Chapter 1

Physics 205 Solution of Home Work Problems

1.1 Problem 1.11

The average lifetime of a π-meson in its own frame of reference is 2.6×10^{-8} s. (This is proper time.) If the meson moves with a speed of $0.95c$, what is:

(a) its mean lifetime as measured by an observer on Earth? and,

(b) the average distance it travels before decaying, as measured by an observer on Earth?

Solution

(a)

$$\tau = \gamma\tau'$$

where $\tau' = 2.6 \times 10^{-8}$ s is the average lifetime in the pion's frame, τ is the average lifetime as measured by an observer at rest, $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = v/c$, we then have:

$$\begin{aligned} \tau &= \frac{\tau'}{\sqrt{1-\beta^2}} \\ &= \frac{\tau'}{\sqrt{1-(\frac{v}{c})^2}} \\ &= \frac{2.6 \times 10^{-8}}{\sqrt{1-(0.95)^2}} \\ &= 8.33 \times 10^{-8}\ s \end{aligned}$$

(b) The distance d travelled by the pions before decaying as measured by an observer at rest is obtain from the rest frame velocity and lifetime, i.e.

$$\begin{aligned} d &= v\tau \\ &= 0.95 \times c \times \tau \\ &= 0.95 \times 3.0 \times 10^8 \times 8.33 \times 10^{-8} \\ &= 24\ m \end{aligned}$$

1.2 Problem 1.15

A rod of length $L_\circ$ moves with a speed of v along the horizontal direction. The rod makes an angle of $\theta_\circ$ with respect to the x' axis.

(a) Show that the length of the rod as measured by a stationary observer is given by:

$$L = L_\circ \sqrt{\left[1 - \frac{v^2}{c^2}\cos^2\theta_\circ\right]}$$

(b) Show that the angle that the rod makes with the x axis is given by the expression:

$$\tan\theta = \gamma \tan\theta_\circ$$

These results show that the rod is both contracted and rotated. (Take the lower end of the rod to be at the origin of the primed coordinate system.)

Solution

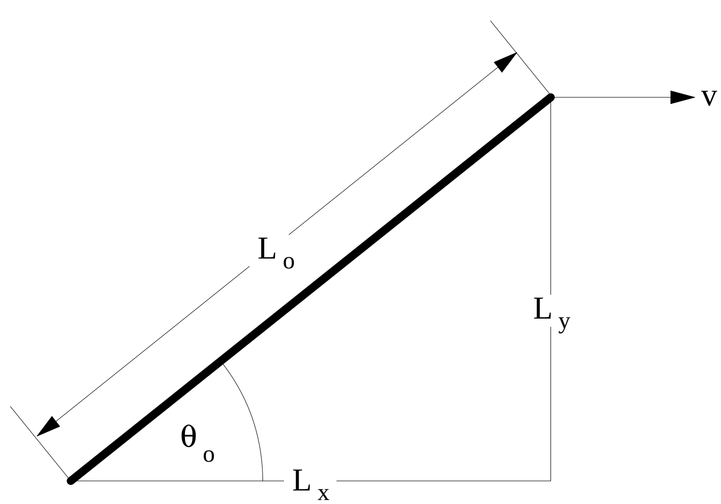

Figure 1.15: A A rod of length $L_\circ$ moving with a speed v along x' axis and makes an angle $\theta_\circ$ with it

(a) Only the x-component of $L_\circ$ contracts as a result of the motion along the x-axis. We have:

$$L_{x'} = L_\circ \cos\theta_\circ$$

and

$$L_{y'} = L_\circ \sin\theta_\circ = L_y$$

a stationary observer will find L_x as:

$$\begin{aligned} L_x &= \frac{L_{x'}}{\gamma} \\ &= \frac{L_\circ \cos\theta_\circ}{\gamma} \end{aligned}$$

The total length of the rod L as measured by a stationary observer is:

$$\begin{aligned} L &= \sqrt{(L_x)^2 + (L_y)^2} \\ &= \sqrt{\left(\frac{L_\circ \cos\theta_\circ}{\gamma}\right)^2 + (L_\circ \sin\theta_\circ)^2} \\ &= L_\circ \sqrt{\cos^2\theta_\circ \left(1 - \frac{v^2}{c^2}\right) + \sin^2\theta_\circ} \\ &= L_\circ \sqrt{\cos^2\theta_\circ - \cos^2\theta_\circ \frac{v^2}{c^2} + \sin^2\theta_\circ} \\ &= L_\circ \sqrt{1 - \frac{v^2}{c^2}\cos^2\theta_\circ} \end{aligned}$$

(b) The angle of the rod makes with the x axis as seen by the stationary observer is:

$$\begin{aligned} \tan\theta &= \frac{L_y}{L_x} \\ &= \frac{L_\circ \sin\theta_\circ}{L_\circ \cos\theta_\circ/\gamma} \\ &= \gamma \tan\theta_\circ \end{aligned}$$

1.3 Problem 1.24

An observer in frame S sees lightning simultaneously strike two points 100 m apart. The first strike occurs at $x_1 = y_1 = z_1 = t_1 = 0$ and the second at $x_2 = 100\ m, y_2 = z_2 = t_2 = 0$.

(a) What are the coordinates of these two events in a frame S' moving in the standard configuration at $0.70c$ relative to S.

(b) How far apart are the events in S'.

(c) Are the events simultaneous in S'? If not, what is the difference in time between the events, and which event occurs first?

Solution

(a) In the moving frame S' the first event have the following coordinates:

$$\begin{aligned} x_1' &= \gamma(x_1 - vt_1) \\ &= 0 \\ y_1' &= y_1 \\ z_1' &= z_1 \\ t_1' &= \gamma\left[t_1 - \frac{v}{c^2}x_1\right] \\ &= 0 \end{aligned}$$

where

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-(0.70)^2}} \\ &= 1.4 \end{aligned}$$

Similarly the second event has the following coordinates:

$$\begin{aligned} x_2' &= \gamma(x_2 - vt_2) \\ &= 1.4 \times (100 - 0) \end{aligned}$$

$$
\begin{aligned}
&= 140 \\
y_2' &= y_2 \\
z_2' &= z_2 \\
t_2' &= \gamma\left[t_2 - \frac{v}{c^2}x_2\right] \\
&= 1.4 \times \left[0 - \frac{0.7}{3 \times 10^8} \times 100\right] \\
&= -0.33\ \mu s
\end{aligned}
$$

(b) Space separation between the events in the moving frame S' is:

$$
\begin{aligned}
\Delta x' &= x_2' - x_1' \\
&= 140 - 0 \\
&= 140\ m
\end{aligned}
$$

(c) The events are not simultaneous in the moving frame S', the second event occurs 0.33 μs earlier than the first event.

1.4 Problem 1.38

Protons in an accelerator at Fermi National laboratory near Chicago are accelerated to an energy of 400 times their rest energy.

(a) What is the speed of these protons?

(b) What is the their kinetic energy in MeV?

(a) The energy of the proton is:

$$\begin{aligned} E &= 400 \times mc^2 \\ &= \gamma mc^2 \end{aligned}$$

so γ must be equal to 400, we then have:

$$400 = \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and

$$\begin{aligned} \frac{v}{c} &= \sqrt{1 - \frac{1}{(400)^2}} \\ &= 0.999997 \end{aligned}$$

(b) The kinetic energy of the proton ($m_p = 938.3\ MeV/c^2$) is then:

$$\begin{aligned} K &= E - m_p c^2 \\ &= 400 \times m_p c^2 - m_p c^2 \\ &= (400 - 1) \times m_p c^2 \\ &= 399 \times 938.3 \\ &= 3.74 \times 10^5\ MeV \\ &= 0.374 \times 10^6\ MeV \\ &= 0.374\ GeV \end{aligned}$$

1.5 Problem 1.47

The K° meson is an uncharged member of the particle "zoo" that decay according to:

$$K^\circ \rightarrow \pi^+ + \pi^-$$

The pions have opposite charges as indicated and the same mass $m_\pi = 140\ MeV/c^2$. Suppose that a K° at rest decays into two pions in a bubble chamber in which a magnetic field of 2.0 T is present (see Figure 1.47). If the radius of curvature of the pions is 34.4 cm, find

(a) the momenta and speeds of the pions, and

(b) the mass of the K° meson.

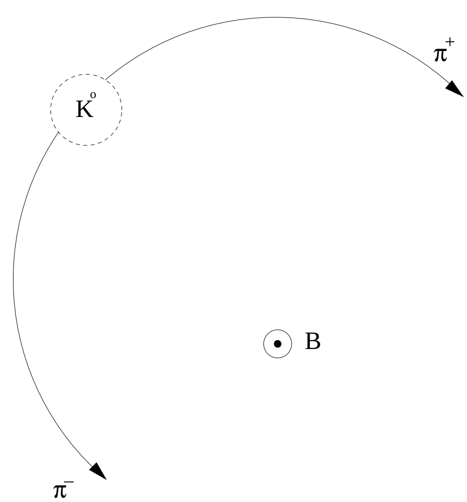

Figure 1.47: A sketch of the tracks made by π^+ and π^-in the decay of K° meson at rest. The pion motion is perpendicular to **B**. (**B** is directed out of the page.)

Solution

(a) A pion with a charge e moving with a velocity v that is perpendicular to a magnetic field **B** will move in a circular path of radius R under a centripetal force of Bev:

$$\frac{m_\pi v_\pi^2}{R} = Bev_\pi$$

and the momentum of the pion is:

$$\begin{aligned} m_\pi v_\pi &= BeR \\ p_\pi &= BeR; \end{aligned} \tag{1.1}$$

if m_π is in kg, v_π in m/s, R in meters, e in coulomb and B in Tesla then the momentum p_π is in $kg \cdot m/s$. The momentum units need to be changed to MeV/c, to do that we use:

$$\begin{aligned} \frac{kg \cdot m}{s} &= \frac{(kg \cdot m) \times c}{c} \\ &= \frac{3 \times 10^8 \; kg \cdot m^2}{s^2 \cdot c} \\ &= 3 \times 10^8 \; \frac{J}{c} \\ &= \frac{3 \times 10^8}{1.6 \times 10^{-19}} \; \frac{eV}{c} \\ &= \frac{3}{1.6} \times 10^{27} \times 10^{-6} \; \frac{MeV}{c} \\ &= \frac{3}{1.6} \times 10^{21} \; \frac{MeV}{c} \end{aligned} \tag{1.2}$$

Substituting in Equation 1.1 by the value of $e = 1.6 \times 10^{-19}$ C, the given values of B and R, and using Equation 1.2 we get:

$$\begin{aligned} p_\pi &= 1.6{-}19BR \; \frac{kg \cdot m}{s} \\ &= 1.6 \times 10^{-19} \times \frac{3}{1.6} \times 1021 \; BR \; \frac{MeV}{c} \\ &= 300 \; BR \; \frac{MeV}{c} \\ &= 300 \times 2.0 \times 0.344 \; \frac{MeV}{c} \\ &= 206 \; \frac{Mev}{c} \end{aligned}$$

Since the K° was at rest at the time of its decay, then momentum before decay is zero. As a result the two pions must have equal and opposite momenta. Since the π^+ and π^- have equal masses, then they must also have equal speeds, v_π, to find this speed we use the relativistic expressions of energy and momentum:

$$p_\pi = \gamma m_\pi v_\pi$$

and

$$E_\pi = \gamma m_\pi c^2$$

we then get:

$$\frac{p_\pi}{E_\pi} = \frac{v_\pi}{c^2}$$

or

$$\frac{v_\pi}{c} = \frac{p_\pi c}{E_\pi}$$

since:

$$E_\pi = \sqrt{p_\pi^2 c^2 + (m_\pi c^2)^2}$$

then:

$$\begin{aligned}
\frac{v_\pi}{c} &= \frac{p_\pi c}{\sqrt{p_\pi^2 c^2 + (m_\pi c^2)^2}} \\
&= \frac{206}{\sqrt{(206)^2 + (140)^2}} \\
&= 0.827
\end{aligned}$$

(b) Conservation of mass-energy requires that:

$$E_{K^\circ} = 2E_\pi$$

$$E_{K^\circ} = m_{K^\circ} c^2$$

or

$$\begin{aligned}
m_{K^\circ} c^2 &= 2E_\pi \\
&= 2\sqrt{p_\pi^2 c^2 + (m_\pi c^2)^2} \\
&= 2\sqrt{(206)^2 + (140)^2} \\
&= 498\ MeV \\
m_{K^\circ} &= 498\ \frac{MeV}{c^2}
\end{aligned}$$