Chapter 12

Superconductivity. Home Work Solutions

12.1 Problem 12.10

A magnetized cylinder of iron has a magnetic field $B = 0.040\ T$ in its interior. The magnet is 3.0 cm in diameter and 20 cm long. If the same magnetic field is to be produced by a 5.0-A current carried by an air-core solenoid with the same dimensions as the cylindrical magnet, how many turns of wire must be on the solenoid?

Solution

The magnetic field produced by a solenoid carrying a current I and has a length ℓ and number of turns N is given by:

$$
\begin{aligned}
B &= \frac{\mu_\circ N}{\ell} I \\
N &= \frac{\ell B}{\mu_\circ I} \\
&= \frac{0.2 \times 0.040}{4\pi \times 10^{-7} \times 5} \\
&= 1273 \text{ Turns}
\end{aligned}
$$

12.2 Problem 12.15

A superconducting solenoid is to be designed to generate a 10-T magnetic field.

(a) If the winding has 2000 turns/m, what is the required current?

(b) What force per meter does the magnetic field exert on the inner windings?

Solution

(a) A solenoid with n turns/m that carries a current I will produce a magnetic field B given by:

$$\begin{aligned} B &= \mu_\circ n I \\ I &= \frac{B}{n\mu_\circ} \\ &= \frac{10}{2000 \times 4\pi \times 10^{-7}} \\ &= 3979\ A \end{aligned}$$

(b) The magnetic force F_B due to a magnetic field B acting on a wire of length ℓ carrying a current I is given by:

$$\begin{aligned} F_B &= I\ell B \\ \frac{F_B}{\ell} &= IB \\ &= 3979 \times 10 \\ &= 39,790\ N/m \end{aligned}$$

If we assume that the loops of the solenoid are in the $x-y$ plane and the current is circulating clockwise then the magnetic field will be directed along the $-ve\ z$ axis and the magnetic force will be directed away from the axis of the solenoid. On the other hand, if the current is circulating anti-clockwise the magnetic field will be directed along the $+ve\ z$ axis and the force will also be directed away from the axis of the solenoid.

12.3 Problem 12.20

Persistent currents. In an experiment carried out by S. C. Collins between 1955 and 1958, a current was maintained in a superconducting lead ring for 2.5 years with no observed loss. If the inductance of the ring was $3.14 \times 10^{-8}\,H$, and the sensitivity of the experiment was 1 part in 10^9, determine the maximum resistance of the ring. (*Hint*: Treat this as a decaying current in an RL circuit, and recall that $e^{-x} \approx 1 - x$ for small x.)

Solution

The current in an RL circuit is given by:

$$\begin{aligned} I(t) &= I_\circ e^{-(R/L)t} \\ \frac{I(t)}{I_\circ} &= e^{-(R/L)t} \end{aligned} \tag{12.1}$$

The sensitivity of the experiment was 1 part in 10^9 and there was no detected change in the current, then the minimum value of the current $I(t)$ is:

$$\begin{aligned} I(t) &= I_\circ - 10^{-9} \times I_\circ \\ \frac{I(t)}{I_\circ} &= 1 - 10^{-9} \end{aligned} \tag{12.2}$$

from equations Equations (12.1) and (12.2) we get the maximum value of the resistance R_{max} that corresponds to the minimum current as:

$$\begin{aligned} 1 - 10^{-9} &= e^{-(R_{max}/L)t} \\ &\approx 1 - \left(\frac{R_{max}}{L}\right) t \\ \left(\frac{R_{max}}{L}\right) t &= 10^{-9} \\ R_{max} &= \frac{L \times 10^{-9}}{t} \\ &= \frac{3.14 \times 10^{-8} \times 10^{-9}}{2.5 \times 365.25 \times 24 \times 60 \times 60} \\ &= 3.94 \times 10^{-24}\,\Omega \end{aligned}$$

12.4 Problem 12.27

Isotope effect. Because of the isotope effect, $T_c \propto M^{-\alpha}$. Use these data for mercury to determine the value of the constant α. Is your result close to what you might expect on the basis of a simple model?

Isotope	**T_c(K)**
^{199}Hg	4.161
^{200}Hg	4.153
^{204}Hg	4.126

Solution

If:

$$T_c = KM^{-\alpha}$$

where K is a constant, then:

$$\ln T_c = \ln K - \alpha M$$

The last equation is that of a straight line with a slope of $-\alpha$ and an intercept $\ln K$. Using the data given and the fact that $M \approx A$, where A is the atomic number and M is the atomic mass, we get:

Isotope	T_c	M	$\ln T_c$	$\ln M$
^{199}Hg	4.161	199	1.4257	5.2933
^{200}Hg	4.153	200	1.4240	5.2983
^{204}Hg	4.126	204	1.4173	5.3181

Plotting $\ln T_c$ *vs* $\ln M$ using the last 2 columns of the table above and fitting them with a straight line, its slope is:

$$\begin{aligned}
\text{Slope} &= -\alpha \\
-\alpha &= \frac{\Delta \ln T_c}{\Delta \ln M} \\
&= \frac{1.4173 - 1.4257}{5.3181 - 5.2933} \\
&= -0.34 \\
\alpha &= 0.34
\end{aligned}$$

α is expected to be $\frac{1}{2}$, which is close to the value obtained using the simple model given in the problem.

12.5 Problem 12.39

Magnetic field inside a wire. A type II superconducting wire of radius R carries current uniformly distributed through it cross section. If the total current carried by the wire is I, show that the magnetic energy per unit length of the wire is $\mu_\circ I^2/16\pi$.

Solution

Let us take a small circular area with a radius r within the cross sectional area of the wire. Since the current is uniform, then the current within this small area is proportional to the area, i.e.

$$I(r) = \frac{\pi r^2}{\pi R^2} I \qquad\qquad r \leq R$$

The magnetic field produced by the current is then,

$$\begin{aligned} B(r) &= \frac{\mu_\circ I(r)}{2\pi r} \\ &= \frac{\mu_\circ I r}{2\pi R^2} \end{aligned}$$

The magnetic energy per unit volume u stored in the magnetic filed is given by:

$$u = \frac{B^2}{2\mu_\circ}$$

The magnetic energy per unit length $\mathcal{U}$ is then given by:

$$\begin{aligned} \mathcal{U} &= \int_0^R u \, dA \\ &= \int_0^R u \, d(\pi r^2) \\ &= \int_0^R 2\pi u r \, dr \\ &= \int_0^R \frac{2\pi r B^2(r)}{2\mu_\circ} dr \\ &= \int_0^R \frac{\pi r}{\mu_\circ} \times \frac{\mu_\circ^2 I^2 r^2}{4\pi^2 R^4} \, dr \end{aligned}$$

$$\begin{aligned}
\mathcal{U} &= \int_0^R \frac{\mu_\circ I^2}{4\pi R^4}\, r^3\, dr \\
&= \frac{\mu_\circ I^2}{4\pi R^4} \times \frac{R^4}{4} \\
&= \frac{\mu_\circ I^2}{16\pi}
\end{aligned}$$

The energy per unit length $\mathcal{U}$ is independent of the radius of the wire.