Chapter 28

Direct Current Circuits. Solutions of Home Work Problems

28.1 Problem 28.14 *(In the text book)*

A 6.00-V battery supplies current to the circuit shown in Figure (28.14). When the double-throw switch S is open, as shown in the figure, the current in the battery is 1.00 mA. When the switch is closed in position 1, the current in the battery is 1.20 mA. When the switch is closed in position 2, the current in the battery is 2.00 mA. Find the resistances R_1, R_2, and R_3.

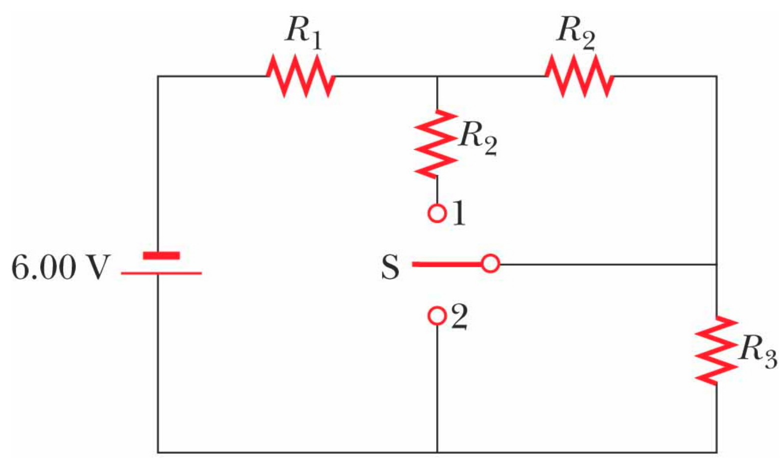

Figure 28.14:

Solution

When S is open R_1, R_2, R_3 are in series with the battery, so:

$$R_1 + R_2 + R_3 = \frac{6}{10^{-3}} = 6\ k\Omega \tag{28.1}$$

When S is closed in position 1, the parallel combination of the two R_2 resitors is in series with R_1, R_2 and the battery, Thus:

$$R_1 + \frac{R_2 R_2}{R_2 + R_2} + R_3 = R_1 + \frac{1}{2}R_2 + R_3 = \frac{6}{1.2 \times 10^{-3}} = 5\ k\Omega \tag{28.2}$$

When S is closed in position 1, R_1 and R_2 are in series with the battery and R_3 is shorted. Thus:

$$R_1 + R_2 = \frac{6}{2 \times 10^{-3}} = 3\ k\Omega \tag{28.3}$$

Substituting for $R_1 + R_2$ from Equation (28.3) in Equation (28.1) we get:

$$3k\Omega + R_3 = 6k\Omega \qquad \text{and} \qquad R_3 = 3k\Omega$$

Now, subtracting Equation (28.2) from Equation (28.1) we get:

$$-\frac{1}{2}R_2 + R_3 = 5k\Omega - 3k\Omega \qquad \text{and} \qquad R_2 = 2k\Omega$$

From Equation (28.3) we get $R_1 = 1k\Omega$

28.2 Problem 28.24 *(In the text book)*

Using Kirchhoffs rules,

(a) find the current in each resistor in Figure (28.24).

(b) Find the potential difference between points c and f. Which point is at the higher potential?

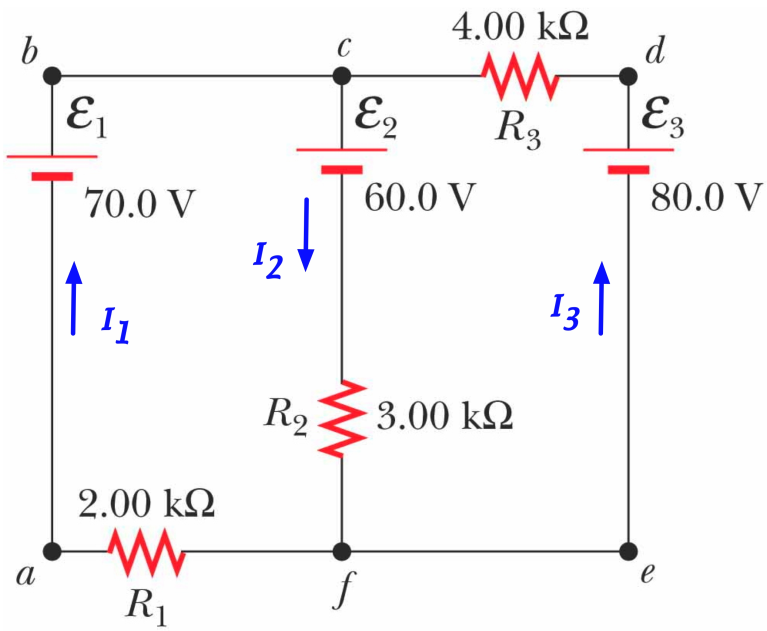

Figure 28.24:

Solution

We name the current I_1, I_2, and I_3 as shown in the Figure (28.14). Now going around loop $abcf$ clockwise and applying Kirchhoff's voltage rule we get:

$$70.0 - 60.0 - I_2(3.00k\Omega) - I_1(2.00k\Omega) = 0 \tag{28.4}$$

Now, applying Kirchhoff's voltage rule to loop $defc$ also clockwise, we get:

$$-80.0 + I_2(3.00k\Omega) + 60.0 + I_3(4.00k\Omega) = 0 \tag{28.5}$$

Apply the current rule at c we get:

$$I_2 = I_1 + I_3 \tag{28.6}$$

(a) Substituting for I_2 from Equation (28.6) in Equation (28.4) ans solving the resulting three equations we get:

$$\begin{aligned} I_1 &= 0.385\, mA \\ I_2 &= 2.69\, mA \\ I_3 &= 3.08\, mA \end{aligned}$$

(b) The potential difference between point c and point f staring from c to f is:

$$\Delta V_{cf} = V_c - V_f = 60.0 + I_3(3.00\ k\Omega) = 60 + 3.08\ mA \times 3.00\ k\Omega = 69.2\ V$$

Point c is at higher potential than f.

28.3 Problem 28.26 *(In the text book)*

In the circuit of Figure (28.26), determine the current in each resistor and the voltage across the 200-Ω resistor.

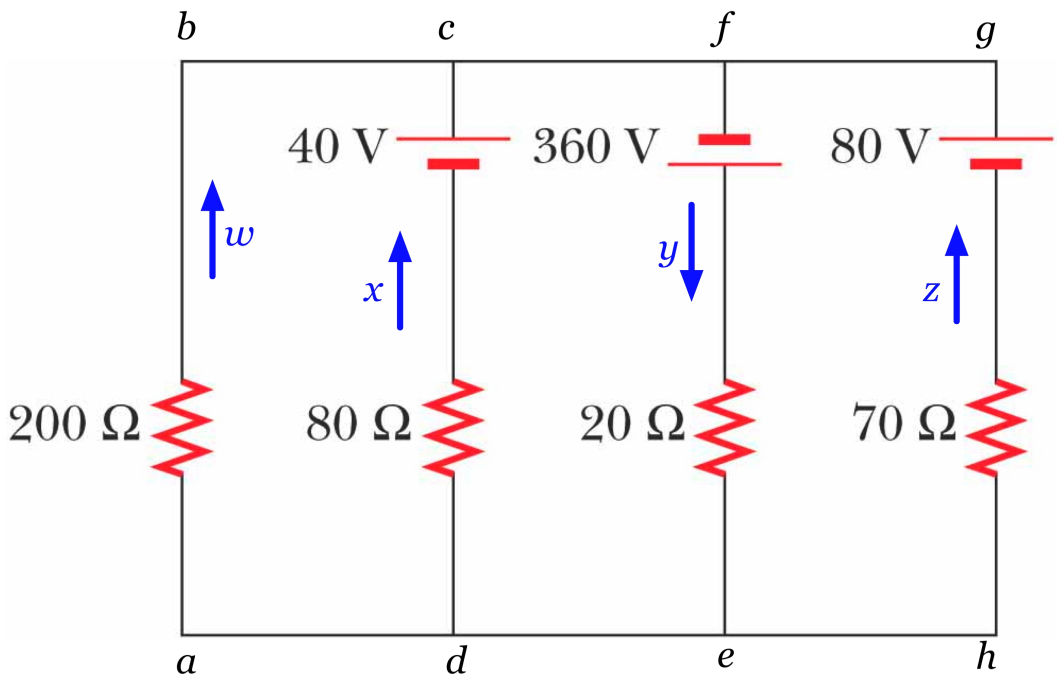

Figure 28.26:

Solution

We have four unknowns, so we need for equations. Naming the currents as shown in Figure (28.26), we find from applying the current rule at d and the voltage rule to loops $abcd$, $dcfe$ and $efgh$ clockwise we get:

$$
\begin{aligned}
w + x + z &= y \\
-200w - 40.0 + 80.0x &= 0 \\
-80.0x + 40.0 + 360 - 20.0y &= 0 \\
+20.0y - 360 - 80.0 + 70.0z &= 0
\end{aligned}
$$

eliminating y by substitution we get

$$\begin{aligned}
2.5w + 0.500 &= x \\
400 - 100x - 20.0w - 20.0z &= 0 \\
440 - 20.0w - 20.0x - 90.0z &= 0
\end{aligned}$$

Eliminating x we get:

$$\begin{aligned}
350 - 270w - 20.0z &= 0 \\
440 - 70.0w - 90.0z &= 0
\end{aligned}$$

Eliminating z we get:

$$z = 17.5 - 13.5w \qquad \text{and} \qquad 430 - 70.0w - 1575 + 1215w = 0$$

and w becomes:

$$w = \frac{1145}{1145} = 1.00\ A$$

and

$$z = 4.00\ A$$

We can find x and y as:

$$x = 3.00\ A$$

and

$$y = 8.00\ A$$

The potential difference across the 200 Ω resistor is then:

$$\Delta V = IR = wR = 1.00\ A \times 200\ \Omega = 200\ V$$

28.4 Problem 28.36 *(In the text book)*

In the circuit of Figure (28.36), the switch S has been open for a long time. It is then suddenly closed. Determine the time constant

(a) before the switch is closed and

(b) after the switch is closed.

(c) Let the switch be closed at $t = 0$. Determine the current in the switch as a function of time.

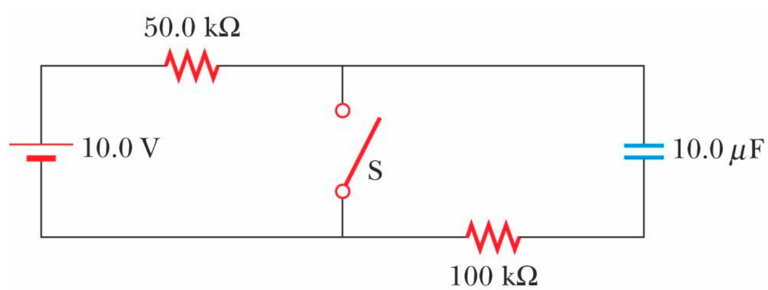

Figure 28.36:

Solution

(a) While the switch is open, the battery charges the capacitor. The current used to charge the battery is determined by the two resistors in series with the battery. So, the time constant τ_{open} is:

$$\tau_{open} = RC = (50.0\ k\Omega + 100\ k\Omega) \times 10\mu F = 1.5\ s$$

(b) When the switch is closed, the capacitor discharges through $100\ k\Omega$ resistor, so the time constant τ_{closed} is:

$$\tau_{closed} = 100\ k\Omega \times 10\ \mu F = 1.0\ s$$

(c) The current through the closed switch will produced by the battery I_b and while the capacitor is discharging a current I_c, the total current trough the switch is then:

$$I = I_b + I_c = \frac{\Delta V_b}{R} + I_\circ e^{-t/\tau_{closed}} = \frac{10}{50.0 \times 10^3} + I_\circ e^{-t/1.00} = 200\ \mu A + I_\circ e^{-t/1.00}$$

To find $I_\circ$ we notice that once the capacitor is fully charged, there will be no current flowing in the circuit an no potential differences on the two resistors. This means that potential difference across the capacitor is the same as that across the battery i.e. 10 V. Once the switch is closed the initial current due to the discharge of the capacitor is determined by the initial potential difference across the capacitor and the resistor through which the discharge takes place. So,

$$I_\circ = \frac{10.0\ V}{100\ k\Omega} = 100\ \mu A$$

and the total current becomes:

$$I = 200 + 100e^{-t/1.00}\ \mu A$$

A long time after the switch is closed the capacitor is fully discharged and the current in the switch will be mainly from the battery.

28.5 Problem 28.42 *(In the text book)*

A typical galvanometer, which requires a current of 1.50 mA for full-scale deflection and has a resistance of 75.0 Ω, may be used to measure currents of much greater values. To enable an operator to measure large currents without damage to the galvanometer, a relatively small shunt resistor is wired in parallel with the galvanometer, as suggested in Figure (28.27). Most of the current then goes through the shunt resistor. Calculate the value of the shunt resistor that allows the galvanometer to be used to measure a current of 1.00 A at full-scale deflection. (Suggestion: use Kirchhoffs rules.)

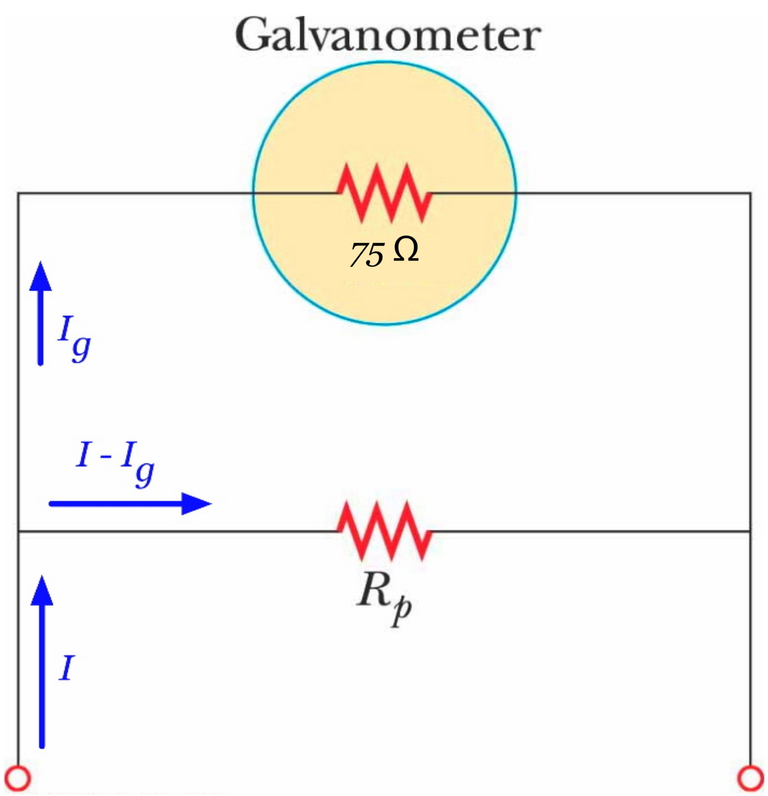

Figure 28.27:

Solution

Applying Kirchhoff's loop rule we get:

$$-I_g \times 75\ \Omega + (I - I_g)R_p = 0$$

The Galvanometer requires 1.5 mA for full deflection, to use it to give full deflection for 1 A we then set $I_g = 1.5\ mA$ and $I = 1\ A$ in the above equation, and R_p becomes:

$$R_p = \frac{75 \times I_g}{I - I_g} = \frac{75 \times 1.5 \times 10^{-3}}{1 - 1.5 \times 10^{-3}} = 0.113\ \Omega$$

28.6 Problem 28.50 *(In the text book)*

Aluminum wiring has sometimes been used instead of copper for economy. According to the National Electrical Code, the maximum allowable current for 12-gauge copper wire with rubber insulation is 20 A. What should be the maximum allowable current in a 12-gauge aluminum wire if the power per unit length delivered to the resistance in the aluminum wire is the same as that delivered in the copper wire?

Solution

Since the power is the same in both wires, then:

$$I_{Al}^2 R_{Al} = I_{Cu}^2 R_{Cu} \qquad \text{or} \qquad I_{Al} = \sqrt{\frac{R_{Cu}}{R_{Al}}} I_{Cu} = \sqrt{\frac{\rho_{Cu}}{\rho_{Al}}} I_{Cu} = \sqrt{\frac{1.70}{2.82}} \times 20.0 = 15.5\ A$$

28.7 Problem 28.68 *(In the text book)*

Switch S has been closed for a long time, and the electric circuit shown in Figure (28.68) carries a constant current. Take $C_1 = 3.00\ \mu F$, $C_2 = 6.00\ \mu F$, $R_1 = 4.00\ k\Omega$, and $R_2 = 7.00\ k\Omega$. The power delivered to R_2 is $2.40\ W$.

(a) Find the charge on C_1.

(b) Now the switch is opened. After many milliseconds, by how much has the charge on C_2 changed?

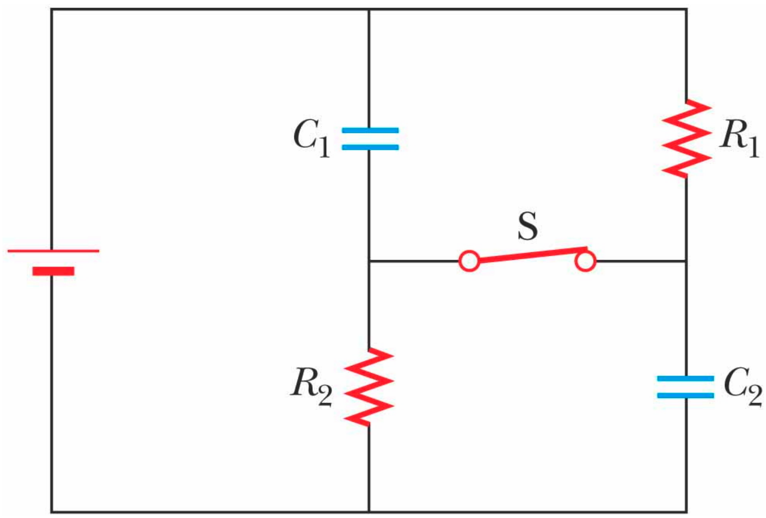

Figure 28.68:

Solution

(a) With the switch closed, current exists in the circuit as shown in the top part of Figure (28.68). The capacitors carry no current. For R_2 we have

$$P = I^2 R_2 \qquad \text{then} \qquad I = \sqrt{\frac{P}{R_2}} = \sqrt{\frac{2.40}{7000}} = 18.5\ mA$$

The potential difference across R_1 and C_1 is

$$\Delta V_1 = IR_1 = 1.85 \times 10^{-2} \times 4000 = 74.1\ V$$

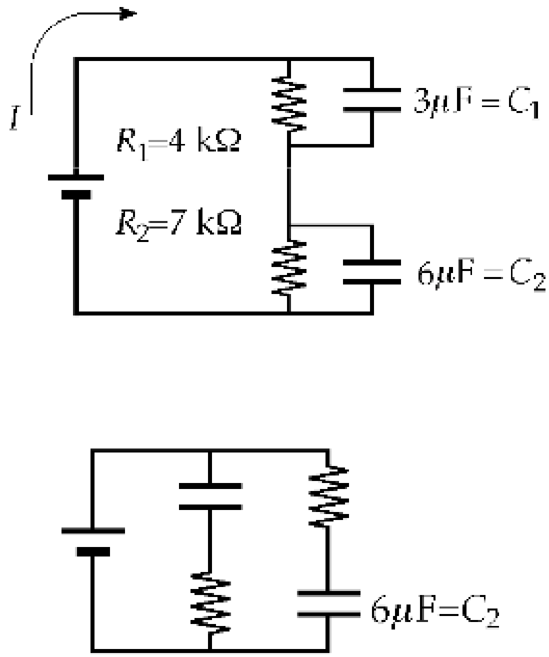

Figure 28.69:

The charge on C_1 is

$$Q = C_1 \Delta V = 3.00 \times 10^{-6} \times 74.1 = 222\ \mu C$$

The potential difference across R_2 and C_2 is:

$$\Delta V_2 = IR_2 = 1.85 \times 10^{-2} \times 7000 = 130\ V$$

The charge on C_2 is:

$$Q = C_2 \Delta V = 6.00 \times 10^{-6} \times 130 = 778\ \mu C$$

The battery emf is:

$$\Delta V = \Delta V_1 + \Delta V_2 = 74.1 + 130 = 204\ V$$

or

$$\Delta V = IR_{eq} = I(R_1 + R_2) = 1.85 \times 10^{-2} \times (4000 + 7000) = 204\ V$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge C_2 is:

$$Q = C_2 \Delta V = 6.00 \times 10^{-6} \times 204 = 1222\ \mu F$$

and the change is $1222 - 778 = 444\ \mu F$