Chapter 25

Electric Potential. Solutions of Selected Problems

25.1 Problem 25.11 *(In the text book)*

Review problem. A block having mass m and charge $+Q$ is connected to a spring having constant k. The block lies on a frictionless horizontal track, and the system is immersed in a uniform electric field of magnitude E, directed as shown in Figure (25.11). If the block is released from rest when the spring is unstretched (at $x = 0$),

(a) by what maximum amount does the spring expand?

(b) What is the equilibrium position of the block?

(c) Show that the block's motion is simple harmonic, and determine its period.

(d) What If? Repeat part (a) if the coefficient of kinetic friction between block and surface is μ_k.

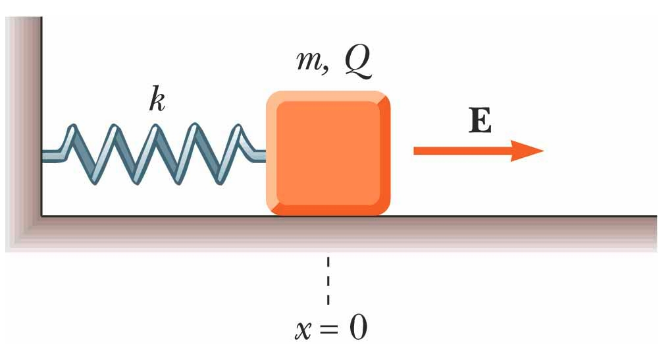

Figure 25.11:

Solution

(a) We can take the electric potential at $x = 0$ to be our reference point and take $V = 0$ at $x = 0$ (like taking the the surface of earth as the reference point for gravitational potential energy). The electric potential V and the electric potential energy U_e at other points are then:

$$V = -Ex \qquad \text{and} \qquad U_e = QV = -QEx$$

The spring potential energy U_s is:

$$U_s = \frac{1}{2}kx_{max}^2$$

The total energy (potential + kinetic) is conserved, i.e. the sum of the potential and kinetic energies at any point along the motion of the block is the same.

$$(K + U_e + U_s)_i = (K + U_e + U_s)_f$$

where K is the kinetic energy of the block, the subscript i refers to some initial position of the block and f refers to a final position. Taking the initial position to be at $x = 0$ and the final position at $x = x_{max}$ and noting that the $K = 0$ at $x = 0$ (since the block was released from rest at $x = 0$) and also at $x = x_{max}$, we get:

$$(0 + 0 + 0)_i = \left(0 + \frac{1}{2}kx_{max}^2 - QEx_{max}\right)_f \qquad \text{or} \qquad x_{max} = \frac{2QE}{k}$$

(b) Let there be equilibrium at a position x_{eq}. At equilibrium the net force on the block along the $x-$axis is zero, so

$$\sum F_x = -F_s + F_e = -kx_{eq} + QE = 0 \qquad \text{or} \qquad x_{eq} = \frac{QE}{k}$$

(c) The blocks moves along the $x-$axis, so the net force along the $x-$axis is the force that accelerates the block and the equation of motion of the block si:

$$\sum F_x = -kx + QE = m\frac{d^2x}{dt^2}$$

Let:

$$x' = x - \frac{QE}{k} \qquad \text{or} \qquad x = x' + \frac{QE}{k}$$

The equation of motion then becomes:

$$-k\left(x' + \frac{QE}{k}\right) + QE = \frac{d^2}{dt^2}\left(x' + \frac{QE}{k}\right) \qquad \text{or} \qquad \frac{d^2x'}{dt^2} = -\left(\frac{k}{m}\right)x'$$

Note that $d^2/dt^2(QE/k) = 0$. So the motion is simple harmonic with:

$$\omega = \sqrt{\frac{k}{m}} \qquad \text{and} \qquad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{k}{m}}$$

(d) With friction, the block will have a smaller x_{max} due to loss of energy as heat because of friction. For final energy to be the same as initial energy we must add the energy loss due to friction to the sum of the potential and kinetic energy at the final position. The loss of energy due to friction is the work done by friction force, i.e. $\mu_k mgx_{max}$, so conservation of energy gives:

$$\begin{aligned}
(K + U_e + U_s)_i &= (K + U_e + U_s)_f + \mu_k mgx_{max} \\
0 &= 0 + \frac{1}{2}kx^2_{max} - QEx_{max} + \mu_k mgx_{max} \\
&= \frac{1}{2}kx_{max} - QE - \mu_k mg \\
x_{max} &= \frac{2(QE - \mu_k mg)}{k}
\end{aligned}$$

25.2 Problem 25.26 *(In the text book)*

Review problem. Two insulating spheres have radii 0.300 cm and 0.500 cm, masses 0.100 kg and 0.700 kg, and uniformly distributed charges of $-2.00\ \mu C$ and $3.00\ \mu C$. They are released from rest when their centers are separated by 1.00 m.

(a) How fast will each be moving when they collide? (Suggestion: consider conservation of energy and of linear momentum.)

(b) What If? If the spheres were conductors, would the speeds be greater or less than those calculated in part (a)? Explain.

Solution

(a) Before release, the the two spheres were at rest, so their momentum and kinetic energies were all zeros. After release, the spheres are moving with velocities v_1 and v_2 in opposite directions. The potential energy before release was the electric potential energy of charges $-q_1$ and q_2 separated by a distance d. The potential energy after release is the electric potential energy of the charges separated by their radii, i.e. by $r_1 + r_2$. Conservation of momentum and energy then give:

$$0 = m_1 v_1 - m_2 v_2 \qquad \text{or} \qquad v_2 = \frac{m_1}{m_2} v_1$$

and

$$0 - \frac{k_e q_1 q_2}{d} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{k_e q_1 q_2}{r_1 + r_2}$$

Eliminating v_2 from the last equation we get:

$$\begin{aligned}
-\frac{k_e q_1 q_2}{d} &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2}\right)^2 v_1^2 - \frac{k_e q_1 q_2}{r_1 + r_2} \\
-\frac{k_e q_1 q_2}{d} + \frac{k_e q_1 q_2}{r_1 + r_2} &= \frac{1}{2} m_1 v_1^2 + \left(\frac{m_1}{m_2}\right) \frac{1}{2} m_1 v_1^2 \\
k_e q_1 q_2 \left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right) &= \frac{1}{2} m_1 v_1^2 \left(1 + \frac{m_1}{m_2}\right) = \frac{1}{2} v_1^2 \left[\frac{m_1(m_1 + m_2)}{m_2}\right] \\
v_1 &= \sqrt{\frac{2 m_2 k_e q_1 q_2}{m_1(m_1 + m_2)} \left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}
\end{aligned}$$

Substituting with the numerical values we get:

$$v_1 = \sqrt{\frac{2 \times 0.7 \times 8.99 \times 10^9 \times 2 \times 10^{-6} \times 3 \times 10^{-6}}{0.1(0.1 + 0.7)} \times \left(\frac{1}{(5+3) \times 10^{-3}} - \frac{1}{1.00} \right)} = 10.8\ m/s$$

and

$$v_2 = \frac{m_1}{m_2} v_1 = \frac{0.1}{0.7} \times 10.8 = 1.54\ m/s$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving faster than calculated in part (a).

25.3 Problem 25.47 *(In the text book)*

A wire having a uniform linear charge density λ is bent into the shape shown in Figure (25.47). Find the electrical potential at point O.

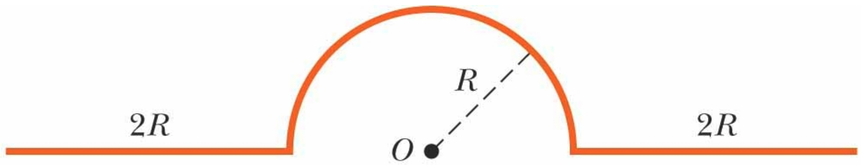

Figure 25.47:

Solution

The electric potential V at O is given by:

$$V = \int k_e \frac{dq}{r}$$

where the integral is carries over the whole length of the wire from $-3R$ to $-R$, over the semicircle, and from $+R$ to $+3R$. It is necessary to divide the wire in these three regions because of the different charge distribution in each region. Taking an element of length dx at a distance x in the first and last regions (note that x is positive in the last region and negative in the first region), then $dq = \lambda dx$. In the middle region we take an element of length ds that is a fixed distance R from O, then $dq = \lambda ds$ and ds varies from 0 to the length of the semicircle πR. The electric potential then becomes:

$$\begin{aligned}
V &= k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int_{0}^{\pi R} \frac{\lambda ds}{R} + k_e \int_{+R}^{+3R} \frac{\lambda dx}{x} \\
&= -k_e \lambda \left[\ln(x)\right]_{-3R}^{-R} + \frac{k_e \lambda}{R} \left[s\right]_0^{\pi R} + k_e \lambda \left[\ln(x)\right]_R^{3R} \\
&= -k_e \lambda \left[\ln(-R) - \ln(-3R)\right] + k_e \lambda \pi + k_e \lambda \left[\ln(3R) - \ln(R)\right] \\
&= k_e \lambda \left[\ln(-3R) - \ln(-R)\right] + k_e \lambda \pi + k_e \lambda \left[\ln(3R) - \ln(R)\right] \\
&= 2 k_e \lambda \ln\left(\frac{3R}{R}\right) + k_e \lambda \pi = k_e \lambda \left(\pi + 2 \ln 3\right)
\end{aligned}$$

25.4 Problem 25.60 *(In the text book)*

Two parallel plates having charges of equal magnitude but opposite sign are separated by 12.0 cm. Each plate has a surface charge density of 36.0 nC/m^2. A proton is released from rest at the positive plate. Determine

(a) the potential difference between the plates,

(b) the kinetic energy of the proton when it reaches the negative plate,

(c) the speed of the proton just before it strikes the negative plate,

(d) the acceleration of the proton, and

(e) the force on the proton.

(f) From the force, find the magnitude of the electric field and show that it is equal to the electric field found from the charge densities on the plates.

Solution

The positive plate by itself produces and electric field E_+ given by:

$$E_+ = \frac{\sigma}{2\epsilon_\circ} = \frac{36.0 \times 10^{-9}}{2 \times 8.85 \times 10^{-12}} = 2.03 \times 10^3 \; N/C$$

The negative plate by itself produces an equal electric field E_-. Since the two field are in the same direction in the region between the plates, then the total in this region E is:

$$E = E_+ + E_- = 2 \times 2.03 = 4.06 \; N/C$$

The total field E is uniform in between the plates.

(a) Take $V = 0$ at the negative plate. The potential at the positive plate is then:

$$V - 0 = -\int_0^d E dx \qquad \text{or} \qquad V = Ed = 4.06 \times 10^3 \times 0.12 = 488 \; V$$

(b) The proton is released from rest at the positive plate, so it kinetic energy there is zero. When the proton reaches the negative plate it will have kinetic energy but its potential energy is zero ($V = 0$), so conservation of energy gives:

$$\left(\frac{1}{2}mv^2 + qV\right)_+ = \left(\frac{1}{2}mv^2 + qV\right)_- \qquad \text{or} \qquad 0 + qV = \frac{1}{2}mv^2 + 0$$

and the kinetic energy of the proton at the negative plate is:

$$\frac{1}{2}mv^2 = qV = 1.602 \times 10^{-19} \times 488 = 7.81 \times 10^{-17}\ J$$

(c) Since $\frac{1}{2}mv^2 = 7.81 \times 10^{-17}\ J$, the velocity of the proton just before it strikes the negative plate is:

$$v = \sqrt{\frac{2 \times 7.81 \times 10^{-17}}{m}} = \sqrt{\frac{2 \times 7.81 \times 10^{-17}}{1.67 \times 10^{-27}}} = 306\ km/s$$

(d) The acceleration of the proton is given by:

$$v_f^2 = v_i^2 + 2a(x_f - x_i) \quad \text{then} \quad a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(3.06 \times 10^5)^2 - o}{2 \times 0.120} = 3.90 \times 10^{11}\ m/s^2$$

(e) The force on the proton is:

$$F = ma = 1.67 \times 10^{-27} \times 3.90 \times 10^{11} = 6.51 \times 10^{-16}\ N$$

(f) The electric field E can be found from the force as:

$$E = \frac{F}{q} = \frac{6.51 \times 10^{-16}}{1.602 \times 10^{-19}} = 4.06 \times 10^3\ N/C$$

25.5 Problem 25.72 *(In the text book)*

A solid sphere of radius R has a uniform charge density ρ and total charge Q. Derive an expression for its total electric potential energy. (Suggestion: imagine that the sphere is constructed by adding successive layers of concentric shells of charge $dq = (4\pi r^2 dr)\rho$and use $dU = V dq$).

Solution

An element of volume as a solid shell with radius r and thickness dr and carries a charge of dq, gives an electric potential energy of:

$$dU_e = V dq \qquad \text{with} \qquad V = \frac{k_e q}{r}$$

The charge on the shell $dq = \rho\times$the volume of the shell, or $dq = \rho(4\pi r^2 dr)$. q is the charge of the sphere of radius r that is enclosed by the shell, i.e. $q = \rho\times$the volume of the sphere, so $q = \rho(4\pi r^3/3)$. The electric potential energy of the whole sphere of radius R is:

$$\begin{aligned}
U_e &= \int_0^R dU_e \\
&= \int_0^R k_e \frac{q}{r}\, dq \\
&= \int_0^R k_e \rho \left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right) \rho \left(4\pi r^2 dr\right) \\
&= \int_0^R k_e \left(\frac{16\pi^2}{3}\right) \rho^2 r^4\, dr \\
&= k_e \left(\frac{16\pi^2}{3}\right) \rho^2 \int_0^R r^4\, dr = k_e \left(\frac{16\pi^2}{3}\right) \rho^2 \left(\frac{R^5}{5}\right) \\
&= k_e \left(\frac{16\pi^2}{15}\right) \rho^2 R^5
\end{aligned}$$

But the total charge Q on the whole sphere is:

$$Q = \rho \frac{4}{3} R^3 \qquad \text{or} \qquad Q^2 = \rho^2 \frac{16}{9} R^6$$

So, the electric potential energy becomes:

$$U_e = \frac{3}{5}\frac{k_e Q^2}{R}$$