Chapter 14

Fluid Mechanics. Solutions of Home Work Problems

14.1 Problem 14.10 *(In the text book)*

(a) A very powerful vacuum cleaner has a hose 2.86 *cm* in diameter. With no nozzle on the hose, what is the weight of the heaviest brick that the cleaner can lift? (Figure (14.10a))

(b) What If? A very powerful octopus uses one sucker of diameter 2.86 *cm* on each of the two shells of a clam in an attempt to pull the shells apart (Figure (14.10b)). Find the greatest force the octopus can exert in salt water 32.3 *m* deep. Caution: Experimental verification can be interesting, but do not drop a brick on your foot. Do not overheat the motor of a vacuum cleaner. Do not get an octopus mad at you.
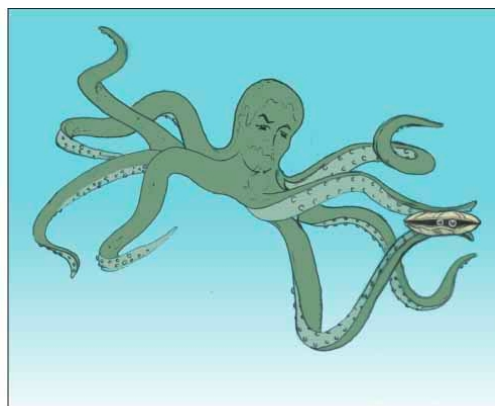
(a)   (b)

Figure 14.10:

Solution

(a) The pressure inside the vacuum clear hose is much smaller than the atmospheric pressure. So, the air will exert a lifting force F_ℓ on the brick. At the same time the weight of the brick will be pulling it downward. The heavies weight w_{max} the clear can lift will be equal to the force exerted by the cleaner. The lifting force is due to air pressure acting over the area of the cleaner's nozzle.

$$F = w_{max} = P_\circ A = 1.013 \times 10^5 \times \pi \left(\frac{2.86 \times 10^{-2}}{2} \right)^2 = 65.1 \ N$$

(b) The octopus can pull the bottom away from the top shell with a force that need not be larger than the force the water pressure on the clam:

$$
\begin{aligned}
F &= PA \\
&= (P_\circ + \rho g h)A \\
&= \left[1.013 \times 10^5 \ (Pa) + 1030 \ (kg/m^3) \times 9.8 \ (m/s^2) \times 32.3 \ (m) \right] \times \left[\pi \times (1.43 \times 10^{-2} m)^2 \right] \\
&= 275 \ N
\end{aligned}
$$

14.2   Problem 14.14 *(In the text book)*

The tank in Figure (14.14) is filled with water 2.00 m deep. At the bottom of one side wall is a rectangular hatch 1.00 m high and 2.00 m wide, which is hinged at the top of the hatch.

(a) Determine the force the water exerts on the hatch.
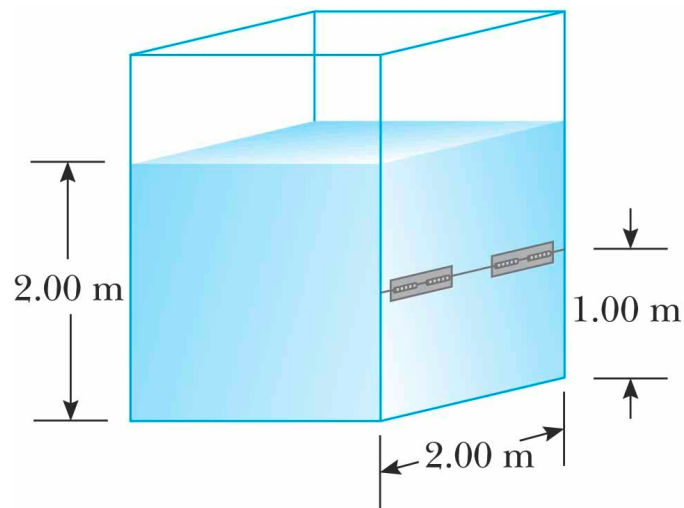
(b) Find the torque exerted by the water about the hinges.

Figure 14.14:

Solution

Solved in the class

14.3   Problem 14.20 *(In the text book)*

A U-tube of uniform cross-sectional area, open to the atmosphere, is partially filled with mercury. Water is then poured into both arms. If the equilibrium configuration of the tube is as shown in Figure (14.20), with $h_2 = 1.00$ cm, determine the value of h_1.
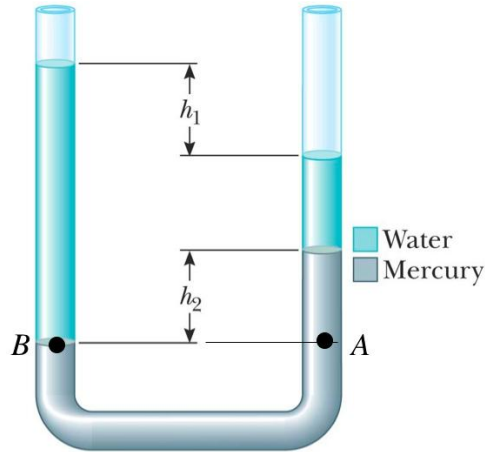

Figure 14.20:

Solution

Applying Pascal's Principle at points A and B in Figure (14.20), noticing that the absolute pressure at the two points are the same:

$$
\begin{aligned}
P_A &= P_B \\
P_\circ + \rho_w gh + \rho_{Hg}gh_2 &= P_\circ + \rho_w(h_1 + h + h_2) \\
\rho_{Hg}gh_2 &= \rho_w g(h_1 + h_2) \\
h_1 &= \left(\frac{\rho_{Hg}}{\rho_w}h_2\right) - h_2 \\
&= \left(\frac{\rho_{Hg}}{\rho_w} - 1\right)h_2 \\
&= \left(\frac{13.6}{1} - 1\right) \times 1 \\
&= 12.6 \ cm
\end{aligned}
$$

14.4   Problem 14.27 *(In the text book)*

A 10.0-*kg* block of metal measuring 12.0 *cm* × 10.0 *cm* × 10.0 *cm* is suspended from a scale and immersed in water as in Figure (14.25b). The 12.0-*cm* dimension is vertical, and the top of the block is 5.00 *cm* below the surface of the water.

(a) What are the forces acting on the top and on the bottom of the block? (Take $P_\circ = 1.0130 \times 10^5 \ N/m^2$.)

(b) What is the reading of the spring scale?

(c) Show that the buoyant force equals the difference between the forces at the top and bottom of the block.
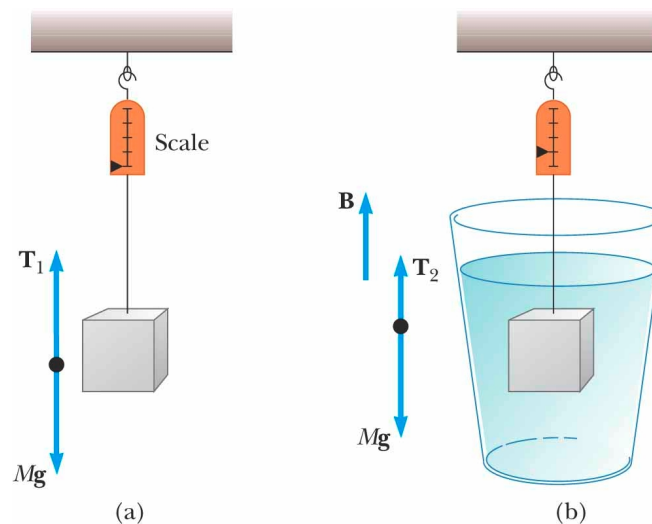

Figure 14.25:

Solution

(a) The top of the block is at $h_t = 5 \ cm$ below the surface of the water. The bottom of the block is at $h_b = 5 + 12 = 17 \ cm$ below the surface of the water. The top and the bottom surfaces of the block each has an are $A = 10 \times 10 = 100 \ cm^2 = 10^{-2} \ m^2$. The pressure on the top surface is given by:

$$
\begin{aligned}
P_t &= P_\circ + \rho g h_t \\
P_t &= 1.013 \times 10^5 \ (Pa) + 1000 \ (kg/m^3) \times 9.8 \ (m/s^2) \times 5 \times 10^{-2} \ (m) \\
&= 1.0179 \times 10^5 \ Pa
\end{aligned}
$$

Similarly, the pressure at the bottom of the block is:

$$
\begin{aligned}
P_b &= P_\circ + \rho g h_b \\
P_b &= 1.013 \times 10^5 \ (Pa) + 1000 \ (kg/m^3) \times 9.8 \ (m/s^2) \times 17 \times 10^{-2} \ (m) \\
&= 1.0297 \times 10^5 \ Pa
\end{aligned}
$$

and the forces F_t and F_b on the top and bottom surfaces of the block are:

$$
\begin{aligned}
F_t &= P_t A \\
&= 1.0179 \times 10^5 \times 10^{-2} \\
&= 1.0179 \times 10^3 \ N \\
F_b &= P_B A \\
&= 1.0297 \times 10^5 \times 10^{-2} \\
&= 1.0297 \times 10^3 \ N
\end{aligned}
$$

(b) The forces acting on the block are; F_t, mg down ward and F_b, T upward, where T is the tension in the string. Since the block is at rest, we get:

$$
\begin{aligned}
T + F_b &= F_t + mg \\
T &= F_t - F_b + mg \\
&= 1.0179 \times 10^3 - 1.0297 \times 10^3 + 10.0 \times 9.8 \\
&= 86.2 \ N
\end{aligned}
$$

The spring scale reads the tension the string which is $86.2 \ N$. Notice that this smaller than the weight of the block which is $98 \ N$.

(c) The buoyant force B is given by:

$$
\begin{aligned}
B &= \rho_w g V_b \\
&= 1000 \ (kg/m^3) \times 9.8 \ (m/s^2) \times (12 \times 10 \times 10) \times 10^{-6} (m^3) \\
&= 11.8 \ N
\end{aligned}
$$

and

$$
F_b - F_t = 1.0297 \times 10^3 - 1.0179 \times 10^3 = 11.8 \ N
$$

The buoyant force equals the difference between the forces on the bottom and the top of the block.

14.5 Problem 14.45 *(In the text book)*

Through a pipe 15.0 *cm* in diameter, water is pumped from the Colorado River up to Grand Canyon Village, located on the rim of the canyon. The river is at an elevation of 564 *m* and the village is at an elevation of 2096 *m*.

(a) What is the minimum pressure at which the water must be pumped if it is to arrive at the village?

(b) If 4500 m^3 are pumped per day, what is the speed of the water in the pipe?

(c) What additional pressure is necessary to deliver this flow? Note: You may assume that the free-fall acceleration and the density of air are constant over this range of elevations.

Solution

(a) The minimum pressure is the pressure that makes the water to just arrive at the village. This means the speed through the pipe is very small and can be ignored. So applying Bernoulli's equation at the river and the village we get:

$$
\begin{aligned}
(P + \frac{1}{2}\rho v^2 + \rho g h)_{river} &= (P + \frac{1}{2}\rho v^2 + \rho g h)_{village} \\
P + 0 + \rho g \times h_{river} &= P_\circ + 0 + \rho g \times h_{village} \\
P &= P_\circ + \rho g (h_{village} - h_{river}) \\
&= P_\circ + 1000 \times 9.8 \times (2096 - 564) \\
&= P_\circ + 15.0 \times 10^6 \ Pa
\end{aligned}
$$

(b) The volume rate of flow $R_V = 4500 \ m^3/day = Av$, where A is the cross sectional area of the pipe and is velocity of the flow,

$$
\begin{aligned}
R_V &= Av \\
v &= \frac{R_V}{A} \\
&= \frac{R_V}{\pi r^2} \\
&= \frac{4500 \ (m^3/d)}{\pi \times (7.50 \times 10^{-2} \ m)^2 \times 24 \ (hr/d) \times 60 \ (min/hr) \times 60 \ (s/min)} \\
&= 2.95 \ m/s
\end{aligned}
$$

(c) Assuming that the pressure is applied to a stationary water at the bottom, then applying Bernoulli's equation at the river and the village again we get:

$$
\begin{aligned}
(P + \tfrac{1}{2}\rho v^2 + \rho g h)_{river} &= (P + \tfrac{1}{2}\rho v^2 + \rho g h)_{village} \\
(P + \rho g h)_{river} &= (P + \tfrac{1}{2}\rho v^2 + \rho g h)_{village} \\
P_{river} &= P_\circ + \tfrac{1}{2}\rho v^2 + \rho g(h_{vilage} - h_{river}) \\
&= P_\circ + \tfrac{1}{2} \times 1000 \times (2.95)^2 + 1000 \times 9.8 \times 1532 \\
&= P_\circ + 15.0 \times 10^6 + 4.35 \times 10^3 \; Pa
\end{aligned}
$$

Comparing the last answer with the answer in part (a), then the extra pressure required to provide the flow of 4500 m^3/day is 4.35 Pa

14.6 Problem 14.59 *(In the text book)*

A light spring of constant $k = 90.0 \ N/m$ is attached vertically to a table (Figure (14.59a)). A 2.00-g balloon is filled with helium (density $= 0.180 \ kg/m^3$) to a volume of 5.00 m^3 and is then connected to the spring, causing it to stretch as in Figure (14.59b). Determine the extension distance L when the balloon is in equilibrium.
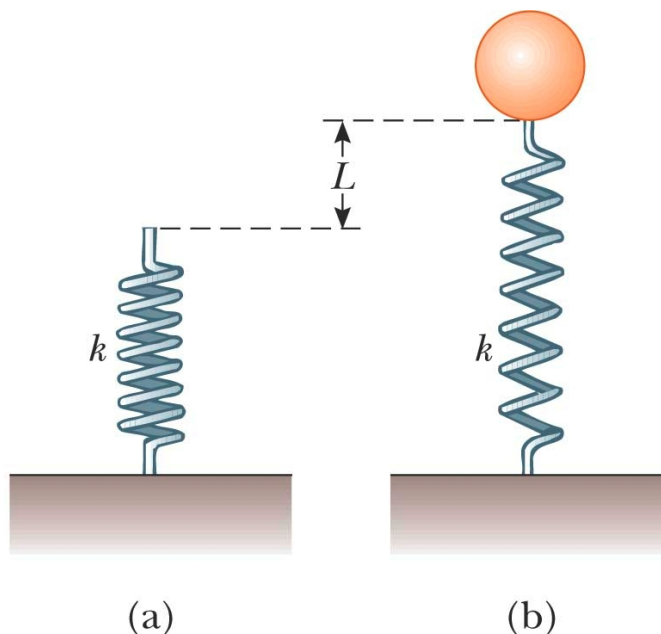

Figure 14.59:

Solution

The forces acting on the balloon are: the buoyant force B of the air on the helium in the balloon, the wight of the balloon w_b, the weight of the helium w_{He} and finally the spring force F_s resulting from extending the spring a distance L. All the forces are pulling the balloon down except the buoyant force which is lifting the balloon up. Since the balloon is in equilibrium we get:

$$
\begin{aligned}
0 &= B - w_b - w_{He} - F_s \\
&= \rho_{air} V_{He} g - m_b g - m_{He} g - kL \\
&= \rho_{air} V_{He} g - m_b g - \rho_{He} V_{He} g - kL \\
L &= \frac{(\rho_{air} - \rho_{He}) V_{He} - m_b}{k} g
\end{aligned}
$$

where k is the spring constant.

$$L = \frac{(1.290 - 0.180) \times 5.00 - 2.00 \times 10^{-3}}{90.0} \times 9.8 = 0.604 \ m$$