## Chapter 16

## Wave Motion. Solutions of Home Work Problems

### 16.1 Problem 16.17 (In the text book)

A transverse wave on a string is described by the wave function

$$
y=(0.120 m) \sin \left[\left(\frac{\pi x}{8}\right)+4 \pi t\right]
$$

(a) Determine the transverse speed and acceleration at $t=0.200 s$ for the point on the string located at $x=1.60 \mathrm{~m}$.
(b) What are the wavelength, period, and speed of propagation of this wave?

## Solution

(a) The transverse velocity of the wave is given by:

$$
\begin{aligned}
v(x, t) & =\frac{\partial y}{\partial t} \\
& =\frac{\partial}{\partial t}\left\{(0.120 m) \sin \left[\left(\frac{\pi x}{8}\right)+4 \pi t\right]\right\} \\
& =(0.120 \times 4 \times \pi) \cos \left[\left(\frac{\pi x}{8}\right)+4 \pi t\right]=(0.480 \times \pi) \cos \left[\left(\frac{\pi x}{8}\right)+4 \pi t\right] \\
v(1.60 m, 0.200 s,) & =0.480 \times \pi \times \cos \left[\left(\frac{1.60 \times \pi}{8}\right)+4 \times 0.200 \times \pi\right] \\
& =0.480 \times \pi \times \cos [0.2 \times \pi+0.800 \times \pi]=0.480 \times \pi \times \cos (\pi) \\
& =-1.5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

The transverse acceleration is given by:

$$
\begin{aligned}
a(x, t) & =\frac{\partial^{2} y}{\partial t^{2}}=\frac{\partial v}{\partial t} \\
& =\frac{\partial}{\partial t}\left\{(0.480 \times \pi) \cos \left[\left(\frac{\pi x}{8}\right)+4 \pi t\right]\right\} \\
& =-\left(0.480 \times 4 \times \pi^{2}\right) \sin \left[\left(\frac{\pi x}{8}\right)+4 \pi t\right] \\
a(1.60 \mathrm{~m}, 0.200 \mathrm{~s}) & =-1.92 \times \pi^{2} \times \sin \left[\left(\frac{1.60 \times \pi}{8}\right)+4 \times 0.200 \times \pi\right] \\
& =-1.92 \times \pi^{2} \times \sin (\pi)=0
\end{aligned}
$$

(b) The general function for a wave is:

$$
y=A \sin (k x+\omega t)
$$

comparing the general wave function with the given wave function we get:

$$
k=\frac{\pi}{8}=\frac{2 \pi}{\lambda} \quad \text { or } \quad \lambda=16.0 \mathrm{~m}
$$

and

$$
\omega=4 \pi=\frac{2 \pi}{T} \quad \text { or } \quad T=0.500 s
$$

The wave speed is then given by:

$$
v=\frac{\lambda}{T}=\frac{16.0}{0.500}=32.0 \mathrm{~m} / \mathrm{s}
$$

### 16.2 Problem 16.24 (In the text book)

A transverse traveling wave on a taut wire has an amplitude of 0.200 mm and a frequency of 500 Hz . It travels with a speed of $196 \mathrm{~m} / \mathrm{s}$.
(a) Write an equation in SI units of the form $y=A \sin (k x-\omega t)$ for this wave.
(b) The mass per unit length of this wire is $4.10 \mathrm{~g} / \mathrm{m}$. Find the tension in the wire.

## Solution

(a) We need to find $\omega$ and $k$ :

$$
\omega=2 \pi f=2 \times \pi \times 500=3142 \mathrm{rad} / \mathrm{s}
$$

and

$$
k=\frac{\omega}{v}=\frac{3142}{196}=16.0 \mathrm{rad} / \mathrm{m}
$$

The wave equation then becomes:

$$
y=\left(2.00 \times 10^{-4} m\right) \sin (16.0 x-3142 t)
$$

(b) The wave velocity is given by:

$$
v=\sqrt{\frac{T}{\rho}}
$$

where $T$ is the tension in the wire and $\rho$ is its linear density, since $v=196 \mathrm{~m} / \mathrm{s}$ and $\rho=4.10 \mathrm{~g} / \mathrm{m}=4.10 \times 10^{-3} \mathrm{~kg} / \mathrm{m}$, then $T$ is:

$$
T=\rho v^{2}=4.10 \times 10^{-3}(\mathrm{~kg} / \mathrm{m}) \times(196 \mathrm{~m} / \mathrm{s})^{2}=158 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}=158 \mathrm{~N}
$$

### 16.3 Problem 16.38 (In the text book)

It is found that a $6.00-m$ segment of a long string contains four complete waves and has a mass of 180 g . The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley distance of 15.0 cm . (The "peak-to-valley" distance is the vertical distance from the farthest positive position to the farthest negative position.)
(a) Write the function that describes this wave traveling in the positive $x$ direction.
(b) Determine the power being supplied to the string.

## Solution

(a) From the given information we can find the wave length $\lambda$, the linear density $\rho$ and the amplitude $A$ and from this and the given frequency we can find $k$ and $\omega$ and be able to write the wave function.

$$
\mu=\frac{180 \mathrm{~g}}{6.00 \mathrm{~m}}=30.0 \mathrm{~g} / \mathrm{m}=3.00 \times 10^{-2} \mathrm{~kg} / \mathrm{m}
$$

and

$$
\lambda=\frac{6.00}{4.00}=1.50 \mathrm{~m}
$$

The amplitude $A$ is:

$$
A=\frac{15.0}{2}=7.5 \mathrm{~cm}=7.5 \times 10^{-2} \mathrm{~m}
$$

The angular frequency $\omega$ and the wave number $k$ are;

$$
\omega=2 \pi f=2 \times \pi \times 50.0=314 \mathrm{rad} / \mathrm{s} \quad \text { and } \quad k=\frac{2 \pi}{\lambda}=\frac{2 \pi}{1.5}=4.19 \mathrm{rad} / \mathrm{m}
$$

The wave function is then:

$$
y=7.5 \times 10^{-2} \sin (41.9 x-314 t)
$$

(b) The wave velocity is:

$$
v=\frac{\omega}{k}=\frac{314}{4.19}=74.9 \mathrm{~m} / \mathrm{s}
$$

and the power supplied to the string is:
$P=\frac{1}{2} \mu \omega^{2} A^{2} v=\frac{1}{2} \times\left(3.00 \times 10^{-2} \mathrm{~kg} / \mathrm{m}\right) \times(314 \mathrm{rad} / \mathrm{s})^{2} \times\left(7.5 \times 10^{-2} \mathrm{~m}\right) \times(74.9 \mathrm{~m} / \mathrm{s})=623 \mathrm{~W}$

### 16.4 Problem 16.46 (In the text book)

(a) Show that the function $y(x, t)=x^{2}+v^{2} t^{2}$ is a solution to the wave equation.
(b) Show that the function in part (a) can be written as $f(x+v t)+g(x-v t)$, and determine the functional forms for $f$ and $g$.
(c) What If? Repeat parts (a) and (b) for the function $y(x, t)=\sin (x) \cos (v t)$.

## Solution

(a) The wave equation is:

$$
\frac{\partial^{2} y}{\partial x^{2}}=\frac{1}{v^{2}} \frac{\partial^{2} y}{\partial t^{2}}
$$

The given function is a solution to the wave equation if the function satisfies the equation, i.e. if the second time derivative of the function divided by the square of the wave sped equals the second $x$ derivative of the function. Taking the derivatives of the function we get:

$$
\frac{\partial y}{\partial t}=\frac{\partial}{\partial t}\left\{x^{2}+v^{2} t^{2}\right\}=2 t v^{2} \quad \text { and } \quad \frac{\partial^{2} y}{\partial t^{2}}=\frac{\partial}{\partial t}\left\{2 t v^{2}\right\}=2 v^{2}
$$

Similarly:

$$
\frac{\partial y}{\partial x}=\frac{\partial}{\partial x}\left\{x^{2}+v^{2} t^{2}\right\}=2 x \quad \text { and } \quad \frac{\partial^{2} y}{\partial x^{2}}=\frac{\partial}{\partial x}\{2 x\}=2
$$

Substituting with the derivatives in the wave equation we get:

$$
2=\frac{1}{v^{2}} \times 2 v^{2}
$$

So, the function $y=x^{2}+v^{2} t^{2}$ is a solution of the wave equation.
(b) Note that:

$$
\frac{1}{2}(x+v t)^{2}+\frac{1}{2}(x-v t)^{2}=\frac{1}{2} x^{2}+\frac{1}{2} v^{2} t^{2}+x v t+\frac{1}{2} x^{2}+\frac{1}{2} v^{2} t^{2}-x v t=x^{2}+v^{2} t^{2}
$$

so,

$$
f(x+v t)=\frac{1}{2}(x+v t)^{2} \quad \text { and } \quad g(x-v t)=\frac{1}{2}(x-v t)^{2}
$$

(c) Using the same procedure followed in (a), we get:

$$
\frac{\partial y}{\partial x}=\cos x \cos v t \quad \text { and } \quad \frac{\partial^{2} y}{\partial x^{2}}=-\sin x \cos v t
$$

similarly:

$$
\frac{\partial y}{\partial t}=-v \sin x \sin v t \quad \text { and } \quad \frac{\partial^{2} y}{\partial t^{2}}=-v^{2} \sin x \cos v t
$$

Substituting in the wave equation we get:

$$
-\sin x \cos v t=\frac{1}{v^{2}} \times-v^{2} \sin x \cos v t
$$

So, once again the function $y=\sin x \cos v t$ is a solution of the wave equation. Similarly, note:

$$
\sin (x+v t)=\sin x \cos v t+\cos x \sin v t \quad \text { and } \quad \sin (x-v t)=\sin x \cos v t-\cos x \sin v t
$$ So,

$$
\sin x \cos v t=\frac{1}{2} \sin (x+v t)+\frac{1}{2} \sin (x-v t)=f(x+v t)+g(x-v t)
$$

### 16.5 Problem 16.52 (In the text book)

A block of mass $M$, supported by a string, rests on an incline making an angle $\theta$ with the horizontal (Figure (16.52)). The length of the string is $L$ and its mass is $m \ll M$. Derive an expression for the time interval required for a transverse wave to travel from one end of the string to the other.


Figure 16.52:

## Solution

Assuming the incline to be frictionless and taking the positive $x$-direction to be up the incline:

$$
\sum F_{x}=T-M g \sin \theta=0
$$

where $T$ is the tension in the string, so:

$$
T=M g \sin \theta
$$

The speed of the transverse wave in the string is:

$$
v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{M g \sin \theta}{\frac{m}{L}}}=\sqrt{\frac{M g L \sin \theta}{m}}
$$

The time interval $\Delta t$ for a pulse to travel from one end of the string to the other is:

$$
\begin{aligned}
\Delta t & =\frac{L}{v} \\
& =L \sqrt{\frac{m}{M g L \sin \theta}} \\
& =\sqrt{\frac{m L^{2}}{M g L \sin \theta}} \\
& =\sqrt{\frac{m L}{M g \sin \theta}}
\end{aligned}
$$

