Chapter 3

Physics 205 Solution of Home Work Problems

3.1 Problem 3.10

A typical rutherford scattering apparatus consists of an evacuated tube containing a polonium -210 α source (2.5 MeV $\alpha's$), collimators, a gold foil target, and a special alpha-detecting film. The detecting film simultaneously measures the alphas scattered over a range from $\phi = 2.5 - 12.5°$. (See Fig. P3.10.) The total number of counts measured over a week's time falling in a specific ring (denoted by its average scattering angle) and the corresponding ring area are given in Table 3.2.

(a) Find the counts per area at each angle and correct these values for the angle independent background. The background correction may be found from a seven-day count taken with the beam blocked with a metal shutter in which 72 counts were measured evenly distributed over the total detector area of 8.50 cm^2.

(b) show that the corrected counts per unit area are proportional to $\sin^{-4}(\phi/2)$ or in terms of the Rutherford formula, Equation 3.16,

$$\frac{\Delta n}{A} = \frac{C}{\sin^4(\phi/2)}$$

Notes: If a plot of $(\Delta n/A)$ versus ϕ will not fit on a single sheet of graph paper, try plotting $\log(\Delta n/A)$ versus $\log\left[1/\sin^4(\phi/2)\right]$. This plot yields a straight line with a slope of of one and an intercept that gives C. Explain why this technique works.

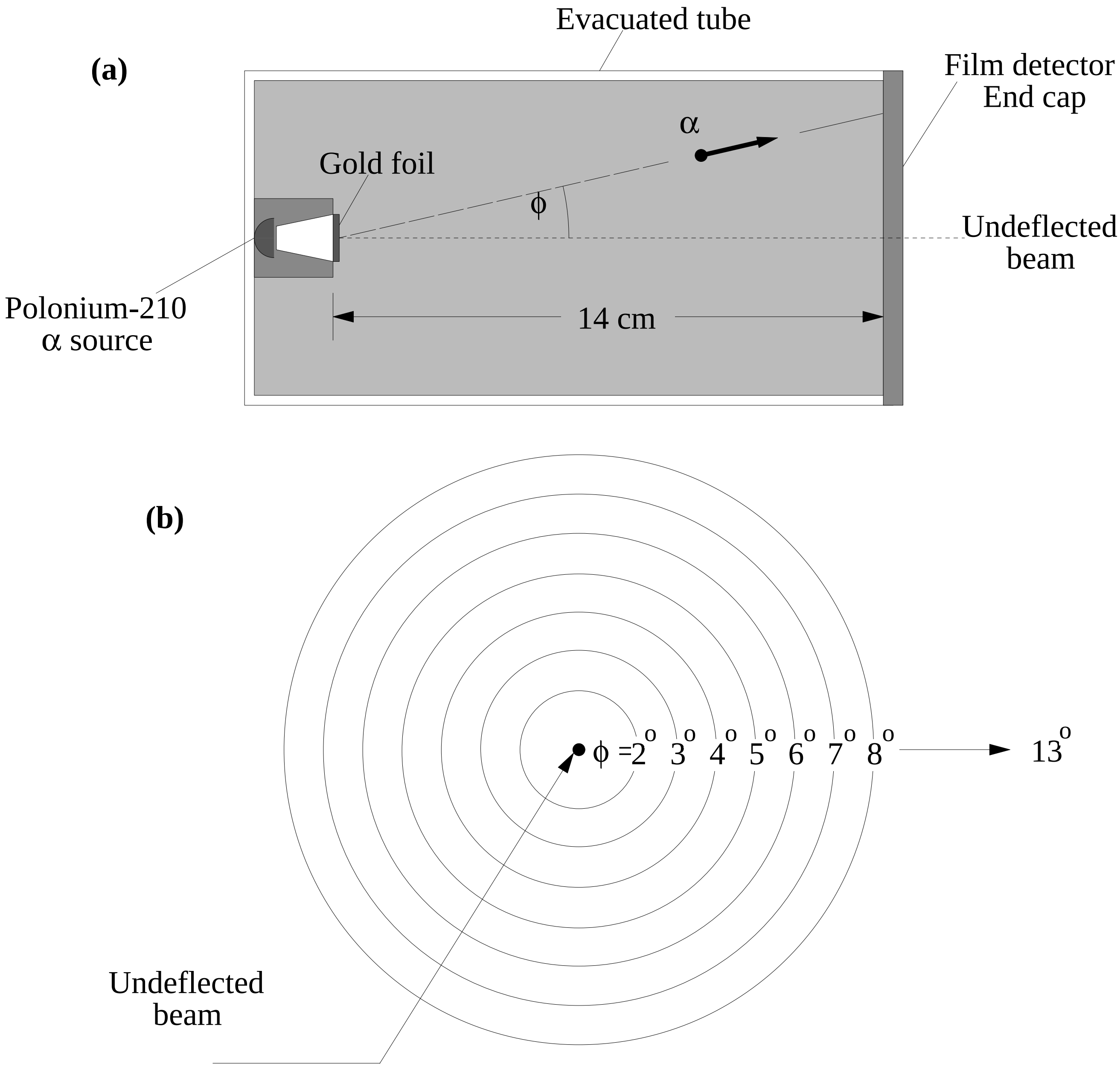

Figure 3.10: **(a)** Side view of Rutherford's scattering apparatus: ϕ is the scattering angle. **(b)** End view of the rutherford apparatus showing the film detector end cap with grid marking the angle ϕ. The α particles damage the film emulsion and after development show up as dots within the rings.

Table 3.1: **Data to be Used in problem 10**

Angle (degrees)	Counts/Ring	Ring Area (cm^2)	Counts/Area
2.5	605	0.257	
3.5	631	0.360	
4.5	520	0.463	
5.5	405	0.566	
6.5	301	0.669	
7.5	201	0.772	
8.5	122	0.875	
9.5	78	0.987	
10.5	65	1.08	
11.5	66	1.18	
12.5	44	1.29	

Solution

(a) The background counts per cm^2 is given by $72/8.50 = 8.47$ Counts/cm^2. So, the corrected Counts/Area = Counts/Area - Background Count/Area. We can then fill the table as follows:

Angle (degrees)	Counts/Ring	Ring Area (cm^2)	Counts/Area	Background-Corrected Counts/Area ($\Delta n/A$)
2.5	2605	0.257	10136	10128
3.5	1031	0.360	2863	2855
4.5	520	0.463	1123	1115
5.5	405	0.566	715.6	707.1
6.5	301	0.669	449.9	441.5
7.5	201	0.772	260.4	251.9
8.5	122	0.875	139.4	131.0
9.5	78	0.987	79.03	70.56
10.5	65	1.08	60.19	51.72
11.5	66	1.18	55.93	47.46
12.5	44	1.29	34.11	25.64

(b) To show the relation between the count rate and the angle we construct the following table:

Angle,ϕ (degrees)	$\sin(\phi/2)$	$\sin^{-4}(\phi/2)$	$\log(\sin^{-4}(\phi/2))$	log (Background Corrected Counts/Area) $\log(\Delta n/A)$
2.5	0.0218	4.43×10^6	6.65	4.01
3.5	0.0305	1.14×10^6	6.06	3.46
4.5	0.0393	4.21×10^5	5.62	3.05
5.5	0.0400	1.89×10^5	5.28	2.85
6.5	0.0567	9.68×10^4	4.99	2.65
7.5	0.0654	5.47×10^4	4.74	2.40
8.5	0.0741	3.32×10^4	4.52	2.12
9.5	0.0828	2.13×10^4	4.33	1.85
10.5	0.0915	1.43×10^4	4.15	1.71
11.5	0.1001	9.98×10^3	4.00	1.68
12.5	0.1089	7.12×10^3	3.85	1.41

The values of $\sin^{-4}(\phi/2)$ have a wide range that scans three orders of magnitude. It will not be practical to plot $\sin^{-4}(\phi/2)$ vs $\Delta n/A$ on a linear graph. Therefore we plot $\log(\sin^{-4}(\phi/2))$ vs $\log(\Delta n/A)$ instead. the plot is shown in Figure 3.11 below.

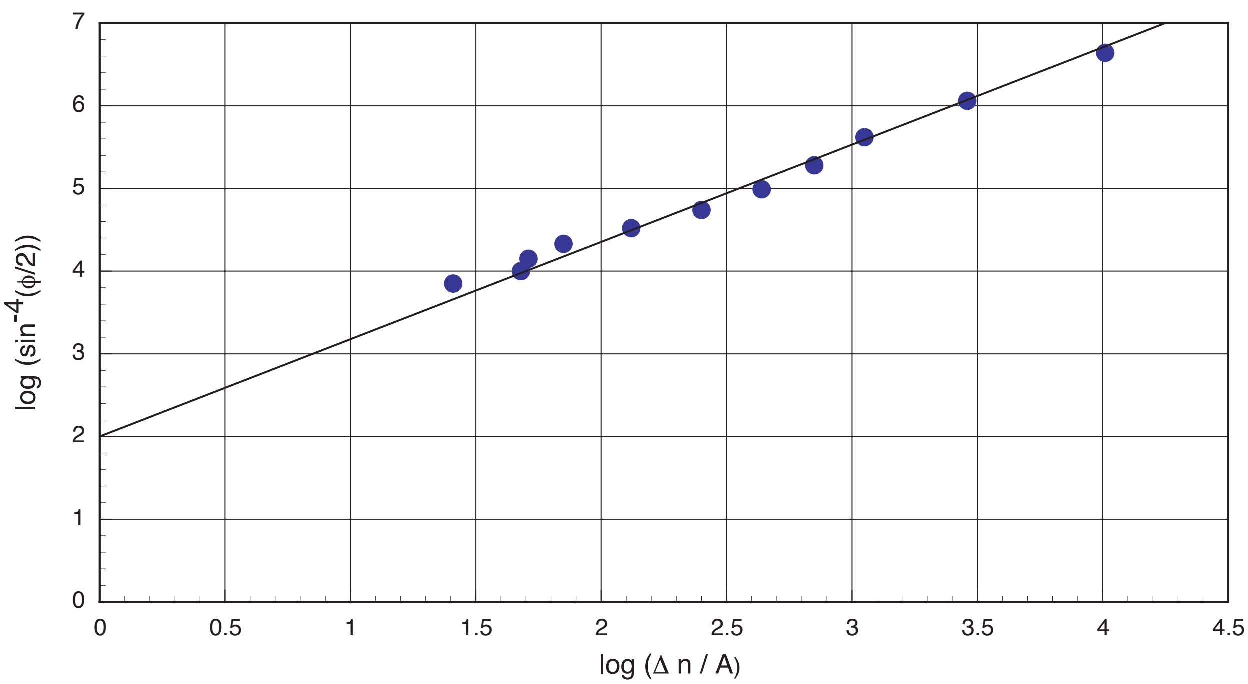

Figure 3.11: A plot of $\log(\Delta n/A) vs \log(\sin^{-4}(\phi/2))$, with a “best fit” straight line. The line has a y-intercept of 2.0 and a slope of ≈ 1.1, confirming the linear relation between $\Delta n/A$ and $\sin^{-4}(\phi/2)$.

This method works since the Rutherford formula is:

$$\frac{\Delta n}{A} = \frac{C}{\sin^{-4}(\phi/2)}$$

taking the log of this equation gives:

$$\log\left(\frac{\Delta n}{A}\right) = \log C + \log\left(\frac{C}{\sin^{-4}(\phi/2)}\right)$$

the last equation is an equation of a straight line like $y = mx + b$. where m is the slope and b is the y-intercept. In Rutherford "log" equation, the "slope = 1" and the "intercept = log C". The straight line in Figure 3.11 is the "best fit" straight line and its equation is $y = 1.1x + 2.0$ where $y = \log\left(\frac{C}{\sin^{-4}(\phi/2)}\right)$ and $x = \log\left(\frac{\Delta n}{A}\right)$.

3.2 Problem 3.23

A hydrogen atom is in its ground state ($n = 1$). Using the Bohr theory of the atom calculate

(a) the radius of the orbit,

(b) the linear momentum of the electron,

(c) the angular momentum of the electron,

(d) the kinetic energy,

(e) the potential energy, and

(f) the total energy.

Solution

(a) The radius of an orbit in a hydrogen atoms is:

$$\begin{aligned} r_n &= \left(\frac{h^2}{4\pi^2 m_e k e^2}\right) n^2 \\ &= 0.0529\, n^2 \;\; nm \\ &= 0.0529 \;\; nm \quad \text{for n = 1} \end{aligned}$$

(b) The linear momentum of the electron in the first orbit is given by:

$$\begin{aligned} \frac{m_e v^2}{r_n} &= \frac{ke^2}{r_n^2} \\ m_e v^2 &= \frac{ke^2}{r_n} \\ (m_e v)^2 &= \frac{m_e k e^2}{r_n} \\ p_n &= \sqrt{\frac{m_e k e^2}{r_n}} \\ p_1 &= \sqrt{\frac{m_e k e^2}{r_1}} \\ &= \sqrt{\frac{9.11 \times 10^{-31} \times 8.988 \times 10^{9} \times (1.602 \times 10^{-19})^2}{0.0529 \times 10^{-9}}} \\ &= 1.99 \times 10^{-24} \;\; kg \cdot m/s \end{aligned}$$

(c) The angular momentum L of the electron is:

$$\begin{aligned} L_n &= p_n r_n \\ L_1 &= p_1 r_1 \\ &= 1.99 \times 10^{-24} \times 0.0529 \times 10^{-9} \\ &= 1.05 \times 10^{-34} \;\; kg \cdot m^2/s \\ &= \frac{h}{2\pi} \end{aligned}$$

(d) The kinetic energy K of the electron is:

$$\begin{aligned} K_n &= \frac{ke^2}{2r_n} \\ K_1 &= \frac{ke^2}{2r_1} \\ &= \frac{8.988 \times 10^9 \times (1.602 \times 10^{-19})^2}{2 \times 0.0529^{-9}} \\ &= 2.18 \times 10^{-18} \;\; Jules \\ &= \frac{2.18 \times 10^{-18}}{1.602 \times 10^{-19}} \;\; eV \\ &= 13.6 \;\; eV \end{aligned}$$

(e) The potential energy PE of the electron is

$$\begin{aligned} PE_n &= -\frac{ke^2}{r_n} \\ PE_1 &= -\frac{ke^2}{r_1} \\ &= -\frac{8.988 \times 10^9 \times (1.602 \times 10^{-19})^2}{\times 0.0529^{-9}} \\ &= -4.36 \times 10^{-18} \;\; Jules \\ &= -\frac{4.36 \times 10^{-18}}{1.602 \times 10^{-19}} \;\; eV \\ &= -27.2 \;\; eV \end{aligned}$$

(f) The total energy E of the electron is:

$$\begin{aligned} E_n &= K_n + PE_n \\ E_1 &= K_1 + PE_1 \\ &= 13.6 - 27.2 \\ &= -13.6 \;\; eV \end{aligned}$$

3.3 Problem 3.28

The Auger process. An electron in chromium makes a transition from the $n = 2$ state to the $n = 1$ state without emitting a photon. Instead, the excess energy is transferred to an outer electron (in the $n = 4$ state), which is ejected by the atom. (This is called an *Auger process*, and the ejected electron is referred to as an *Auger electron.*) Use the Bohr theory to find the kinetic energy of the Auger electron.

Solution

The chromium nucleus has 24 protons. In a simple picture, orbiting electrons see a nucleus with a charge of $Ze = 24e$. The energy E_n an electron is then:

$$E_n = 13.6 Z^2 \left(\frac{1}{n^2}\right)$$

the energy ΔE that is available from the $n = 2$ to $n = 1$ transition is:

$$\begin{aligned}
\Delta E &= 13.6 Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\
&= 13.6 \times (24)^2 \left(\frac{1}{1} - \frac{1}{4}\right) \\
&= 5875 \;\; eV \\
&= 5.875 \;\; keV
\end{aligned}$$

The kinetic energy K of the Auger electron is 5.875 keV minus the ionization energy $E_{ionization}$ that is required to to remove the electron from the $n = 4$ state to infinity.

$$\begin{aligned}
E_{ionization} &= E_4 \\
&= 13.6 \left(\frac{Z}{4}\right)^2 \\
&= 13.6 \left(\frac{24}{4}\right)^2 \\
&= 0.4896 \;\; keV \\
K &= \Delta E - E_{ionization} \\
&= 5.875 - 0.4896 \\
&= 5.385 \;\; keV
\end{aligned}$$

3.4 Problem 3.37

Use Bohr's model of the hydrogen atom to show that when the atom makes a transition from the state n to the state $n-1$, the frequency of the emitted light is given by

$$\nu = \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n-1}{(n-1)^2 n^2}\right]$$

Show that as $n \to \infty$, the expression above varies as $1/n^3$ and reduces to the classical frequency one would expect the atom to emit. (*Hint:* To calculate the classical frequency, note that the frequency of revolution is $v/2\pi r$, where r is given by Equation 3.28.) This an example of the correspondence principle, which requires that the classical and quantum models agree for large values of n.

Solution

The energy ΔE that is emitted in a transition from a state n to a state $n-1$ is:

$$\begin{aligned}
\Delta E &= h\nu \\
h\nu &= \Delta E \\
&= \left(\frac{4\pi^2 m_e k^2 e^4}{2h^2}\right)\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) \\
\nu &= \left(\frac{2\pi^2 m_e k^2 e^4}{h^3}\right)\left(\frac{n^2-(n-1)^2}{(n-1)^2 n^2}\right) \\
&= \left(\frac{2\pi^2 m_e k^2 e^4}{h^3}\right)\left(\frac{n^2-n^2+2n-1}{(n-1)^2 n^2}\right) \\
&= \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n-1}{(n-1)^2 n^2}\right]
\end{aligned}$$

When n becomes very large i.e. $n \gg 1$ the above equation becomes:

$$\begin{aligned}
\nu &= \frac{2\pi^2 m_e k^2 e^4}{h^3}\left(\frac{2}{n^3}\right) \\
&= \frac{4\pi^2 m_e k^2 e^4}{h^3}\left(\frac{1}{n^3}\right)
\end{aligned}$$

Classically the emitted frequency is the frequency of revolution $\nu = 1/T$, where T is the period of the revolution. The speed of the circling electron $v = 2\pi r/T = 2\pi r\nu$. The velocity and radius of the electron are given by:

$$
\begin{aligned}
v &= \sqrt{\frac{ke^2}{m_e r}} \\
\frac{1}{r} &= \frac{4\pi^2 m_e k e^2}{h^2}\frac{1}{n^2}
\end{aligned}
$$

Using the last two equations we get:

$$
\begin{aligned}
\nu &= \frac{v}{2\pi r} \\
&= \frac{\sqrt{\frac{ke^2}{m_e r}}}{2\pi r} \\
&= \sqrt{\frac{ke^2}{4\pi^2 m_e r^3}} \\
&= \sqrt{\frac{ke^2}{4\pi^2 m_e} \times \left(\frac{4\pi^2 m_e k e^2}{h^2}\frac{1}{n^2}\right)^3} \\
&= \sqrt{\frac{16\pi^4 m_e^2 k^4 e^8}{h^6 n^6}} \\
&= \frac{4\pi^2 m_e k^2 e^4}{h^3}\left(\frac{1}{n^3}\right)
\end{aligned}
$$

The last equation is identical to the quantum equation for $n \gg 1$.

3.5 Problem 3.43

Four possible transition for a hydrogen atom are listed below.

$$\begin{array}{lllllll}
(A) & n_i & = & 2; & n_f & = & 5 \\
(B) & n_i & = & 5; & n_f & = & 3 \\
(C) & n_i & = & 7; & n_f & = & 4 \\
(D) & n_i & = & 4; & n_f & = & 7
\end{array}$$

(a) Which transitions emits the photons having the shortest wavelength?

(b) For which transition does the atom gain the most energy?

(c) For which transition(s) does the atom lose energy?

Solution

In a hydrogen atom, the energy ΔE in eV involved in a transition from a state n_i to a state n_f is given by:

$$\Delta E = 13.6 \left[\frac{1}{n_i^2} - \frac{1}{n_f^2} \right]$$

When $\Delta E > 0$ we have absorption and when $\Delta E < 0$ we have emission. So,

(A) for $n_i = 2$ and $n_f = 5$, $\Delta E = +2.86\ eV$ (absorption)

(B) for $n_i = 5$ and $n_f = 3$, $\Delta E = -0.97\ eV$ (emission)

(C) for $n_i = 7$ and $n_f = 4$, $\Delta E = -0.57\ eV$ (emission)

(D) for $n_i = 4$ and $n_f = 7$, $\Delta E = +0.57\ eV$ (absorption)

using these results we get:

(a) Since $E = hc/\lambda$, then the shortest wavelength is emitted in transition B.

(b) The atom gains most energy in transition A.

(c) The atom loses energy in transitions B and C.