Chapter 13

Nuclear Structure. Home Work Solutions

13.1 Problem 13.10

(a) find the radius of the ${}^{12}_{6}C$ nucleus.

(b) find the force of repulsion between a proton at the surface of a ${}^{12}_{6}C$ nucleus and the remaining five protons.

(c) How much work (in MeV) must be done to overcome this electrostatic repulsion and put the last proton into the nucleus?

(d) Repeat (a),(b), and (c) for ${}^{238}_{92}U$.

Solution

(a) the nuclear radius R is given by

$$\begin{aligned} R &= R_\circ A^{1/3} \\ &= 1.2 \times 10^{-15}\ A^{1/3} \\ &= 1.2 \times 10^{-15} \times (12)^{1/3} \\ &= 2.75 \times 10^{-15}\ m \\ &= 2.75\ fm \end{aligned}$$

(b) The force of repulsion between a proton at the the surface of a nucleus and the rest of the protons is:

$$F = \frac{k(Z-1)e^2}{R^2}$$

For ^{12}C, $Z = 6$ and R is given above in part (a),

$$\begin{aligned} F &= \frac{8.988 \times 10^9 \times 5 \times (1.602 \times 10^{-19})^2}{(2.75 \times 10^{-15})^2} \\ &= 152.5\ N \end{aligned}$$

(c) The work done to overcome the force of repulsion given in part (b) is given by the potential energy between the proton at the surface of the nucleus and the rest of the protons inside:

$$\begin{aligned} U &= \frac{kq_1q_2}{R} \\ &= \frac{k(Z-1)e^2}{R} \\ &= \frac{8.988 \times 10^9 \times 5 \times (1.602 \times 10^{-19})^2}{2.75 \times 10^{-15}} \\ &= 4.194 \times 10^{-13}\ J \\ &= \frac{4.194 \times 10^{-13}}{1.602 \times 10^{-19}} \\ &= 2.618 \times 10^6\ eV \\ &= 2.618\ MeV \end{aligned}$$

(d) For $^{238}_{92}U$, $A = 238$ and $Z = 92$,

$$\begin{aligned} R &= 1.2 \times 10^{-15} \times (238)^{1/3} \\ &= 7.437 \times 10^{-15}\ m \\ F &= \frac{8.988 \times 10^9 \times 91 \times (1.602 \times 10^{-19})^2}{(7.437 \times 10^{-15})^2} \\ &= 379.5\ N \\ U &= \frac{8.988 \times 10^9 \times 1.602 \times 10^{-19}}{7.437 \times 10^{-15}} \\ &= 1.762 \times 10^7\ eV \\ &= 17.62\ MeV \end{aligned}$$

13.2 Problem 13.14

Two nuclei with the same mass number are known as *isobars*. If the two nuclei also have interchanged atomic and neutron numbers, such as $^{23}_{11}Na$ and $^{23}_{12}Mg$, the nuclei are *mirror isobars*. Binding-energy measurements on these nuclei can be used to obtain evidence of the charge independence of nuclear forces (that is, proton-proton, proton-neutron and neutron-neutron forces are approximately equal).

(a) calculate the difference in binding energy for the two mirror nuclei $^{15}_{8}O$ and $^{14}_{7}N$.

(b) Calculate the difference in binding energy per nucleon for the mirror isobars $^{23}_{11}Na$ and $^{23}_{12}Mg$. How do you account for the difference?

Solution

The masses used in this problems are:

$$
\begin{array}{lcll}
M(H) & = & 1.007825 & amu \\
m_n & = & 1.008665 & amu \\
M(^{15}_{8}O) & = & 15.003066 & amu \\
M(^{15}_{7}N) & = & 15.000109 & amu \\
M(^{23}_{11}Na) & = & 22.98977 & amu \\
M(^{23}_{12}Mg) & = & 22.99412 & amu
\end{array}
$$

(a) The biding energy is:

$$E_b(MeV) = \left[ZM(H) + Nm_n - M(^A_ZX)\right] \times 931.494$$

for $^{15}_{8}O$, $A = 15$, $Z = 8$, and $N = 7$, its binding energy is:

$$
\begin{aligned}
E_b &= [8 \times 1.007825 + 7 \times 1.008665 - 15.003066] \times 931.494 \\
&= 111.96\ MeV
\end{aligned}
$$

for $^{15}_{7}N$, $A = 15$, $Z = 7$, and $N = 8$, its binding energy is:

$$
\begin{aligned}
E_b &= [7 \times 1.007825 + 8 \times 1.008665 - 15.000109] \times 931.494 \\
&= 115.49\ MeV
\end{aligned}
$$

The difference in the binding energy is:

$$\Delta E_b = 3.573\ MeV$$

(b) For ${}^{23}_{11}Na$, $A = 23$, $Z = 11$, and $N = 12$, its binding energy per nucleon is:

$$\begin{aligned} E_b &= [11 \times 1.007825 + 12 \times 1.008665 - 22.98977] \times 931.494 \\ &= 186.56 \; MeV \\ \frac{E_b}{A} &= \frac{186.56}{23} \\ &= 8.11 \; MeV \end{aligned}$$

for ${}^{23}_{12}Mg$, $A = 23$, $Z = 12$, and $N = 11$, its binding energy per nucleon is:

$$\begin{aligned} E_b &= [12 \times 1.007825 + 11 \times 1.008665 - 22.99412] \times 931.494 \\ &= 181.73 \; MeV \\ \frac{E_b}{A} &= \frac{181.73}{23} \\ &= 7.90 \; MeV \end{aligned}$$

The difference in the binding energy per nucleon is:

$$\frac{\Delta E_b}{A} = 0.210 \; MeV$$

The isobars with more protons experience more Coulomb repulsion and consequently are less tightly bound than those with less protons.

13.3 Problem 13.20

(a) Use the following equation for the total binding energy E_b;

$$E_b = C_1 A - C_2 A^{2/3} - C_3 \frac{Z(Z-1)}{A^{1/3}} - C_4 \frac{(N-Z)^2}{A} \tag{13.1}$$

where A is the mass number, Z is the atomic number, and N is the number of neutrons. C_1, C_2, C_3 and C_4 are constants given by:

$$C_1 = 15.7\ MeV \qquad C_2 = 17.8\ MeV \qquad C_3 = 0.71\ MeV \qquad C_4 = 23.6\ MeV$$

to calculate the binding energy per nucleon for the isobars ${}^{64}_{29}Cu$ and ${}^{64}_{30}Zn$.

(b) Compare these values to the binding energy per nucleon calculated with:

$$E_b(MeV) = \left[ZM(H) + Nm_n - M({}^{A}_{Z}X)\right] \times 931.494 \tag{13.2}$$

where $M(H)$ is the mass of hydrogen atom, $M({}^{A}_{Z}X)$ is the mass of atom X, m_n is the mass of the neutron. All masses are in atomic mass units.

Solution

The masses needed for this problem are:

$$\begin{array}{lcll} M(H) & = & 1.007825 & amu \\ m_n & = & 1.008665 & amu \\ M({}^{64}_{29}Cu) & = & 63.929766 & amu \\ M({}^{64}_{30}Zn) & = & 63.929146 & amu \end{array}$$

(a) For ${}^{64}_{29}Cu$, $A = 64$, $Z = 29$, and $N = 35$ and using Equation (13.1) its binding energy per nucleon is:

$$\begin{aligned} \frac{E_b}{A} &= C_1 - C_2 \frac{1}{A^{1/3}} - C_3 \frac{Z(Z-1)}{A^{4/3}} - C_4 \frac{(N-Z)^2}{A^2} \\ &= 15.7 - \frac{17.8}{(64)^{1/3}} - 0.71 \frac{29 \times 28}{(64)^{4/3}} - 23.6 \frac{(35-29)^2}{(64)^2} \\ &= 8.791\ MeV \end{aligned}$$

For ${}^{64}_{30}Zn$, $A = 64$, $Z = 30$, and $N = 34$ and using Equation (13.1) its binding energy per nucleon is:

$$\begin{aligned} \frac{E_b}{A} &= 15.7 - \frac{17.8}{(64)^{1/3}} - 0.71 \frac{30 \times 29}{(64)^{4/3}} - 23.6 \frac{(34-30)^2}{(64)^2} \\ &= 8.745\ MeV \end{aligned}$$

(b) Using Equation (13.2) for ${}^{64}_{29}Cu$, we get for the binding energy per nucleon:

$$\begin{aligned} \frac{E_b}{A} &= [29 \times 1.007825 + 35 \times 1.008665 - 63.929766] \times \frac{931.494}{64} \\ &= 8.739\ MeV \end{aligned}$$

and for ${}^{64}_{30}Zn$ we get:

$$\begin{aligned} \frac{E_b}{A} &= [30 \times 1.007825 + 34 \times 1.008665 - 63.929146] \times \frac{931.494}{64} \\ &= 8.736\ MeV \end{aligned}$$

The differences between the two equations are:

$$\begin{aligned} \Delta E_b(Cu) &= \frac{8.791 - 8.739}{8.791} \times 100 \\ &= 0.59\% \\ \Delta E_b(Zn) &= \frac{8.745 - 8.736}{8.745} \times 100 \\ &= 0.10\% \end{aligned}$$

13.4 Problem 13.30

During the manufacture of a steel engine component, radioactive iron (^{56}Fe) is included in the total mass of 0.2 kg. The component is placed in a test engine when the activity due to this isoptope is 20 μCi. After 1000-h test period, oil is removed from the engine and found to contain enough ^{56}Fe to produce 800 disintegrations/min/liter of oil. The total volume of oil is the engine is 6.5 L. Calculate the total mass worn from the engine component per hour of operation. (The half-life of ^{56}Fe is 45.1 days)

Solution

The initial activity per unit mass $(R/m)_\circ$ of ^{59}Fe in the component of the steel engine is given by:

$$\begin{aligned}
(\frac{R}{m})_\circ &= \frac{20}{0.2} \\
&= 100\ \mu Ci/kg \\
&= 100 \times 10^{-6} \times 3.7 \times 10^{10} \\
&= 3.7 \times 10^{6}\ Bq/kg
\end{aligned}$$

The decay constant λ of^{59}Fe is given by the half-life,

$$\begin{aligned}
\lambda &= \frac{\ln 2}{T_{1/2}} \\
&= \frac{0.693}{45.1 \times 24} \\
&= 6.40 \times 10^{-4}\ hr
\end{aligned}$$

The remaining activity per unit mass R/m after 1000 hours is:

$$\begin{aligned}
\frac{R}{m} &= (\frac{R}{m})_\circ\, e^{-\lambda t} \\
&= 3.7 \times 10^{6} \times e^{-(6.40 \times 10^{-4} \times 1000)} \\
&= 1.95 \times 10^{6}\ Bq/kg
\end{aligned}$$

The total activity in the oil R_{oil} can be obtained from the disintegrations/min/liter as (noting that 1 Bq = 1 disintegration/sec):

$$\begin{aligned}
R_{oil} &= \frac{800}{60} \times 6.5 \\
&= 86.7\ Bq
\end{aligned}$$

The total mass of ^{59}Fe in oil, $m_{in\ oil}$ is:

$$
\begin{aligned}
m_{in\ oil} &= \frac{R_{oil}}{R/m} \\
&= \frac{86.7}{1.95 \times 10^{6}} \\
&= 4.45 \times 10^{-5}\ kg
\end{aligned}
$$

The rate of wear w of the engine component is then:

$$
\begin{aligned}
w &= \frac{m_{in\ oil}}{t} \\
&= \frac{4.45 \times 10^{-5}}{1000} \\
&= 4.45 \times 10^{-8}\ kg/hr \\
&= 44.5\ \mu g/hr
\end{aligned}
$$

13.5 Problem 13.54

(a) Can ^{57}Co decay by β^+ emission? Explain.

(b) Can ^{14}C decay by β^- emission? Explain.

(c) If either answer is yes, what is the range of kinetic energies available for the β particles?

Solution

(a) $^{57}_{27}Co$ can decay by β^+ if the the Q-value of the decay process is positive. The decay process is:

$$^{57}_{27}Co \longrightarrow {}^{57}_{26}Fe + {}^{0}_{1}e^+ + {}^{0}_{0}\nu$$

The Q-value of the decay is:

$$\begin{aligned}
Q &= M(^{57}_{27}Co) - M(^{57}_{26}Fe) - 2M(^{0}_{1}e^+) - M(^{0}_{0}\nu) \\
&= 56.936294 - 56.935396 - 2 \times 5.485 \times 10^{-4} - 0 \\
&= -1.990 \times 10^{-4}\ amu \\
&= -1.990 \times 10^{-4} \times 931.494 \\
&= -0.185\ MeV
\end{aligned}$$

Since $Q < 0$, this decay can proceed spontaneously.

(b) As for $^{14}_{6}C$ we have:

$$^{14}_{6}C \longrightarrow {}^{14}_{7}N + {}^{0}_{-1}e^- + {}^{0}_{0}\overline{\nu}$$

The Q-value of this decay is:

$$\begin{aligned}
Q &= M(^{14}_{6}C) - M(^{14}_{7}N) - M(^{0}_{0}\nu) \\
&= 14.003242 - 14.003074 - 0 \\
&= 1.680 \times 10^{-4}\ amu \\
&= 1.680 \times 10^{-4} \times 931.494 \\
&= 0.165\ MeV
\end{aligned}$$

This decay can occur spontaneously.

(c) The energy of this decay (the Q-value) is shared between the $^{14}_{7}N$, the electron and the neutrino. The $^{14}_{7}N$ takes very little energy due to its relatively large mass, so

$$\begin{aligned}
Q &= K_e + K_{\overline{\nu}} \\
0.165 &= K_e + K_{\overline{\nu}}
\end{aligned}$$

The electron energy K_e can vary between 0 to 0.156 MeV.