Chapter 3

Quantum Theory of Light. Solutions of Selected Problems

3.1 Problem 3.4 (In the text book)

(a) Use Stefan’s law to calculate the total power radiated per unit area by a tungsten filament at a temperature of 3000 K. (Assume that the filament is an ideal radiator.)

(b) If the tungsten filament of a lightbulb is rated at 75 W, what is the surface area of the filament? (Assume that the main energy loss is due to radiation.)

Solution

(a) From Stefan’s Law we get:

\[
\frac{P}{A} = \sigma T^4 = 5.7 \times 10^{-8} \times (3000)^4 = 4.62 \times 10^6 \text{ W/m}^2
\]

(b) The area can be obtained from the power rating and the total power per unit area we calculated in part (a):

\[
A = \frac{74}{4.62 \times 10^6}
= 1.62 \times 10^{-5} \text{ m}^2
= 1.62 \times 10^{-5} \times 10^6 \text{ mm}^2
= 16.2 \text{ mm}^2
\]
3.2 Problem 3.16 *(In the text book)*

When cesium metal is illuminated with light of wavelength 300 nm, the photoelectrons emitted have a maximum kinetic energy of 2.23 eV. Find

(a) the work function of cesium and

(b) the stopping potential if the incident light has a wavelength of 400 nm.

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**Solution**

(a) In photoelectric effect, the maximum kinetic energy $K_{\text{max}}$ of the emitted electron is given by:

\[
K_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi
\]

\[
\phi = \frac{hc}{\lambda} - K_{\text{max}} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.602 \times 10^{-19} \times 300 \times 10^{-9}} - 2.23 = 4.13 - 2.23 = 1.90 \text{ eV}
\]

(b) The energy lost in the stopping potential $eV_s$ should be equal to the maximum kinetic energy $K_{\text{max}}$ of the emitted electrons:

\[
eV_s = \frac{hc}{\lambda} - \phi = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.602 \times 10^{-19} \times 400 \times 10^{-9}} - 1.90 = 3.10 - 1.90 = 1.20 \text{ eV}
\]

\[
V_s = 1.20 V
\]
3.3 Problem 3.31 \textit{(In the text book)}

A photon of initial energy 0.1 \textit{MeV} undergoes Compton scattering at an angle of \(60^\circ\). Find

(a) the energy of the scattered photon,
(b) the recoil kinetic energy of the electron, and
(c) the recoil angle of the electron.

\textbf{Solution}

(a) The wavelength \(\lambda_o\) of the incoming photon related to its energy \(E_o\) by:

\[
E_o = \frac{hf_o}{hc} = \frac{\lambda_o}{\lambda_o} = \frac{hc}{E_o}
\]

\[
= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.602 \times 10^{-19} \times 0.1 \times 10^6}
= 1.241 \times 10^{-11} \text{ m}
= 1.241 \times 10^{-2} \text{ nm}
\]

The wavelength shift \(\Delta \lambda\) due Compton scattering at \(60^\circ\) is:

\[
\Delta \lambda = \left(\frac{hc}{m_ec^2}\right) (1 - \cos \theta)
= \left(\frac{1.240 \times 10^3}{511 \times 10^3}\right) (1 - \cos 60)
= 1.213 \times 10^{-3} \text{ nm}
= 1.213 \times 10^{-12} \text{ m}
\]
Note that I used the value of $hc$ in units of $eV \cdot nm$ and the rest mass energy of the electron in $eV$ units. The wavelength $\lambda'$ of the scattered photon is then:

$$\lambda' = \lambda_o + \Delta \lambda = 1.241 \times 10^{-2} + 1.213 \times 10^{-3} = 1.362 \times 10^{-2} \text{nm} = 1.362 \times 10^{-11} \text{m}$$

The energy $E'$ of the scattered photon is then:

$$E' = \frac{hc}{\lambda'} = \frac{1.240 \times 10^{3} (eV \cdot nm)}{1.362 \times 10^{-2} (nm)} = 9.104 \times 10^{4} eV = 91.04 keV$$

(b) The kinetic energy of the electron $K_e$ is the difference between the the kinetic energies of the incident and scattered photons:

$$K_e = E - E' = 100(keV) - 91.04(keV) = 8.96 keV$$

(c) let assume that the scattered electron moves with a velocity $u$, the conservation of momentum along the $x$-direction requires:

$$\frac{h}{\lambda_o} = \frac{h}{\lambda'} \cos \theta + m_e \gamma u \cos \phi$$

and conservation along the $y$-direction requires:

$$\frac{h}{\lambda_o} - \frac{h}{\lambda'} \cos \theta = m_e \gamma u \cos \phi$$

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1$hc = 6.626 \times 10^{-34} (J \cdot s) \times 3 \times 10^8 (m/s) = 1.988 \times 10^{-25} (J \cdot m) = 1.988 \times 10^{-25} / 1.602 \times 10^{-19} (eV \cdot m) = 1.240 \times 10^{-6} (eV \cdot m) = 1.240 \times 10^{-6} \times 10^9 (eV \cdot nm) = 1.240 \times 10^3 (eV \cdot nm)$. 

\[
\frac{h}{\lambda'} \sin \theta = m_c \gamma u \sin \phi \tag{3.2}
\]

Now dividing Equation (3.2) by Equation (3.1) we get:

\[
\tan \phi = \frac{(h/\lambda') \sin \theta}{(h/\lambda) - (h/\lambda') \cos \theta} = \frac{(1/\lambda') \sin \theta}{(1/\lambda) - (1/\lambda') \cos \theta} = \frac{1}{\lambda(1/\lambda) - \lambda'(1/\lambda') \cos \theta} = \frac{1.241 \times 10^{-11} \sin 60}{1.362 \times 10^{-11} - 1.241 \times 10^{-11} \cos 60} = 1.449
\]

\[
\phi = 55.4^\circ
\]
3.4 Problem 3.38 (In the text book)

As a single crystal is rotated in an x-ray spectrometer Figure (3.1), many parallel planes of atoms besides $AA$ and $BB$ produce strong diffracted beams. Two such planes are shown in Figure (3.2).

(a) Determine geometrically the interplanar spacings $d_1$ and $d_2$ in terms of $d_o$.

(b) Find the angles (with respect to the surface plane $AA$) of the $n = 1, 2, \text{ and } 3$ intensity maxima from planes with spacing $d_1$. Let $\lambda = 0.626 \text{ Å}$ and $d_o = 4.00 \text{ Å}$. Note that a given crystal structure (for example, cubic) has interplanar spacings with characteristic ratios, which produce characteristic diffraction patterns. In this way, measurement of the angular position of diffracted x-rays may be used to infer the crystal structure.

Solution

(a) To determine geometrically the interplanar spacings $d_1$ and $d_2$ in terms of $d_o$ we make use of Figure (3.3). From triangle $abc$ we find that the angle $\phi_1 = 45^\circ$. Using triangle $efg$ we find $\phi_2$ as:

$$\sin \phi_2 = \frac{d_o}{\sqrt{d_o^2 + (2d_o)^2}} = \frac{1}{\sqrt{5}}$$
and using the same triangle we get:

\[ d_1 = d_o \sin \phi_1 = \frac{d_o}{\sqrt{2}} \]

\[ d_2 = d_o \sin \phi_2 = \frac{d_o}{\sqrt{3}} \]

(b) Consider two parallel rays one scatters from plane \( B - B \) and the other from plane \( BB - BB \) the rays make angle \( \Theta \) with the the \( B - B \) and \( BB - BB \) planes. The rays make angle \( \Theta \) with the \( A - A \) plane as shown in Figure (3.4).

To find the angles \( \Theta \) that correspond to the first three maximum intensity \( (n = 1, 2, 3, \cdots) \), we apply Bragg’s law:

\[ n\lambda = 2d_1 \sin \Theta \]
Using $\lambda = 0.626 \, \text{Å}$ and $d_1 = 4.00 \, \text{Å}$ and $n = 1, 2, 3, \cdots$ we get:

\[
\theta_1 = \sin^{-1}\left(\frac{n_1 \lambda}{2d_1}\right) = \sin^{-1}\left(\frac{n_1 \lambda}{2d_0/\sqrt{2}}\right) = \sin^{-1}\left(\frac{1 \times 0.626\sqrt{2}}{2 \times 4}\right) = \sin^{-1}(0.111) = 6.37^\circ
\]

\[
\theta_2 = \sin^{-1}\left(\frac{n_2 \lambda}{2d_0/\sqrt{2}}\right)
\]
\[ \begin{align*}
\theta_3 &= \sin^{-1} \left( \frac{3 \times 0.626 \sqrt{2}}{2 \times 4} \right) \\
&= \sin^{-1}(0.332) \\
&= 19.4^\circ
\end{align*} \]

Since plane \( A - A \) makes an angle of \( 45^\circ \) with \( B - B \) or \( BB - BB \) then the angles \( \Theta \) with respect to \( A - A \) that correspond to the first three maxima are:

\[ \begin{align*}
\Theta_1 &= \theta_1 + 45 \\
&= 6.73 + 45 \\
&= 51.7^\circ \\
\Theta_2 &= 57.8^\circ \\
\Theta_3 &= 64.4^\circ
\end{align*} \]
3.5 Problem 3.43 \textit{(In the text book)}

In a Compton collision with an electron, a photon of violet light \((\lambda = 4000 \text{ Å})\) is backscattered through an angle of 180°.

(a) How much energy \((eV)\) is transferred to the electron in this collision?

(b) Compare your result with the energy this electron would acquire in a photoelectric process with the same photon.

(c) Could violet light eject electrons from a metal by Compton collision? Explain.

\textbf{Solution}

(a) The energy lost by the photon equals the energy gained by the electron. The energy lost \(\Delta E\) by the photon is related to the change in its wave length \(\Delta \lambda\) by:

\[
E = hf = \frac{hc}{\lambda} \\
dE = \frac{hc}{\lambda^2} d\lambda \\
\Delta E = \frac{hc}{\lambda^2} \Delta \lambda \\
= -\frac{hc}{\lambda^2} \frac{hc}{m_e c^2} (1 - \cos \theta) \\
= -\left(\frac{hc}{\lambda}\right)^2 \frac{1 - \cos \theta}{m_e c^2}
\]

Using the numerical values \(\lambda = 4000 \text{ Å} = 400 \text{ nm}, \theta = 180^\circ, hc = 1.240 \times 10^3 \text{ eV} \cdot \text{nm}\) and \(m_e c^2 = 5.11 \times 10^5 \text{ eV}\) we get;

\[
\Delta E = -\left(\frac{1.240 \times 10^3}{400}\right)^2 \times \frac{1 - (-1)}{5.11 \times 10^5} \\
= -3.76 \times 10^{-5} \text{ eV}
\]

This is the energy lost by the photon, so the energy transferred to the electron is 3.76 \times 10^{-5} \text{ eV}.
(b) In a photoelectric effect with a photon with $\lambda = 400 \text{ nm}$, the electron acquires all of the photon energy, i.e.

$$E_e = hf$$

$$= \frac{hc}{\lambda}$$

$$= \frac{1.240 \times 10^3}{400}$$

$$= 3.10 \text{ eV}$$

Comparing with the energy acquired by the electron in a Compton scattering, gives;

$$\frac{\Delta E}{E_e} = \frac{3.76 \times 10^{-5}}{3.1} = 1.21 \times 10^{-5}$$

So the maximum energy an electron can get from a 400 nm photon in Compton scattering is $\approx 10^{-5}$ times smaller than the energy it can get from the photon in a photoelectric effect.

(c) The answer is NO. A photon in the violet region of the electromagnetic spectrum has a wavelength of about 400 nm. In a Compton scattering the maximum energy an electron can get is when the photon back scatter at $\theta = 180^o$. As we have seen in part (a) this energy is $\approx 10^{-5} \text{ eV}$, which is much smaller than the work function of all metals, typically several electron volts.