Chapter 15

Oscillatory Motion. Solutions of Home Work Problems

15.1 Problem 15.16 *(In the text book)*

A 200-g block is attached to a horizontal spring and executes simple harmonic motion with a period of 0.250 s. If the total energy of the system is 2.00 J, find

(a) the force constant of the spring and

(b) the amplitude of the motion.

Solution

(a) The force constant k can be calculated as follows:

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

so,

$$k = m\omega^2 = m\left(\frac{2\pi}{T}\right)^2 = 0.200kg \times \left(\frac{2\pi}{0.250s}\right)^2 = 126\ N/m$$

(b) The amplitude of the oscillation A can be calculated from:

$$E = \frac{1}{2}kA^2$$

so,

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2 \times 2.00J}{126N/m}} = 0.178\ m$$

15.2 Problem 15.28 *(In the text book)*

A "seconds pendulum" is one that moves through its equilibrium position once each second. (The period of the pendulum is precisely 2 s.) The length of a seconds pendulum is 0.992 7 m at Tokyo, Japan and 0.994 2 m at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations?

Solution

The period in Tokyo T_T is:

$$T_T = 2\pi\sqrt{\frac{L_T}{g_T}}$$

where L_T and G_T are the length of the Tokyo pendulum and the acceleration due to gravity in Tokyo. Similarly, in Cambridge we have:

$$T_C = 2\pi\sqrt{\frac{L_C}{g_C}}$$

Since the pendulums are a one second pendulums, then $T_T = T_C = 2\ s$. Squaring the above two equayions and divide them we get:

$$\frac{L_T}{g_T} = \frac{L_C}{g_C}$$

or,

$$\frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.9942}{0.9927} = 1.0015$$

or

$$g_C = 1.0015 g_T$$

15.3 Problem 15.38 *(In the text book)*

A torsional pendulum is formed by taking a meter stick of mass 2.00 kg, and attaching to its center a wire. With its upper end clamped, the vertical wire supports the stick as the stick turns in a horizontal plane. If the resulting period is 3.00 minutes, what is the torsion constant for the wire?

Solution

The period of sticks oscillation is given by:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

where I is the moment of inertial of the stick around it's center. So,

$$\kappa = \frac{4\pi^2 I}{T^2}$$

and the moment of inertial is:

$$\frac{1}{12}mL^2$$

so

$$\kappa = \frac{4\pi^2 mL^2}{12T^2} = \frac{4\pi^2 \times 2.00\ kg \times (1.00\ m)^2}{12\ (3 \times 60\ s)^2} = 2.03 \times 10^{-4} N \cdot m$$

15.4 Problem 15.44 *(In the text book)*

The front of her sleeper wet from teething, a baby rejoices in the day by crowing and bouncing up and down in her crib. Her mass is 12.5 kg and the crib mattress can be modeled as a light spring with force constant 4.30 kN/m.

(a) The baby soon learns to bounce with maximum amplitude and minimum effort by bending her knees at what frequency?

(b) She learns to use the mattress as a trampoline losing contact with it for part of each cycle when her amplitude exceeds what value?

Solution

(a) To maximize the bouncing with minimum effort, the baby must achieve resonance condition, i.e she should bounce with the same frequency as the natural frequency of the mattress' spring or she should bounce with,

$$f_\circ = \frac{\omega_\circ}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.3 \times 10^3\ N/m}{12.5\ kg}} = 2.95\ Hz$$

(b) The forces acting on the baby are her weight pushing down and the spring force. In one half cycle the two forces act downward and during the other half of the cycle the spring force acts upward and the weight acts downward. When the maximum spring force is equal and opposite to the weight the baby looses contact with the mattress. In other words she looses contact if the maximum acceleration of the spring equals the acceleration due to gravity, i.e.

$$a_{max} = A\omega^2 = g$$

or

$$A = \frac{g}{\omega^2} = \frac{g}{2\pi f_\circ} = \frac{9.8\ m/s^2}{(2\pi \times 2.95\ s^{-1})^2} = 2.85 \times 10^{-2}\ m$$

15.5 Problem 15.58 *(In the text book)*

After a thrilling plunge, bungee-jumpers bounce freely on the bungee cord through many cycles. After the first few cycles, the cord does not go slack. Your little brother can make a pest of himself by figuring out the mass of each person, using a proportion which you set up by solving this problem: An object of mass m is oscillating freely on a vertical spring with a period T. An object of unknown mass m' on the same spring oscillates with a period T'. Determine

(a) the spring constant and

(b) the unknown mass.

Solution

(a) The angular frequency is:

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

and the spring constant k is:

$$k = m\omega^2 = \frac{4\pi^2 m}{T^2}$$

(b) The mass of the object attached to the spring is:

$$m' = \frac{kT'^2}{4\pi^2}$$

Using k from part (a) we get:

$$m' = \frac{kT'^2}{4\pi^2} = \frac{4\pi^2 m}{T^2} \times \frac{T'^2}{4\pi^2} = m\left(\frac{T'}{T}\right)^2$$