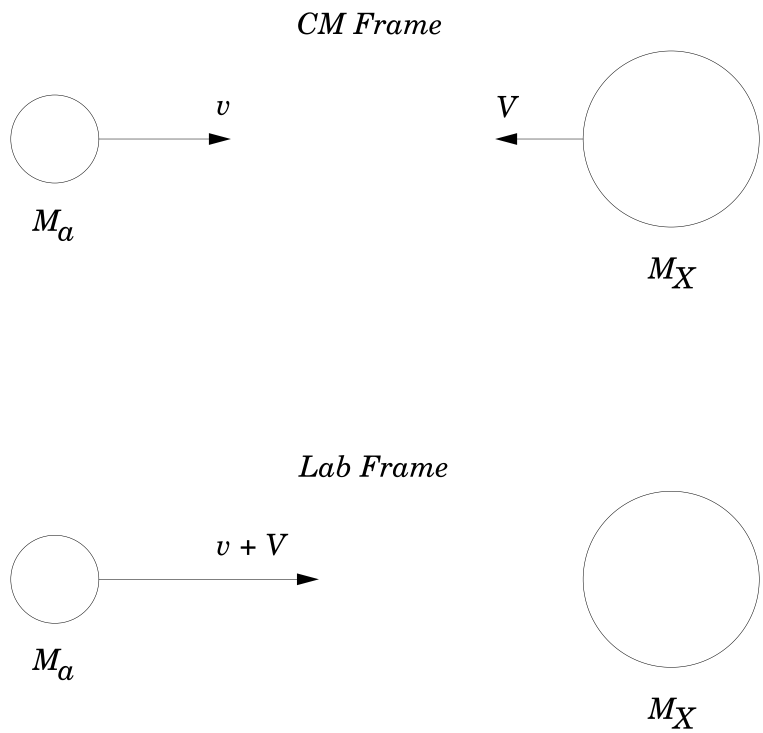

Figure 14.1:

(a) According to the problem, in the CM frame mass a is moving with a velocity v and mass X is moving with a velocity V in the opposite direction to v. To convert this to the lab frame where mass X is at rest, we find that mass a is moving with a velocity $v + V$, as shown in Figure (14.1) In the CM frame we have:

$$\begin{aligned} p &= M_a v \\ &= M_X V \\ K_{CM} &= \frac{p^2}{2M_a} + \frac{p^2}{2M_X} \\ &= \frac{p^2}{2}\left(\frac{M_a + M_X}{M_a M_X}\right) \end{aligned}$$

In the Lab frame, using $v = p/M_a$ and $V = p/M_X$, we get:

$$\begin{aligned} p_{lab} &= M_a(v + V) \\ &= M_a\left(\frac{p}{M_a} + \frac{p}{M_X}\right) \\ &= M_a p\left(\frac{M_a + M_X}{M_a M_X}\right) \end{aligned}$$

$$\begin{aligned}
p_{lab} &= p\left(\frac{M_a + M_X}{M_X}\right) \\
K_{lab} &= \frac{p_{lab}^2}{2M_a} \\
&= \frac{p^2}{2M_a}\left(\frac{M_a + M_X}{M_X}\right)^2 \\
&= \frac{p^2}{2}\left(\frac{M_a + M_X}{M_a M_X}\right)\left(\frac{M_a + M_X}{M_X}\right) \\
&= K_{CM}\left(\frac{M_a + M_X}{M_X}\right)
\end{aligned}$$

When in the CM frame $K_{th} = -Q$ then, $K_{lab} = K_{th}$, we then get:

$$\begin{aligned}
K_{th} &= -Q\left(\frac{M_a + M_X}{M_X}\right) \\
&= -Q\left(1 + \frac{M_a}{M_X}\right)
\end{aligned}$$

(b) The Q-value of the reaction is given by:

$$\begin{aligned}
Q &= \left[M(^{14}N) + M(^4He) - M(^{17}O) - M(^1H)\right](931.5) \\
&= (14.003074 + 4.002603 - 16.999132 - 1.007825)(931.5) \\
&= -1.19 MeV
\end{aligned}$$

Now we can calculate the threshold energy:

$$\begin{aligned}
K_{th} &= -Q\left[1 + \frac{M(^4He)}{M(^{14}N)}\right] \\
&= -(-1.19)\left[1 + \frac{4.002603}{14.003074}\right] \\
&= 1.53\ MeV
\end{aligned}$$

14.2 Problem 14.12

The density of lead is 11.35 g/cm^3, and its atomic weight is 207.2. Assume that 1.000 cm of lead reduces a beam of 1-MeV gamma rays to 28.65% of its initial intensity.

(a) How much lead is required to reduce the beam to 10^{-4} of its initial intensity?

(b) What is the effective cross section of a lead atom for a 1-MeV photon?

Solution

(a) The intensity of a gamma ray beam is reduced through matter (in this case it is lead) according to:

$$I = I_\circ e^{-n\sigma x}$$

where $I_\circ$ is the incident intensity, I is the intensity after traveling a distance x in lead, n is the number of lead nuclei per unit volume and σ is the cross section of the absorption process. We can then use the information given to calculate $n\sigma$:

$$\begin{aligned} n\sigma &= \frac{1}{x}\ln\frac{I_\circ}{I} \\ &= \ln\frac{I_\circ}{0.2865 I_\circ} \\ &= 1.25\ cm^{-1} \end{aligned}$$

The thickness of lead that reduces the beam intensity to $10^{-4} I_\circ$ is then:

$$\begin{aligned} x &= \frac{1}{n\sigma}\ln\frac{I_\circ}{I} \\ &= \frac{1}{1.25}\ln\frac{I_\circ}{10^{-4} I_\circ} \\ &= 7.37\ cm \end{aligned}$$

the number of lead nuclei per unit volume can be calculated from volume density of lead d, Avogadro's number A_v and the atomic weight w:

$$n = \frac{dA_v}{w}$$

the cross section σ is then given by:

$$\begin{aligned}
\sigma &= \frac{1.25}{n} \\
&= \frac{1.25w}{dA_v} \\
&= \frac{1.25 \times 207.2}{11.35 \times 6.02 \times 10^{-23}} \\
&= 3.79 \times 10^{-23}\ cm^2 \\
&= 37.9\ b
\end{aligned}$$

14.3 Problem 14.21

Find the energy released in the fission reaction

$$^{1}_{0}n \;+\; ^{235}_{92}U \;\longrightarrow\; ^{141}_{56}Ba \;+\; ^{92}_{36}Kr \;+\; 3(^{1}_{0}n)$$

the requred masses are

$$\begin{array}{lcrl} \mathrm{M}(^{1}_{0}n) & = & 1.008665 & \mathrm{u} \\ \mathrm{M}(^{235}_{92}U) & = & 235.043915 & \mathrm{u} \\ \mathrm{M}(^{141}_{56}Br) & = & 140.913900 & \mathrm{u} \\ \mathrm{M}(^{92}_{36}Kr) & = & 91.897300 & \mathrm{u} \end{array}$$

Solution

The energy ΔE released in the given fission process is:

$$\begin{aligned} \Delta E &= (m_U + m_n - m_{Br} - m_{Kr} - 3m_n) \\ &= (235.043915 + 1.008665 - 140.913900 - 91.897300 - 3 \times 1.008665) \times 931.5 \\ &= 200.6 \; MeV \end{aligned}$$

14.4 Problem 14.25

An electrical power plant operates on the basis of thermal energy generated in a pressurized-water reactor. The electrical power output of the plant is 1 GW, and its efficiency is 30%.

(a) Find the total power generated by the reactor.

(b) How much power is discharged to the environment as waste heat?

(c) Calculate the rate of fission events in the reactor core.

(d) Calculate the mass of ^{235}U used up in one year.

(e) Using the results from (a), determine the rate at which fuel is converted to energy (in kg/s) in the reactor core, and compare your answer with the result from (d).

Solution

(a) The power generated by the reactor P_r can be obtain form the efficiency e and electric power P_e produced:

$$\begin{aligned} e &= \frac{P_e}{P_r} \\ P_r &= \frac{P_e}{e} \\ &= \frac{1000}{0.3} \\ &= 3333\ MW \end{aligned}$$

(b) The power lost as heat P_h is:

$$\begin{aligned} P_h &= P_r - P_e \\ &= 3333 - 1000 \\ &= 2333\ MW \end{aligned}$$

(c) Every fission event produces energy equal to the Q-value of the fission process $\approx 200\ MeV$, so the rate of fission events R in the reactor core is:

$$\begin{aligned} R &= \frac{P_r}{Q} \\ &= \frac{3.333 \times 10^9 (J/s)}{200(MeV) \times 1.602 \times 10^{-13}(J/MeV)} \\ &= 1.04 \times 10^{20}\ events/s \end{aligned}$$

(d) Every fission event consumes one ^{235}U nucleus. In the reactor core 1.04×10^{20} nuclei of ^{235}U are consumed every second, so the mass M of ^{235}U consumed in a year is then:

$$\begin{aligned} M &= R \times \frac{235 \times 10^{-3} (kg/mole)}{6.02 \times 10^{23}\ nuclei/mole} \times \text{time} \\ &= 1.04 \times 10^{20} \times \frac{235 \times 10^{-3}}{6.02 \times 10^{23}} \times (60 \times 60 \times 24 \times 365) \\ &= 1.28 \times 10^{3}\ kg/year \end{aligned}$$

The rate at which the mass is converted to energy dM/dt is given by:

$$\begin{aligned} \frac{dM}{dt} &= \frac{P_r}{c^2} \\ &= \frac{3.333 \times 10^{9}}{(3 \times 10^{8})^2} \\ &= 3.70 \times 10^{-8}\ kg/s \end{aligned}$$

To compare this answer to (d) we find dM/dt in one year:

$$\begin{aligned} \frac{dM}{dt} &= 3.70 \times 10^{-8} \times 60 \times 60 \times 24 \times 365 \\ &= 1.17\ kg/year \end{aligned}$$

This is only 0.1% of the mass calculated in part (d). This is of course because not all the mass in part (d) is converted to energy, it is converted to energy as well as to the masses of the fission products.

14.5 Problem 14.33

The carbon cycle, first proposed by Bethe in 1939, is another cycle by which energy is released in stars and hydrogen is converted to helium. The carbon cycle requires higher temperature than the proton-proton cycle. The series of reactions is

$$\begin{array}{ccccccc} {}^{12}C & + & {}^{1}H & \longrightarrow & {}^{13}N & + & \gamma \\ & & {}^{13}N & \longrightarrow & {}^{13}C & + & e^{+} + \nu \\ {}^{13}C & + & {}^{1}H & \longrightarrow & {}^{14}N & + & \gamma \\ {}^{14}N & + & {}^{1}H & \longrightarrow & {}^{15}O & + & \gamma \\ & & {}^{15}O & \longrightarrow & {}^{15}N & + & e^{+} + \nu \\ {}^{15}N & + & {}^{1}H & \longrightarrow & {}^{12}C & + & {}^{4}He \end{array}$$

(a) If the proton-proton cycle requires a temperature of $1.5 \times 10^{7}\ K$, estimate the temperature required for the first step in the carbon cycle.

(b) Calculate the Q-value for each step in the carbon cycle and the overall energy released.

(c) Do you think the energy carried off by the neutrinos is deposited in the stars? Explain.

Solution

(a) In a proton-proton interaction, one needs enough energy to overcome the Coulomb energy of two charges, while in carbon-proton interaction one needs to overcome the Coulomb of seven charges. So the temperature required for carbon-proton cycle T_{cp} is $\approx 7/2\times$ the temperature required for the proton-cycle T_{pp}, i.e.:

$$\begin{aligned} T_{cp} &\approx \frac{7}{2} \times T_{pp} \\ &= \frac{7}{2} \times 1.5 \times 10^{7} \\ &= 5.2 \times 10^{7}\ K \end{aligned}$$

(b) The Q-Values of various reactions are:

$$\begin{aligned}
Q_1 &= M(^{12}C) + M(H) - M(^{13}N) \\
&= (12.000000 + 1.007825 - 13.005739) \times 931.5 \\
&= 1.943\ MeV \\
Q_2 &= M(^{13}N) - M(^{13}C) - 2m_e \\
&= (13.005739 - 13.003355) \times 931.5 - 2 \times 0.511 \\
&= 1.198\ MeV \\
Q_3 &= M(^{13}C) + M(H) - M(^{14}N) \\
&= (13.003355 + 1.007825 - 14.003074) \times 931.5 \\
&= 7.551\ MeV \\
Q_4 &= M(^{14}N) + M(H) - M(^{15}O) \\
&= (14.003074 + 1.007825 - 15.003066) \times 931.5 \\
&= 7.296\ MeV \\
Q_5 &= M(^{15}O) - M(^{15}N) - 2m_e \\
&= (15.003066 - 15.000109) \times 931.5 - 2 \times 0.511 \\
&= 1.732\ MeV \\
Q_6 &= M(^{15}N) + M(H) - M(^{12}C) - M(^4He) \\
&= (15.000109 + 1.007825 - 12.000000 - 4.002603) \times 931.5 \\
&= 4.966\ MeV
\end{aligned}$$

The total energy produced in full cycle is the sum of all the Q-Values of th individual reactions in the cycle:

$$\begin{aligned}
Q &= Q_2 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 \\
&= 1.943 + 1.198 + 7.551 + 7.296 + 1.732 + 4.966 \\
&= 24.686\ MeV
\end{aligned}$$

The total energy can also be calculated from the net effect of the whole cycle:

$$^{12}C\ +\ 4^1H\ \longrightarrow\ ^{12}C\ +\ ^4He$$

which has a Q-value given by (taking into account the emission of two positrons):

$$\begin{aligned}
Q &= M(^{12}C) + 4M(^1H) - M(^{12}C) - M(^4He) - 4m_e \\
&= (4 \times 1.007825 - 4.002603) \times 931.5 - 4 \times 0.511 \\
&= 24.687\ MeV
\end{aligned}$$

(c) Most of the energy of the neutrinos will be lost due to the fact that the neutrino interactions with matter are very weak because it has very small mass and no charge.