Chapter 27

Origins of Quantum Theory. Home Work Solutions

27.1 Problem 27.20

What is the minimum voltage required in an x-ray tube to produce photons whose wavelength is 0.10 nm

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Solution

- **Given:** x-ray photons with wavelength $\lambda = 0.10 \times 10^{-9}\, m$
- **Required:** Minimum voltage required to produce these x-ray photons.

The energy carried by a photon $E = hf$ where f is the frequency of the photon and h is Planck's constant.

$$\begin{aligned} E &= hf \\ &= \frac{hc}{\lambda} \\ &= \frac{(6.626 \times 10^{34}\, J \cdot s) \times (3.00 \times 10^{8}\, m/s)}{0.10 \times 10^{-9}\, m} \\ &= 1.988 \times 10^{-15}\, J \\ &= \frac{1.988 \times 10^{-15}\, J}{1.602 \times 10^{-19}\, J/eV} \\ &= 12.4 \times 10^{3}\, eV \end{aligned}$$

So the minimum voltage required to produce x-ray photons with $\lambda = 0.10\, nm$ is 14.2 kV.

27.2 Problem 27.29

A photoflood lamp operates at a temperature of 3400 K

(a) What is the wavelength of the peak in its blackbody spectrum?

(b) What is the energy of a photon with with the wavelength found in (a)? Give your answer in eV

Solution

- **Given:** A photoflood lamp at a temperature of $T = 3400\ K$
- **Required:** The peak wavelength λ_m of the emitted spectrum assuming the lamp behaves as a black body and the energy of photons with wavelength of λ_m

(a) The peak wavelength λ_m is related to the temperature by Wien displacement law:

$$\begin{aligned}
\lambda_m T &= 2.90 \times 10^{-3}\, m \cdot K \\
\lambda_m &= \frac{2.90 \times 10^{-3}}{T} \\
&= \frac{2.90 \times 10^{-3}}{3400} \\
&= 8.53 \times 10^{-7}\, m \\
&= 853\, nm
\end{aligned}$$

(b) The energy of such a photon is:

$$\begin{aligned}
E &= hf \\
&= \frac{hc}{\lambda_m} \\
&= \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{853 \times 10^{-9}} \\
&= 2.33 \times 10^{-19}\, J \\
&= \frac{2.33 \times 10^{-19}}{1.602 \times 10^{-19}} \\
&= 1.45\, eV
\end{aligned}$$

27.3 Problem 27.33

As the integer n_2 in the Rydberg equation (Eq. 27.2) approaches infinity, the the spectral lines converge to a minimum wavelength known as the series limit. Calculate the series limit for the Balmer lines.

Solution

- **Given:** Balmer series spectral lines.
- **Required:** The series limit of the Balmer lines.

The Rydberg formula is:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $R = 1.097 \times 10^7\, m^{-1}$ is the Rydberg constant. For Balmer series $n_1 = 2$ and $n_2 = 3, 4, 5, \cdots \infty$. The wavelength corresponding to $n_2 = \infty$ is called the series limit. We then have:

$$\begin{aligned}
\frac{1}{\lambda} &= R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) \\
&= \frac{R}{4} \\
\lambda &= \frac{4}{R} \\
&= \frac{4}{1.097 \times 10^7} \\
&= 3.646 \times 10^{-7}\, m \\
&= 364.6\, nm
\end{aligned}$$