1. A 240 kV power transmission line carrying 500 A is hung from grounded metal towers by ceramic insulators, each having a $1.00 \times 10^9 \Omega$ resistance.
   (a) what is the resistance to ground of 100 of these insulators?
   (b) calculate the power dissipated by 100 of them.
   (c) What fraction of the power carried by the line is this?

Solution

Given:

$$\begin{aligned} V &= 240\ kV \\ I &= 500\ A \\ \text{The resistance of one insulator} &= 1.00 \times 10^9\ \Omega \end{aligned}$$

Required:

(a) Total resistance of 100 insulators.

(b) Total power lost in 100 of insulator P_{lost}.

(c) The fraction of the power lost in the 100 insulators of the total power on the transmission line.

(a) Each of the 100 insulators connects the transmission line to the ground through the metal tower. This means that the top end of each insulator is connected to the transmission line and the bottom end to the ground. The resistance of the transmission line is extremely small compared to the resistance of the insulator. In other words, the top ends of all insulators are connected together and so are the bottom ends. The voltage across each insulator is the same as the potential difference between the transmission line and the ground. Each insulator carries a very small current of $I = 2.4 \times 10^5 / 1.00 \times 10^9 = 2.4 \times 10^{-4}\ A$. This means that all 100 insulators are connected in parallel.

$$\begin{aligned}\frac{1}{R} &= \sum_{i=1}^{i=100} \frac{1}{R_i} \\ &= \frac{100}{1.00 \times 10^9} \\ R &= 1.00 \times 10^7\ \Omega\end{aligned}$$

(b) The total power lost in the 100 insulators is the same power lost in the equivalent resistor:

$$\begin{aligned}P_{lost} &= \frac{V^2}{R} \\ &= \frac{2.4 \times 10^5}{1.00 \times 10^7} \\ &= 5.76\ kW\end{aligned}$$

(c) The fraction of the power lost in the insulators to the total power is:

$$\begin{aligned}\frac{P_{lost}}{P_{total}} &= \frac{\frac{V^2}{R}}{IV} \\ &= \frac{V}{IR} \\ &= \frac{2.4 \times 10^5}{500 \times 1.00 \times 10^7} \\ &= 4.8 \times 10^{-5}\end{aligned}$$

Notice that the term IR in the equation above does not equal to the voltage V. This is because the current I is the current carried by the transmission line (500 A) and not the current ($100 \times 2.4 \times 10^{-4}\ A$) through the equivalent resistor of the insulators R.

2. A child's electronic toy is supplied by three 1.58 V alkaline cells having internal resistance of 0.0200 Ω in series with a 1.53 V carbon-zinc dry cell having a 0.100 Ω internal resistance. The load resistance 10.0 Ω.

   (a) What current flows?

   (b) How much power is supplied to the load?

   (c) What is the internal resistance of the the dry cell if it goes bad, resulting in only 0.500 Wbeing supplied to the load?

Solution

Given:

- Three alkaline batteries with an emf of 1.58 V and internal resistance of 0.0200 Ω each, connected in series to a dry cell with emf 1.53 V and internal resistance of 0.100 Ω.
- 10 Ω load resistance.
- When the dry cell goes bad, the power delivered to the load resistor drops to 0.5 W.

Required:

(a) Current through the load.

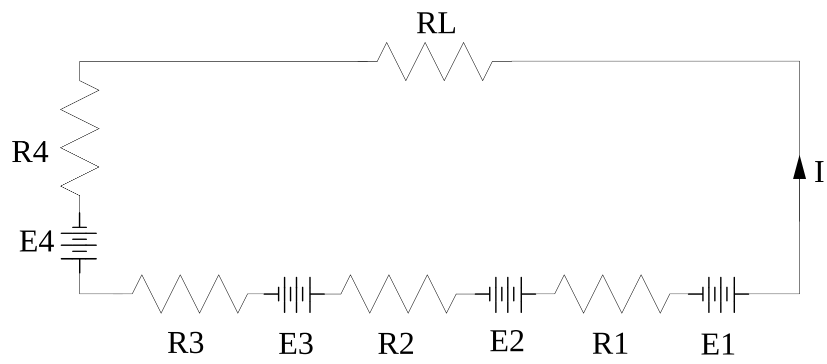
(b) Power delivered to the load.

(c) Internal resistance of a bad dry cell when the power level in the load resistor drops to 0.500 W.

The circuit is shown in Figure (1). The four batteries are connected in series. The total emf E of the circuit is then:

$$
\begin{aligned}
E &= E_1 + E_2 + E_3 + E_4 \\
&= 3 \times 1.58 + 1.53 \\
&= 6.27\ V
\end{aligned}
$$

All resistors in the circuit are connected in series. The total resistance in the circuit is then the sum of all of them, i.e. $R = 10.16\ \Omega$.



E1 = E2 = E3 = 1.58
VR1 = R2 = R3 =0.02
ΩE4 = 1.53 VR4 = 0.10
ΩRL = 10.0 Ω

Figure 1: Circuit diagram.

(a) the current in the circuit is given by Ohm's law:

$$\begin{aligned} I &= \frac{E}{R} \\ &= \frac{6.27}{10.16} \\ &= 0.617\ A \end{aligned}$$

(b) The power given to the load is:

$$\begin{aligned} P &= I^2 R_L \\ &= (0.617)^2 \times 10 \\ &= 3.81\ W \end{aligned}$$

(c) We can replace the circuit in Figure (1) by the equivalent circuit shown in Figure (2). The equivalent circuit contains the load resistor R_L and one battery with emf E and internal resistance R. The bad dry cell now has unknown terminal voltage V_b and its internal resistance R_b is also unknown. The total equivalent internal resistance R is:

$$R = R_1 + R_2 + R_3 + R_b$$

The emf and the internal resistance of the three alkaline batteries remain unchanged. The emf of the dry cell also remain unchanged while its terminal voltage and internal resistance change to V_b and R_b

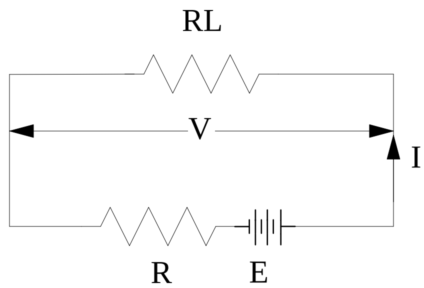


Figure 2: Equivalent circuit diagram of the circuit shown in Figure (1).

The power delivered to the load resistor is given as 0.5 W. The voltage across the load resistor is the terminal voltage of the equivalent battery V and the current I through the circuit and the load resistor are then :

$$
\begin{aligned}
P &= I^2 R_L \\
I &= \sqrt{\frac{P}{R_L}} \\
&= \sqrt{\frac{0.5}{10}} \\
&= 0.2236\ A \\
P &= \frac{V^2}{R_L} \\
V &= \sqrt{P R_L} \\
&= \sqrt{0.5 \times 10} \\
&= 2.2361\ V \\
&= E - IR \\
&= E - I \times (R_1 + R_2 + R_3 + R_b) \\
R_b &= \frac{E - V}{I} - (R_1 + R_2 + R_3) \\
&= \frac{6.27 - 2.2361}{0.2236} - (3 \times 0.0200) \\
&= 17.9801\ \Omega
\end{aligned}
$$

3. Find the currents flowing in the circuit in Figure (3).

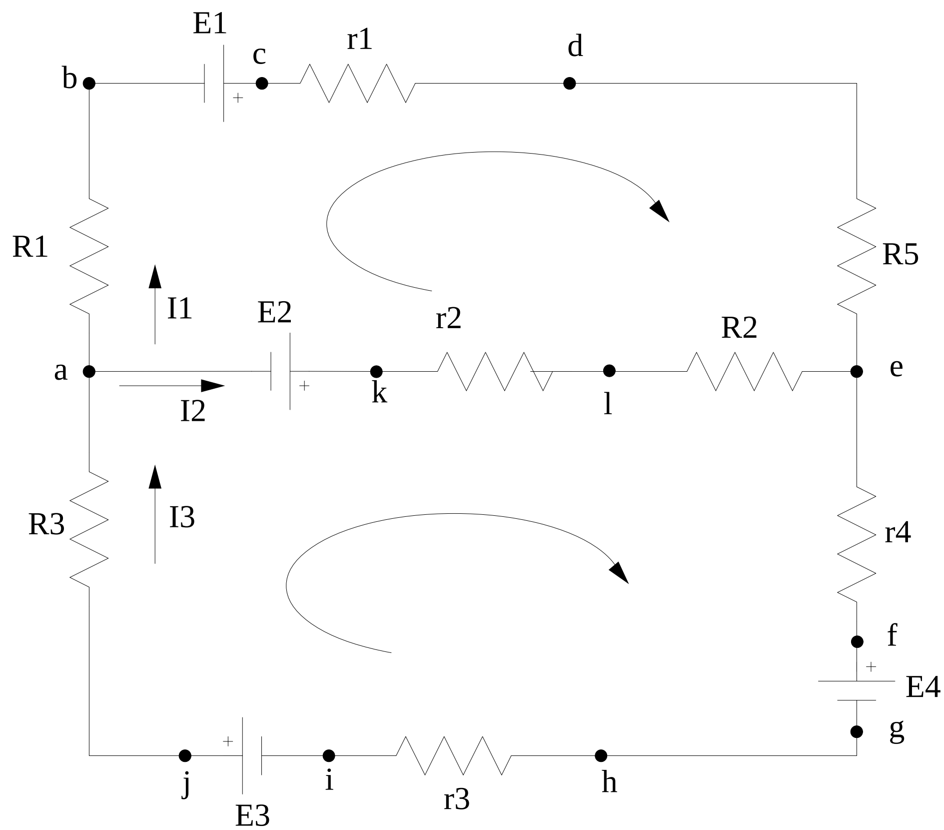


Figure 3: Circuit diagram.

Solution

Given:

- The circuit Shown in Figure (3).
-

$$
\begin{aligned}
E_1 &= 24.0\ V \\
E_2 &= 48.0\ V \\
E_3 &= 6.0\ V \\
E_4 &= 36.0\ V
\end{aligned}
$$

$$
\begin{aligned}
r_1 &= 0.10\ \Omega \\
r_2 &= 0.50\ \Omega \\
r_3 &= 0.05\ \Omega \\
r_4 &= 0.20\ \Omega \\
R_1 &= 5.0\ \Omega \\
R_2 &= 40\ \Omega \\
R_3 &= 78\ \Omega \\
R_5 &= 20\ \Omega
\end{aligned}
$$

Required:

- The current I_1, I_2, and I_3 flowing in the circuit.

We need to apply Kirchhoff's rules to the circuit. Since we have three unknowns we, then need three equations. We get the required equations by applying the current rule at junction point a, and the voltage rule to the loops $egjae$ and $abdea$. We traverse these two loops in clockwise direction.

$$
\begin{aligned}
I_3 &= I_1 + I_2 && (1) \\
0 &= E_2 - I_2R_2 - I_2r_2 - I_3r_4 - E_4 - I_3r_3 + E_3 - I_3R_3 && (2) \\
0 &= -I_1R_1 + E_1 - I_1r_1 - I_1R_5 + I_2R_2 + I_2r_2 - E_2 && (3)
\end{aligned}
$$

Rearranging Equation (2) and Equation (3) we get:

$$
\begin{aligned}
0 &= -I_3(r_4 + r_3 + R_3) + (E_2 + E_3 - E_4) - I_2(R_2 + r_2) && (4) \\
0 &= -I_1(R_1 + r_1 + R_5) + I_2(R_2 + r_2) + (E_1 - E_2) && (5)
\end{aligned}
$$

From Equation (4) and Equation (5) we get:

$$
\begin{aligned}
0 &= -78.25I_3 - 40.5I_2 + 18.0 && (6) \\
0 &= -25.1I_1 + 40.5I_2 - 24 && (7)
\end{aligned}
$$

Using Equation (1), and rearranging Equation (6), and Equation (7) we get:

$$\begin{aligned} I_1 &= -1.517I_2 + 0.230 \quad (8) \\ I_1 &= 1.613I_2 - 0.956 \quad (9) \end{aligned}$$

Subtracting Equation (9) from Equation (8) we get

$$\begin{aligned} 0 &= -3.449I_2 + 1.186 \\ I_2 &= 0.379\ A \quad (10) \end{aligned}$$

Substituting for I_2 from Equation (10) into Equation (9) we get

$$I_1 = -0.345\ A \quad (11)$$

Substituting for I_1 from Equation (11) and I_2 from Equation (10) we get:

$$I_3 = 0.034\ A$$

4. A 1.00 $M\Omega$ resistor is placed in parallel with a 75.0 $k\Omega$ resistor in a circuit.
   (a) What is the resistance of the combination?
   (b) If the voltage across the combination is kept the same as it was across the 75.0 $k\Omega$ resistor alone, what is the percent increase in current?
   (c) If the current through the combination is kept the same as it was through the 75.0 $k\Omega$ resistor alone, what is the percent decrease in voltage?

Solution

Given:

- A 1.00 $M\Omega$ resistor is connected in parallel to a 75.0 $k\Omega$.

Required:

- The resistance of the combination of the 1.00 $M\Omega$ and the 75 $k\Omega$ resistors.
- Percent change in current if the voltage across the combination is the same as across the 75.0 $k\Omega$.
- Percent change in voltage if the current through the combination is the same as through the 75.0 $k\Omega$.

(a) The 1.00 $M\Omega$ resistance is connected in parallel with the the 75 $k\Omega$, the resistance of the combination R is:

$$\begin{aligned} \frac{1}{R} &= \frac{1}{1.00 \times 10^6} + \frac{1}{75.0 \times 10^3} \\ R &= 69.8\ k\Omega \end{aligned}$$

(b) Let the current through the 75.0 $k\Omega$ resistor be I_R and the current through the combination I. The voltage across the 75.0 $k\Omega$ is then $75.0 \times 10^3 \times I_R$ and the voltage across the combination is IR, since these two voltages are the same we get:

$$\begin{aligned} 75.0 \times 10^3 \times I_R &= 69.8 \times 10^3 \times R \\ I - I_R &= \left(\frac{75.0}{69.8} I_R\right) - I_R \\ \frac{I - I_R}{I_R} \times 100 &= \left(\frac{75.0}{69.8} - 1\right) \times 100 \\ &= 7.45\% \end{aligned}$$

(c) Keeping the current through the combination the same as the current through the 75.0 $k\Omega$ resistor changes the voltages. Let the voltage across the 75.0 $k\Omega$ resistor be V_R and the voltage across the combination be V, we then have:

$$\begin{aligned}
V_R &= 75.0 \times 10^3 \times I_R \\
V &= 69.8 \times 10^3 \times I_R \\
\frac{V_R - V}{V_R} \times 100 &= \frac{75.0 - 69.8}{75} \times 100 \\
&= 6.93\%
\end{aligned}$$

5. The batteries of a submerged nonnuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

Solution

Given:

$$\begin{aligned} I &= 1000\ m \\ \text{Number of electrons moved, } N &= 6.02 \times 10^{23}\ s \\ \text{Electron charge, e} &= 1.602^{-19}\ C \end{aligned}$$

Required:

Time required to move N electrons.

The total charge moved is given by $Q = Ne$, using the definition of electric current we get:

$$\begin{aligned} I &= \frac{Q}{t} \\ t &= \frac{Q}{I} \\ &= \frac{Ne}{I} \\ &= \frac{6.02 \times 10^{23} \times 1.602 \times 10^{-19}}{1000} \\ &= 96.4\ s \end{aligned}$$

6. SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator, has 20 A circulating beam of electrons that are moving nearly the speed of light. How many electrons are in the beam?

Solution

Given:

$$\begin{aligned} d &= 72\ m \\ I &= 20.0\ A \\ v &\approx 3 \times 10^8\ m/s \\ \text{Electron charge, e} &= 1.602^{-19}\ C \end{aligned}$$

Required:

Number of electrons N in the beam.

N electrons are moving around the circumference of the ring with a speed very close to the speed of light, constituting a current I. Since $I = Qt$ and we know I we can then find Q; if we can find t. N can then be found from $Q = Ne$. The time T can be found from the circumference l and the speed v:

$$\begin{aligned} v &= \frac{l}{t} \\ &= \frac{2\pi r}{t} \\ &= \frac{\pi d}{t} \\ t &= \frac{\pi d}{v} \\ &= \frac{Q}{I} \\ Q &= \frac{\pi d I}{v} \end{aligned}$$

$$
\begin{aligned}
N &= \frac{Q}{e} \\
&= \frac{\pi d I}{e v} \\
&= \frac{\pi \times 72.0 \times 20.0}{3 \times 10^{8} \times 1.602 \times 10^{-19}} \\
&= 9.41 \times 10^{13} \text{ electrons}
\end{aligned}
$$

7. An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs 9.00 $c/kW.h$?

Solution

Given:

$$\begin{aligned} \text{Power consumption of the heater} P &= 5.00\ kW \\ \text{Time heater used} &= 2.00\ h\ per\ day \\ \text{Cost of electricity} &= 9.00\ c/kW.h \end{aligned}$$

Required:

Cost C of using the heater for one year.

Electric energy E consumed by the heater is the product of the power consumption times the the total time (in hours) of using the heater.

$$\begin{aligned} E &= P \times t\ kW.h \\ C &= P \times t \times \text{cost per } kW.h \\ &= 5.00 \times 2 \times 365 \times 0.09\ \$ \\ &= \$329 \end{aligned}$$

8. Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at 1.00 $A.h$ and 1.58 V keep a 1.00 W flash light burning?

Solution

Given:

$$\begin{aligned} \text{Battery rating } R &= 1.00\ A.h \\ \text{Terminal Voltage } V &= 1.58\ V \\ \text{Powe consumption } P &= 1.00\ W \end{aligned}$$

Required: The total time t the battery can keep the flash light on.

The rating of the battery as 1.00 $A.h$ means that the battery can provide one ampere of current at a voltage of 1.58 V for one hour, or two amperes for half an hour, or 10 amperes for 6 minutes and so on. In other words, a battery can provide a total amount of energy given by $R \times V$. If R is in$A.h$ then the total energy stored in the battery is RV in $W.h$. This means that the battery can provide RV watts for one hour or $2RV$ in half an hour and so on.

The battery in the problem can provide a total energy of 1.58 $W.h$, that is it can provide 1.58 W for one hour or can provide 1.00 W for 1.58 hours. This means that the battery can keep the flash light, rated at 1.00 W, burning for 1.58 hours.