

## Chapter 14

# Fluid Mechanics. Solutions of Selected Problems

### 14.1 Problem 14.18 (*In the text book*)

Mercury is poured into a U-tube as in Figure (14.18a). The left arm of the tube has cross-sectional area  $A_1$  of  $10.0 \text{ cm}^2$ , and the right arm has a cross-sectional area  $A_2$  of  $5.00 \text{ cm}^2$ . One hundred grams of water are then poured into the right arm as in Figure Figure (14.18b).

- (a) Determine the length of the water column in the right arm of the U-tube.
- (b) Given that the density of mercury is  $13.6 \text{ g/cm}^3$ , what distance  $h$  does the mercury rise in the left arm?

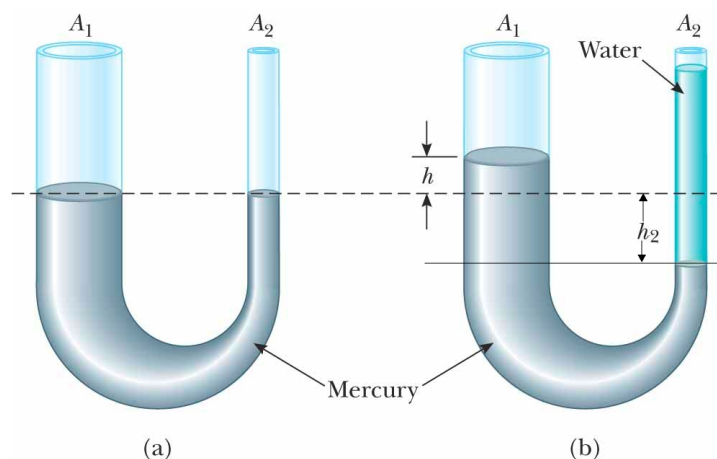


Figure 14.18:

## Solution

(a) The volume of the water in the right arm is:

$$\begin{aligned}
 V_w &= \frac{m_w}{\rho_w} \\
 &= \frac{100 \text{ (g)}}{1.00 \text{ (g/cm}^3\text{)}} \\
 &= 100 \text{ cm}^3 \\
 &= h_w A_2 \\
 h_w &= \frac{V_w}{A_2} \\
 &= \frac{100 \text{ (cm}^3\text{)}}{5 \text{ cm}^2} \\
 &= 20 \text{ cm}
 \end{aligned}$$

(b) Figure (14.18b) represents the situation after the water is added. A volume  $A_2 h_2$  of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is  $A_1 h$ . Since the total volume of mercury has not changed, then

$$A_2 h_2 = A_1 h \quad \text{or} \quad h_2 = \frac{A_1}{A_2} h \quad (14.1)$$

At the level of the mercurywater interface in the right tube, we may write the absolute pressure  $P_r$  as:

$$P_r = P_o + \rho_w g h_w$$

The pressure  $P_\ell$  at this same level in the left tube is given by:

$$P_\ell = P_o + \rho_{Hg} g (h + h_2)$$

Since  $P_\ell = P_r$  we get:

$$P_o + \rho_{Hg} g (h + h_2) = P_o + \rho_w g h_w \quad (14.2)$$

Substituting for  $h_2$  from Equation (14.1) into Equation (14.2) we get:

$$\begin{aligned}\rho_w g h_w &= \rho_{Hg} g (h + h_2) \\ &= \rho_{Hg} g \left[ h + \frac{A_1}{A_2} h \right] \\ &= \rho_{Hg} g h \left[ 1 + \frac{A_1}{A_2} \right]\end{aligned}$$

We then find  $h$  as:

$$\begin{aligned}h &= \frac{\rho_w h_w}{\rho_{Hg} (1 + A_1/A_2)} \\ &= \frac{1.00 \text{ (g/cm}^3\text{)} \times 20.0 \text{ (cm)}}{13.6 \text{ (g/cm}^3\text{)} \times (1 + 10.0/5.00)} \\ &= 0.490 \text{ cm}\end{aligned}$$

## 14.2 Problem 14.26 (*In the text book*)

The weight of a rectangular block of low-density material is  $15.0\text{ N}$ . With a thin string, the center of the horizontal bottom face of the block is tied to the bottom of a beaker partly filled with water. When 25.0% of the blocks volume is submerged, the tension in the string is  $10.0\text{ N}$ .

- Sketch a free-body diagram for the block, showing all forces acting on it.
- Find the buoyant force on the block.
- Oil of density  $800\text{ kg/m}^3$  is now steadily added to the beaker, forming a layer above the water and surrounding the block. The oil exerts forces on each of the four side walls of the block that the oil touches. What are the directions of these forces?
- What happens to the string tension as the oil is added? Explain how the oil has this effect on the string tension.
- The string breaks when its tension reaches  $60.0\text{ N}$ . At this moment, 25.0% of the blocks volume is still below the water line; what additional fraction of the blocks volume is below the top surface of the oil?
- After the string breaks, the block comes to a new equilibrium position in the beaker. It is now in contact only with the oil. What fraction of the blocks volume is submerged?

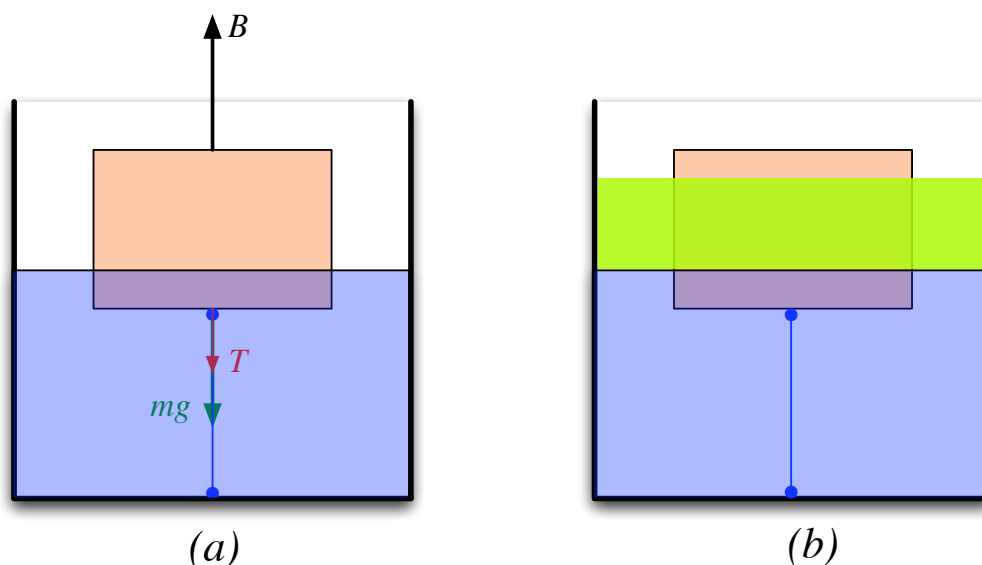


Figure 14.19:

## Solution

- (a) The free body diagram for the block showing all forces acting on it is shown in Figure (14.19a)
- (b) Since the body is at rest then the sum of all forces must be zero, i.e.

$$B - mg - T = 0 \quad \text{or} \quad B = mg + T = 15 + 10 = 25 \text{ N}$$

- (c) Since the block is not submerged, the oil will then push inward on each side of the block. This situation is shown in Figure (14.19b)
- (d) The tension in the string increases as the oil is added. This is because the oil increases the pressure on the water surface and the buoyant force on the block increases.
- (e) The situation just before the string breaks is shown in Figure (14.20a), where the forces are  $mg = 15 \text{ N}$  as before, the tension  $T = 60 \text{ N}$ , the buoyant force due to water  $B_w = 25 \text{ N}$  since the block is still only 25% under the water, and finally buoyant force due to oil  $B_{oil}$  which is unknown. The block still under equilibrium, so the sum of all

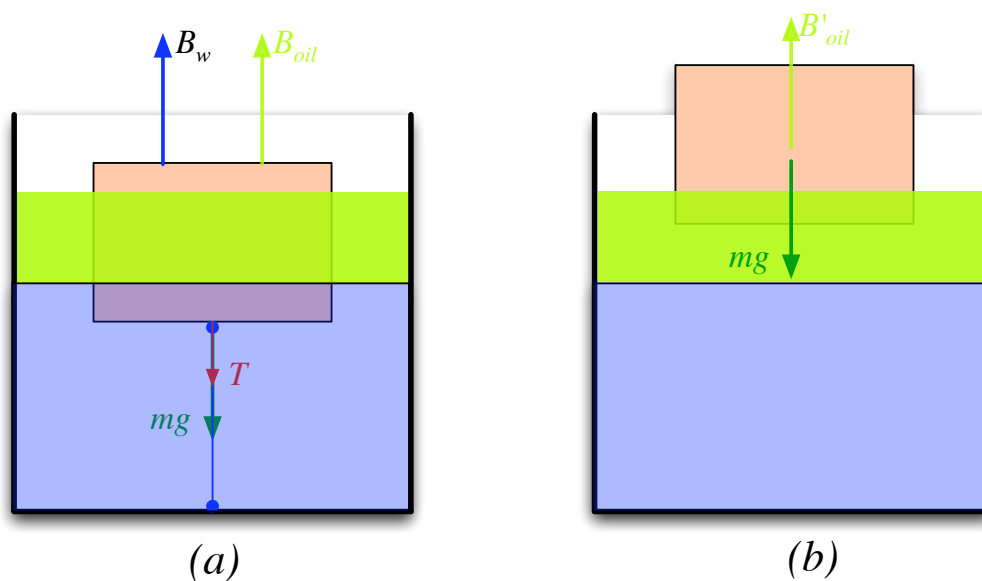


Figure 14.20:

forces shown in Figure (14.20a) is zero:

$$B_w + B_{oil} - mg - T = 0 \quad \text{or} \quad B_{oil} = mg + T - B_w = 15 + 60 - 25 = 50 \text{ N}$$

If the volume of the block is  $V_b$  then the buoyant force due to water is:

$$B_w = \rho_w \times 0.25V_bg \quad \text{or} \quad V_b = \frac{B_w}{0.25\rho_w g} = \frac{25}{0.25 \times 1.00 \times 9.8} = 1.02 \times 10^{-2} \text{ m}^3$$

Let  $f_{oil}$  is the fraction of the block covered by the oil, then the buoyant force due to oil is:

$$B_{oil} = \rho_{oil}f_{oil}V_bg \quad \text{or} \quad f_{oil} = \frac{B_{oil}}{\rho_{oil}V_bg} = \frac{50}{800 \times 1.02 \times 10^{-2} \times 9.8} = 0.625 = 62.5\%$$

- (f) When the string breaks the block will be under equilibrium above the water surface and only partially submerged under the the oil. Let the fraction submerged now be  $f'_{oil}$ . The forces acting in this case as shown in Figure (14.20)b are the  $mg = 15 \text{ N}$  and the new buoyant force due to the oil  $B'_{oil} = \rho_{oil}f'_{oil}V_bg$ , we then have:

$$mg = B'_{oil} = \rho_{oil}f'_{oil}V_bg \quad \text{or} \quad f'_{oil} = \frac{mg}{\rho_{oil}V_bg} = \frac{15}{800 \times 1.02 \times 10^{-2} \times 9.8} = 0.187 = 18.7\%$$

### 14.3 Problem 14.40 (*In the text book*)

A village maintains a large tank with an open top, containing water for emergencies. The water can drain from the tank through a hose of diameter  $6.60\text{ cm}$ . The hose ends with a nozzle of diameter  $2.20\text{ cm}$ . A rubber stopper is inserted into the nozzle. The water level in the tank is kept  $7.50\text{ m}$  above the nozzle.

- Calculate the friction force exerted on the stopper by the nozzle.
- The stopper is removed. What mass of water flows from the nozzle in  $2.00\text{ h}$ ?
- Calculate the gauge pressure of the flowing water in the hose just behind the nozzle.

### Solution

The tank and the hose set up is shown in Figure (14.40).

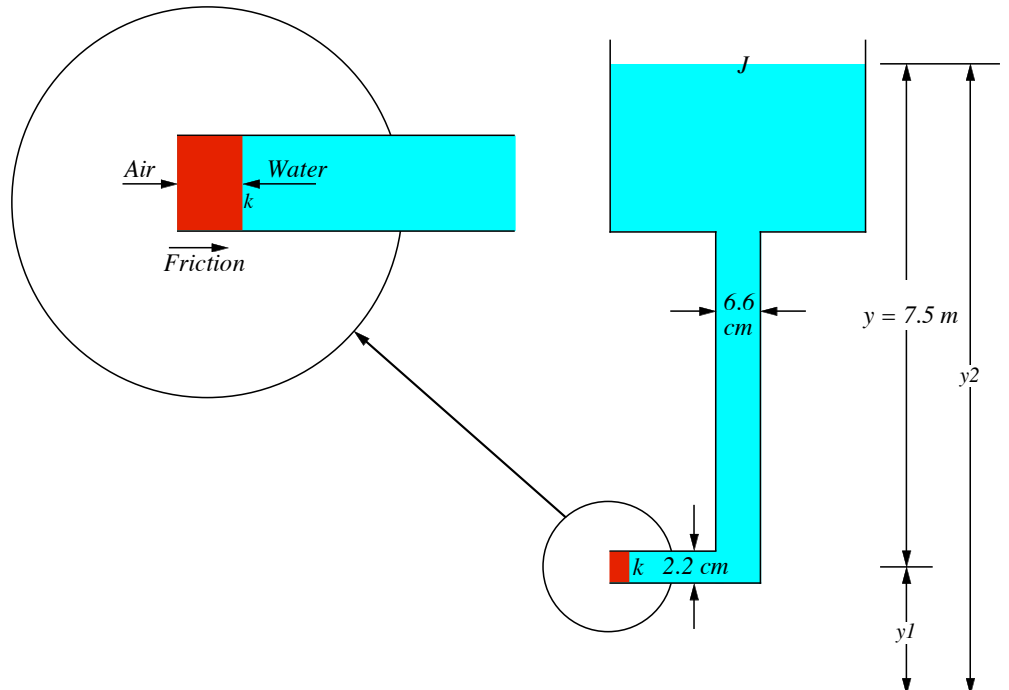


Figure 14.40:

- (a) The water pressure  $P_k$  to the right of the rubber stopper at point  $k$  is given by:

$$\begin{aligned} P_k &= P_o + \rho g y \\ P_k - P_o &= \rho g y \end{aligned} \quad (14.3)$$

The stopper is at rest, so the forces acting on it must sum up to zero. The forces on the stopper are the  $F_w = P_k A$  produced by the water pressure on the right side of the stopper and directed to the left,  $F_a = P_o A$  produced by the air pressure on the left side of the stopper and directed to the right, and in addition there is the frictional force  $F_f$  acting to the right.  $F_a$  and  $F_f$  both act in the same direction to balance  $F_w$  and keep the stopper in place, we then have:

$$\begin{aligned} F_f &= F_w - F_a \\ &= P_k A - P_o A \\ &= (P_k - P_o) A \end{aligned} \quad (14.4)$$

where  $A$  is the cross sectional area of the hose at the stopper. Using Equation (14.3), Equation (14.4) becomes:

$$F_f = (P_k - P_o) A = \rho g y (\pi r^2)$$

Using numerical values,  $h = 7.5 \text{ m}$ ,  $\rho = 1000 \text{ kg/m}^3$ , and  $r = 1.10 \text{ cm} = 1.10 \times 10^{-2} \text{ m}$ , we get

$$F_f = 1000 \text{ (kg/m}^3\text{)} \times 9.8 \text{ (m/s}^2\text{)} \times 7.5 \text{ (m)} \times \pi \times (1.10 \times 10^{-2} \text{ (m)})^2 = 7.35 \times 10^4 \text{ N}$$

- (b) After removing the nozzle, we apply Bernoulli's equation taking point "1" at the nozzle and point "2" at the top of the tank, and get:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

Since the tank holding the water is large then its diameter must be much larger than the diameter of the nozzle. Then velocity of the water at the top of the tank is very small compared to the velocity at the nozzle. So,  $P_1 = P_o$ ,  $v_2 = 0$ ,  $P_2 = P_o$ , and  $y_2 - y_1 = 7.5 \text{ m}$ , and Bernoulli's equation then becomes:

$$\begin{aligned} P_o + \frac{1}{2} \rho v_1^2 + \rho g y_1 &= P_o + 0 + \rho g y_2 \\ v_1 &= \sqrt{2g(y_2 - y_1)} \\ &= \sqrt{2 \times 9.8 \times 7.5} \\ &= 12.1 \text{ m/s} \end{aligned}$$



If the volume of the water flowing out of the nozzle is  $V$  and the area of the nozzle is  $A_n$  and its radius is  $r_n$ , the mass of the water that flows from the nozzle in two hours is:

$$\begin{aligned}
 M &= \rho V \\
 &= \rho A_n \times v_1 \times t \\
 &= \rho \pi r_n^2 \times v_1 \times t \\
 &= 1000 \text{ (kg/m}^3\text{)} \times \pi \times (0.011 \text{ m})^2 \times 12.1 \text{ (m/s)} \times 2 \text{ (hr)} \times 3600 \text{ (sec/hr)} \\
 &= 3.31 \times 10^4 \text{ kg}
 \end{aligned}$$

- (c) Applying the continuity and Bernoulli's equations to point "1" just outside the nozzle and point "2" at the bottom of the wide hose , we get:

$$\begin{aligned}
 A_1 v_1 &= A_2 v_2 \\
 v_2 &= \frac{A_1 v_1}{A_2} \\
 &= \frac{\pi r_1^2 v_1}{\pi r_2^2} \\
 &= \left( \frac{r_1}{r_2} \right)^2 v_1
 \end{aligned} \tag{14.5}$$

and

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

Since,  $P_1 = P_2$ ,  $y_1 = y_2$ ,  $r_1 = 0.011 \text{ m}$ ,  $r_2 = 0.033 \text{ m}$  and using Equation (14.5) we get;

$$\begin{aligned}
 P_2 - P_1 &= \frac{1}{2} \rho (v_1^2 - v_2^2) \\
 &= \frac{1}{2} \rho \left[ v_1^2 - \left( \frac{r_1}{r_2} \right)^4 v_1^2 \right] \\
 &= \frac{1}{2} \rho v_1^2 \left[ 1 - \left( \frac{r_1}{r_2} \right)^4 \right] \\
 &= \frac{1}{2} \times 1000 \times (12.1)^2 \times \left[ 1 - \left( \frac{0.011}{0.033} \right)^4 \right] \\
 &= 7.23 \times 10^4 \text{ Pa}
 \end{aligned}$$

### 14.4 Problem 14.44 (*In the text book*)

A legendary Dutch boy saved Holland by plugging a hole in a dike with his finger, 1.20 cm in diameter. If the hole was 2.00 m below the surface of the North Sea (density 1 030 kg/m<sup>3</sup>),

1. what was the force on his finger?
2. If he pulled his finger out of the hole, how long would it take the released water to fill 1 acre of land to a depth of 1 foot, assuming the hole remained constant in size? (A typical U.S. family of four uses 1 acre-foot of water, 1 234 m<sup>3</sup>, in 1 year.)

### Solution

- (a) With the hole clogged, there is no flow of water. Applying Bernoulli's equation at the sea surface as point "2" and at the clogged hole as point "1" and taking the we get:

$$\begin{aligned}
 P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \\
 P_1 - 0 + \rho g y_1 &= P_o + 0 + \rho g y_2 \\
 P_1 - P_o &= \rho g (y_2 - y_1) \\
 &= 1030 \times 9.8 \times 2 \\
 &= 2.02 \times 10^4 \text{ Pa}
 \end{aligned} \tag{14.6}$$

The pressure given in Equation (14.6) is pressure acting on the boy's finger, since on his side the pressure is atmospheric and on the sea side it is  $P_1$ . The net force acting on the boy's hand is:

$$F = (P_1 - P_o)A = 2.02 \times 10^4 \times \pi \times (6 \times 10^{-3})^2 = 2.28 \text{ N}$$

- (b) With the hole now unclogged, we apply Bernoulli's equation at point "1" at a location at the hole just on the dry side of the dike and at point "2" at the sea level. The pressure at the sea surface and at the dry side of the dike is one atmosphere i.e.  $P_o$ , we then have:

$$\begin{aligned}
P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \\
P_o + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= P_o + 0 + \rho g y_2 \\
v_1 &= \sqrt{2g(y_2 - y_1)} \\
&= \sqrt{4g} \\
&= 6.26 \text{ m/s}
\end{aligned}$$

where  $v_1$  is the velocity of the water spilling out of the hole. Let the volume rate of flow be  $R_V$ , then:

$$R_V = A_1 v_1 = \pi r_1^2 v_1 = \pi (6 \times 10^{-3})^2 \times 6.26 = 7.08 \times 10^{-4} \text{ m}^3/\text{s}$$

A acre is an imperial unit of area, such that 1 acre = 4047  $\text{m}^2$  and 1 foot = 0.3048  $\text{m}$ , so a volume of one acre  $\times$  foot =  $1.234 \times 10^3 \text{ m}^3$ . We want to find the time “ $T$ ” it takes to fill a volume of  $1.234 \times 10^3 \text{ m}^3$  at the rate of  $7.08 \times 10^{-4} \text{ m}^3/\text{s}$ :

$$T = \frac{1.234 \times 10^3}{7.08 \times 10^{-4}} = 1.74 \times 10^6 \text{ s} = \frac{1.74 \times 10^6}{60 \times 60 \times 24} = 20.1 \text{ days}$$

### 14.5 Problem 14.51 (*In the text book*)

A siphon is used to drain water from a tank, as illustrated in Figure (14.51). The siphon has a uniform diameter. Assume steady flow without friction.

- If the distance  $h = 1.00\text{ m}$ , find the speed of outflow at the end of the siphon.
- What If? What is the limitation on the height of the top of the siphon above the water surface? (For the flow of the liquid to be continuous, the pressure must not drop below the vapor pressure of the liquid.)

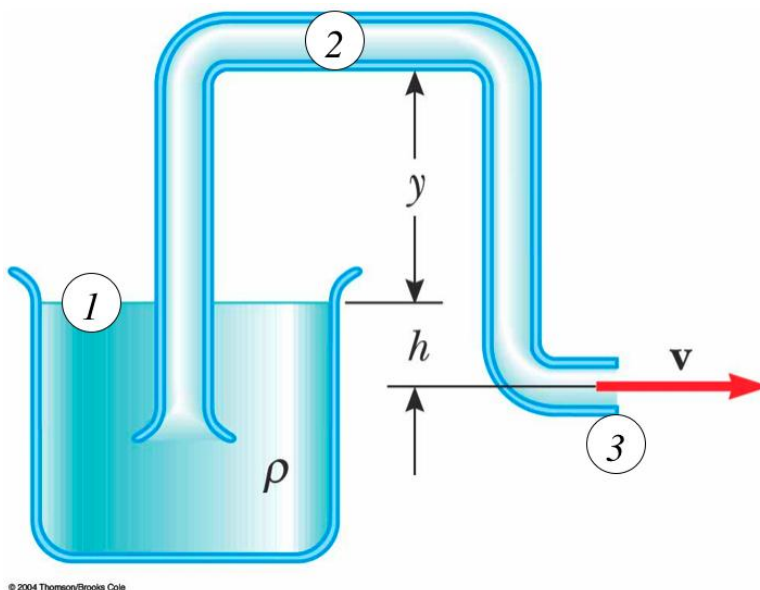


Figure 14.51:

### Solution

- We take the center of exit tube at point “3” as our reference line. Applying Bernoulli’s equation at points “1” and “3” we get (notice that  $v_1 = 0$ ,  $P_1 = P_3 = P_o$  and  $h = 1.00\text{ m}$ ):

$$\begin{aligned}
 P_1 + \frac{1}{2}\rho v_1^2 + \rho gh &= P_3 + \frac{1}{2}\rho v_3^2 + 0 \\
 P_o + 0 + \rho gh &= P_o + \frac{1}{2}\rho v_3^2 + 0 \\
 v_3 &= \sqrt{2gh} \\
 &= \sqrt{2 \times 9.8} \\
 &= 4.43 \text{ m/s}
 \end{aligned}$$

(b) Now applying Bernoulli's equation at "2" and "3", noticing that  $v_2 = v_3$ , we get:

$$\begin{aligned}
 P_2 + \frac{1}{2}\rho v_2^2 + \rho gy &= P_3 + \frac{1}{2}\rho v_3^2 + 0 \\
 P_2 &= P_o - \rho gy
 \end{aligned}$$

for the flow to continue  $P_2 \geq 0$  or  $\rho gy \leq P_o$  or

$$y \leq \frac{P_o}{\rho g} = \frac{1.013 \times 10^5 \text{ (Pa)}}{9.8 \text{ (m/s}^2\text{)} \times 1000 \text{ (kg/m}^3\text{)}} = 10.3 \text{ m}$$

### 14.6 Problem 14.54 (*In the text book*)

Figure (14.54) shows a water tank with a valve at the bottom. If this valve is opened, what is the maximum height attained by the water stream coming out of the right side of the tank? Assume that  $h = 10.0\text{m}$ ,  $L = 2.00\text{m}$ , and  $\theta = 30.0^\circ$ , and that the cross-sectional area at  $A$  is very large compared with that at  $B$ .

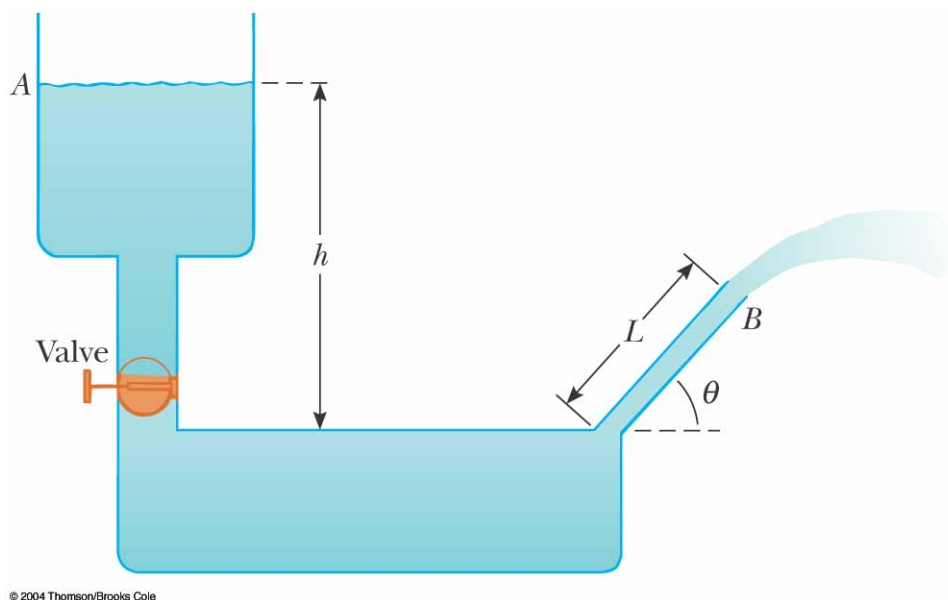


Figure 14.54:

### Solution

Let us take the level at point  $B$  in Figure (14.54) to be where  $y = 0$ . Since the area at level  $A$  is much larger than the area at  $B$ , then the velocity of the water at  $A$ ,  $v_A$ , is essentially zero. In addition  $P_A = P_B = P_o = \text{atmospheric pressure}$ .

Applying Bernoulli's equation at  $A$  and  $B$  when the valve is open gives:

$$\begin{aligned}
P_A + \frac{1}{2}\rho v_A^2 + \rho g(h - L \sin \theta) &= P_B + \frac{1}{2}\rho v_B^2 + \rho g \times 0 \\
P_o + 0 + \rho g(h - L \sin \theta) &= P_o + \frac{1}{2}\rho v_B^2 + 0 \\
v_B &= \sqrt{2g(h - L \sin \theta)} \\
&= \sqrt{2 \times 9.8 \times (10.0 - 2.00 \times \sin 30.0)} \\
&= 13.3 \text{ m/s}
\end{aligned}$$

Now the problem reduces to a projectile motion with initial velocity in the vertical direction,  $v_{iy}$  of:

$$v_{iy} = v_B \sin 30.0 = 6.65 \text{ m/s}$$

This velocity is related to the final velocity  $v_{fy}$  and the height of the water above  $B$ ,  $\Delta y$  by:

$$v_{fy}^2 = v_{iy}^2 + 2a\Delta y = v_{fy}^2 - 2g\Delta y$$

$\Delta y$  becomes maximum when  $v_{fy} = 0$ , so:

$$(\Delta y)_{max} = \frac{v_{iy}^2}{2g} = \frac{(6.65)^2}{2 \times 9.8} = 2.26 \text{ m above the level at } B$$