Master of Science Thesis Proposal:
Model of Heat and Moisture Movement in Logs

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1 Research Statement

The proposed thesis project involves the analysis of heat and moisture movement within log segments during conditioning. The conditioning process is one of the steps involved in plywood manufacturing. The analysis of the process will consist of developing or augmenting an existing model of time-dependent heat and moisture transport within a cylindrical timber segment. It is anticipated that an exact solution will not be possible and therefore a numerical solution will be employed. The numerical results will then be compared with experimental data obtained from local industry.

The thesis project is partially funded by CANFOR's North Central Plywood. It is hoped that the study will give insight into the process and help mill operators understand, and therefore, employ a more energy efficient and cost effective process. Once developed and verified the model will be able to accurately simulate heat and moisture movement within a single log during conditioning. In this fashion parameters can be *tweaked* in the simulation and multiple runs made in a fraction of the time it would take in the actual conditioning chest. It will also make it possible to test ideas that may require relatively extensive changes to be made to the current conditioning chest itself or the process in general. Currently the mill controls conditioning by timing according to the species of wood.

North Central Plywood routinely utilizes beetle infested wood as well as frozen wood in the winter months. The beetle infested wood is thought to have different water absorption properties. Additionally, frozen wood is also thought to have a much different "warm up" time then un-frozen wood. Heat and moisture transfer in beetle infested or frozen wood is an area that remains untouched in the literature. A desirable feature of the model will be to able to account for these particular situations through model parameters.

The experiments required for the model verification will also be completed as part of the thesis work. The experiments will be conducted at North Central Plywood as well as in the lab. The experimental program will be designed in such a way as to gather all required data for verification while keeping the number of experiments to a minimum.

2 Background

2.1 Wood Structure

A brief discussion of wood structure and properties is necessary before discussing model theories. The following summary of wood structure is adapted from standard textbooks on wood structure, such as; [7], [8], and [15]. Looking at a cross section of wood it is apparent that it is not a homogeneous material. There is a definite color difference between the outermost layers of wood and the inner portions. The outermost layers, usually lighter in colour, are called sapwood, and the inner darker layers are called heartwood. The main difference between sapwood and heartwood is that heartwood is biologically dead. What this means is that nutrients and water moving within the tree move only through the sapwood, the heartwood acts as a structural support. Sapwood has a higher moisture content than heartwood at a given relative humidity and temperature. At equal moisture content, sapwood and heartwood have the same density and structural strength characteristics. Generally, as a tree ages, the thickness of sapwood decreases. These properties of sapwood and heartwood discussed here are evident in most of the time, although there are exceptions in some cases. For instance, sapwood content of Douglas fir is approximately 20% where as for some tropical species, sapwood can be over 50% of the total wood content of the tree. The properties of interest to this thesis project are the different heat and moisture transfer properties of sapwood and heartwood. These differences will most likely be reflected in the model parameters.

Taking a closer look at a cross section of wood it can be seen to be made up of concentric rings called growth rings. The growth rings are the amount of new material added to the tree through one growing season. An even closer look an individual growth ring shows that there is a definite difference between the wood added early in the growing season and the wood added late in the growing season. These are called late wood and early wood. The main difference between late wood and early wood is that late wood is denser then early wood. The transition from early wood to late wood ranges in species from very abrupt to gradual. It is anticipated that a sufficient first approximation in modeling heat and moisture transfer in wood is not to differentiate early wood and late wood.

On a microscopic scale wood is composed mainly of relatively long tubular cells, or fibers, called tracheids. The walls of the tracheids are on the same order of thickness as the diameter of the voids, know as lumen, inside the tubular tracheids. The rounded shape and tubular structure of the tracheids results in voids around the tracheids as well as within the tracheids, and wood can therefore be classified as a porous material. As a result of porosity, moist air can diffuse into the voids in and around the tracheids. The tracheid walls are made up mainly of cellulose. The chemical nature of the cell wall causes water molecules in the moist air to bond to the cell wall. The water molecules that have bound to the cell walls are referred to as *bound* water or *hygroscopic* water. At high enough humidity the cell walls reach a saturated state where there is no more room for water molecules to bind. This point is called the fiber saturation point, or FSP. The FSP results in two different regimes with respect moisture transport and distribution within the wood. Below the FSP moisture in the wood is either in the vapour or bound form. Above FSP bound moisture still exists but the net amount of bound moisture becomes fixed and moisture in the wood begins to accumulate in liquid and vapour form.

Capillary forces arise when liquid water is in a very thin tube or capillary. The attraction of the water molecules on the surface of a liquid is greater then the attraction of the molecules within the liquid. In a thin enough tube the surface tension will be enough to hold the liquid in place against gravity and therefore moisture remains bound as free water. From this it can be seen that moisture movement above and below FSP are slightly different processes. In the case of this thesis project we are dealing with wood above FSP ($> 30\%$ moisture content), therefore appropriate steps must be taken to account for this in the model.

2.2 Conditioning Process

The plywood making process requires thin veneers of wood to be laminated together. The veneers are cut using a lathe. A cross section of this cutting process can be seen in figure 1. In order for the lathe to operate effectively the wood must be soft enough to cut with the lathe. The conditioning process is designed to maximize these properties of the wood through pouring water at approximately 70^0C over the log segments. By using heated water the internal temperature of the timber segments can be raised without drying the wood. At NCP the conditioning chests consist of long cement tunnels approximately 30 meters long by 2 meters wide by 3 meters high. The log segments are stacked horizontally and sprinklers spaced about 2 meters

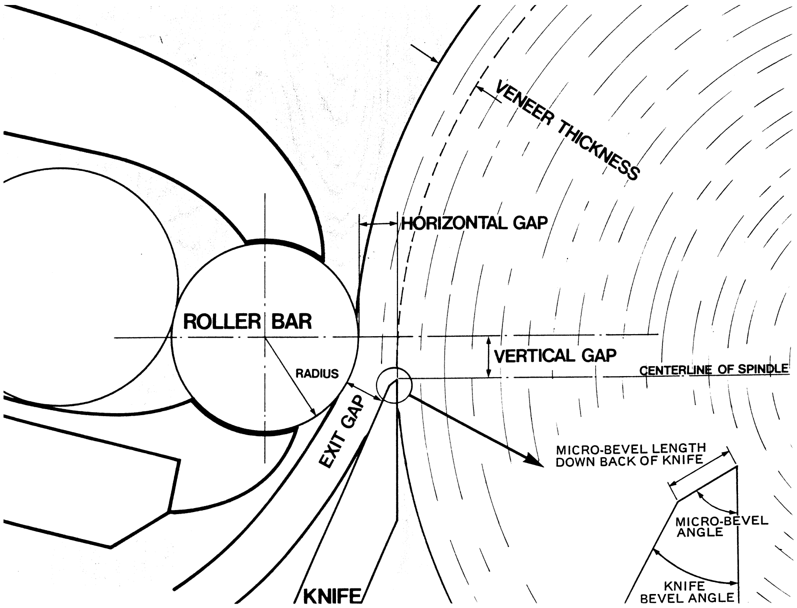

Figure 1: Cross section of veneer cutting lathe.

apart along the center of the roof are used to douse the logs with heated water. Figure 2 shows a typical orientation of timber segments. Currently, conditioning times range from 7 to 72 hours depending on species.

In this process heat energy and moisture are transfered from the water to the timber segments. It is well accepted that heat and moisture transport in wood is a coupled process, [1], [4], [7], and [11]. For instance, the rate at which heat transfers through the wood depends on the current moisture content. It is also believed that, even though it is a coupled process, moisture transport in wood is a much slower process then heat transfer, [1]. This suggests that moisture transport may play a less significant role in the modeling process. The significance of moisture distribution on heat transport in wood will be one of the questions to be addressed in this study. For this reason moisture transport will be included in the modelling process.

2.3 Current Methods

The current method used to judge conditioning time is a software package called LOGCON. LOGCON is based upon an empirical model, which was

Figure 2: Inside of conditioning tunnel, North Central Plywood.

developed in the 1970's. Since NCP routinely uses beetle infested and frozen wood the LOGCON software often gives incorrect predictions. The effect of the beetle infested and frozen wood on conditioning times is an observation raised by NCP staff. A more accurate model of conditioning will allow more accurate predictions of conditioning times. This will give the possibility for a more efficient conditioning schedule to be designed based on a range of parameters, such as outside temperature, outside relative humidity, log diameter, species, and health of the wood (beetle infested or not).

3 Drying Models

Almost all the current literature concerning heat and moisture transport in wood refers to drying. Wood drying is a well studied subject because of its applications in the industry to kiln drying, which is a much more common process then a combination of heating and soaking in plywood manufacture. In drying the models must account for conditions, below the fiber saturation point, and in some instances above the fibers saturation point as well. In the case of conditioning for plywood making the timber segments begin the conditioning process green and therefore never go below the fiber saturation point. Initial measurements of moisture content in pine log segments before and after conditioning back up this assumption. This is advantages since the net amount of bound water remains constant and the model complexities arising from the hygroscopic range can be safely ignored.

To this time the models employed in the literature vary in their level of complexity. This varying level of complexity partially occurs from the range of moisture content being modelled, as well as the number of assumptions and simplifications that can be made. Using past studies as a guide an appropriate model can be chosen and adapted to this particular case. There are essentially four different models described in the literature. A brief discussion of each will be given.

3.1 Diffusive Model

Another problem with a large proportion of the existing literature is that it refers to the simpler case of isothermal wood drying. This is because moisture transport in wood is know to be a much slower process then heat transport, therefore isothermal conditions are assumed throughout the drying process. The approach taken by Baronas et al. [12], Liu et al. [11], and Cheng et al. [13], employed Fick's law of mass transfer to varying degrees of success to model isothermal moisture transport. It is obvious that this approach cannot be used since the area of interest is the heating of the timber segments. One approach to deal with the coupled nature of heat and moisture transport in wood is to combine Fourier's law of heat conduction and Fick's law of mass transfer where the coupling occurs through the molecular diffusion and thermal conduction coefficients.

$$\frac{\partial M}{\partial t} = \nabla \cdot D_m(M,T)\nabla M \tag{1}$$

and,

$$\frac{\partial T}{\partial t} = \nabla \cdot D_q(M,T)\nabla T \tag{2}$$

where D_q is the heat conduction coefficient, D_m is the moisture diffusion coefficient, M is moisture content, and T is temperature. In this approach the actual physical processes such as capillary forces are not individually considered. Instead a more macroscopic look is taken of the process and the diffusion coefficients are tuned through experiment. In this way, the diffusion coefficients result in relatively complex function of temperature and moisture content as well as experimentally obtained parameters. This also results in a much more empirical model which relies on many accurate experiments to be conducted rather then a robust theoretical model. This effectively causes the model to lose its flexibility to adapt to things such as different species, beetle infested wood, frozen wood and different drying conditions. No literature has been found where this model has been applied above the FSP. The complex nature of thick cylindrical timber segments in the conditioning style environment may not be sufficiently represented by a strictly diffusive model. Whitaker [2] notes that the diffusive models of wood drying are somewhat accurate below FSP only but above FSP becomes inaccurate.

3.2 Siau's Model

Onsager's theorem which is described by Luikov [3] as pertaining to coupled heat and mass transfer in gases, was adapted to wood by Siau [7]. The model of heat and mass transport potentials given here was derived by Luikov [3] to describe heat and mass transport in the mixing of a binary gas. The justification for applying this model to heat and mass transport to a porous material such as wood is not clear, and is therefore one of the reasons for not using this model. Although Siau's model will not be employed in this thesis project, it is useful as a guide and is worth mentioning. This approach is founded on the principles of irreversible thermodynamics by considering the increase of entropy of the system. The second law of thermodynamics can be stated in the form of Gibbs equation,

$$TdS = dU + PdV - \sum_{i=1}^{n} \mu_i dN \tag{3}$$

where dU is the infinitesimal increase in energy, P is pressure, dV is an infinitesimal increase in volume, $\sum_i^n \mu_i dN$ denotes the chemical potential of the ith type of particle, and dS is an infinitesimal increase in entropy. Using Onsager's fundamental equation which says that the product of the rate of increase of entropy and the absolute temperature is equal to the sum of the products of the potentials and corresponding thermodynamic forces. Gibbs equation (3) can therefore be written as,

$$T\frac{dS}{dt} = \sum_{i=1}^{n} \mathbf{j}_i X_i \tag{4}$$

where $\mathbf{j}_i$ are the potentials (*mass transport potential, thermal transport potential...*), X_i are the corresponding thermodynamic forces, and n is the number of individual thermodynamic forces involved in the process. By this definition, a set of coupled differential equations can be developed to describe the potentials associated with simultaneous heat and mass transport. The thermodynamic forces of heat and mass transfer are derived by Luikov [3] from equation (3) to be,

$$X_{heat} = -\frac{1}{T}\nabla T \qquad X_{moisture} = -T\nabla\left(\frac{\mu}{T}\right) \tag{5}$$

where, μ is is the chemical potential, and T is the temperature. The potentials associated with a coupled process such as simultaneous heat and mass transfer are determined by the action of all the corresponding thermodynamic forces.

$$\mathbf{j}_i = \sum_{k=1}^{n} L_{ik} X_i \qquad i = 1, 2, 3, ..., n \tag{6}$$

where the X_i's are the thermodynamic forces, $L'_{ik}s$ are the kinetic coefficients or coefficients of proportionality, and n is the number of individual simultaneous processes. This is known as Onsager's system of equations, and therefore the potentials associated with heat and mass transfer are given as,

$$\mathbf{j}_{q(heat)} = -L_{qq}\left(\frac{1}{T}\right)\nabla T - L_{qm} T\nabla\left(\frac{\mu}{T}\right) \tag{7}$$

$$\mathbf{j}_{m(moisture)} = -L_{mq}\left(\frac{1}{T}\right)\nabla T - L_{mm} T\nabla\left(\frac{\mu}{T}\right) \tag{8}$$

Where L_{mq}, L_{qm}, L_{mm}, and L_{qq} are kinetic coefficients. It this model heat transfer occurs by conduction, in the form of the L_{qq} term, and convection in the form of the L_{qm} term. Additionally, mass transfer occurs by way of molecular diffusion in the form of the L_{mm} term, and convective mass transfer in the form of the L_{mq} term. Although the main heat and mass mechanisms are considered, they are given in a relatively simplistic form, relying heavy on the coefficients. Although it is not clear the continuity equation,

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \mathbf{v} \tag{9}$$

which will be discussed in more detail in later sections, seems to be used in constructing a time-dependent form of the transport equations given above. The result is the following time-dependent transport equations,

$$\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{j}_q \tag{10}$$

$$\frac{\partial M}{\partial t} = -\nabla \cdot \mathbf{j}_m \tag{11}$$

therefore,

$$\frac{\partial T}{\partial t} = \nabla \cdot \left[L_{qq}\left(\frac{1}{T}\right)\nabla T - L_{qm}T\nabla\left(\frac{\mu}{T}\right)\right] \tag{12}$$

$$\frac{\partial M}{\partial t} = \nabla \cdot \left[L_{mq}\left(\frac{1}{T}\right)\nabla T - L_{mm}T\nabla\left(\frac{\mu}{T}\right)\right] \tag{13}$$

where M is the percent moisture content, given as the percentage of green mass to dry mass of the material. The above equations can also be manipulated into the following form,

$$\frac{\partial T}{\partial t} = \nabla \cdot \left[L_{qq}\left(\frac{1}{T}\right)\nabla T - L_{qm}\nabla M\right] \tag{14}$$

$$\frac{\partial M}{\partial t} = \nabla \cdot \left[L_{mq}\left(\frac{1}{T}\right)\nabla T - L_{mm}\nabla M\right] \tag{15}$$

This model does not account for capillary transport but since most wood drying studies are conducted for wood well below the fiber saturation point, capillary moisture movement is not a concern. This model was solved for a one dimensional case of wood drying by Avramidis et al.[4]. A combination of theoretical and empirical expressions were used as the coefficients of proportionality. Theoretical moisture content and core temperature were compared to experiment and matched reasonably well. It is interesting to note that if the Soret effect and Dufour effect are neglected here the result is the diffusive model described earlier.

3.3 Luikov's Model

The majority of work done in developing models of heat and moisture transport in wood has almost exclusively relied on the more empirical models mentioned earlier, such as the diffusive model and Siau's model. In the latter, and in some situations the former, experiment matched theory reasonably well. Additionally, models such as the diffusive model and those described by Siau [],[] have been applied to wood in a very intuitive and empirical way. The justification for utilizing these models for wood is not clear. One of the main problems in applying the previously discussed models is the lack of clear justification for them to be applied to wood. This study requires a more general approach to be applied. The approach to be taken here is to work from the more general theory of heat and mass transport in porous capillary media such as the ones described by Luikov [3], or Whitaker [2], to develop a model to suit the needs of the study.

Luikov [3] considers the distribution of moisture in capillary-porous bodies to be either in the form of non-condensible gases, vapour, liquid and possibly solid. The major difference in Luikov's model compared to the models previously discussed is that capillary forces are considered, and a differentiation between gases, vapour, liquids and solid is made. Luikov describes the two phenomena associated with the transport of gas/vapour and liquids through the porous media as *molecular transport* and *molar transport*. A brief discussion of each is given below.

3.3.1 Molecular Transport

Molecular transport is the movement of mass on the molecular level. An example of molecular transport is diffusion of one substance into another. Molecular diffusion can be thought of in a way analogous to thermal conduction. The random molecular motion of the molecules of the substances, in this case the wood and water, tend to mix with each other with no external force until equilibrium is achieved. Molecular diffusion can be described by Fick's law which is identical to the heat equation.

$$\begin{aligned} \mathbf{j}_m &= D_m \nabla M \qquad \text{(Fick's Law of Mass Transfer)} \\ \mathbf{j}q &= D_q \nabla T \qquad \text{(Fourier's Law of Heat Transfer)} \end{aligned}$$

Where, $\mathbf{j}_m$ is the moisture transport potential, $\mathbf{j}_q$ is the heat transport potential, M is moisture content, T is temperature, and $D_{m,q}$ are the coefficients of proportionality.

3.3.2 Molar Transport

Molar transport differs from molecule transport because in molar transport small parcels of fluid move throughout the material due to external forces. One form of molar transport is convective, where vapour and liquid is forced through some media under the influence of a pressure gradient. When this occurs vapour or liquid is essentially pulled or pushed through the material by the pressure differential. For instance, this is the driving force behind thermally induced mass transfer or the Soret effect in gases and vapours. The pressure gradient is a result of the thermal gradient introduced by external or internal heating of the object. Since pressure is directly proportional to temperature in a fixed volume, a pressure gradient results in vapour or gases being driven from high to low pressure or from the hot to the cool. Molar movement of liquid moisture may also be driven by gravitational forces.

Capillary transport is considered a molar process. Capillary forces are present only in the liquid phase. Capillary absorption occurs when the surface tension of a liquid is enough to hold it in place against gravity. Surface tension is a result of molecules or atoms at the surface of a liquid missing bonds to undergo hydrogen bonding with. As a result, the molecules at the surface bond more strongly to the neighboring surface molecules then the rest, and form somewhat of a membrane on the surface. For instance, in a tube containing liquid water and gas/vapour, the surface tension can act as a support arch and depending on the radius of the tube, to hold itself up against external forces such as gravity. Since tracheids are essentially long narrow tubes, capillary forces will tend to hold liquid water within them.

3.3.3 Continuity Equation

The framework of Luikov's model is derived from conservation principles. Consider a macroscopic volume V. The volume must be small enough to assume a constant distribution of the quantity being analyzed throughout the volume, but large enough to contain a very large number of molecules and satisfy the continuum hypothesis. For instance if we consider mass moving through the volume element, the change in mass over time is given by,

$$\left(\frac{\Delta\text{mass}}{\Delta\text{time}}\right)_V = \text{inflow - outflow} \tag{16}$$

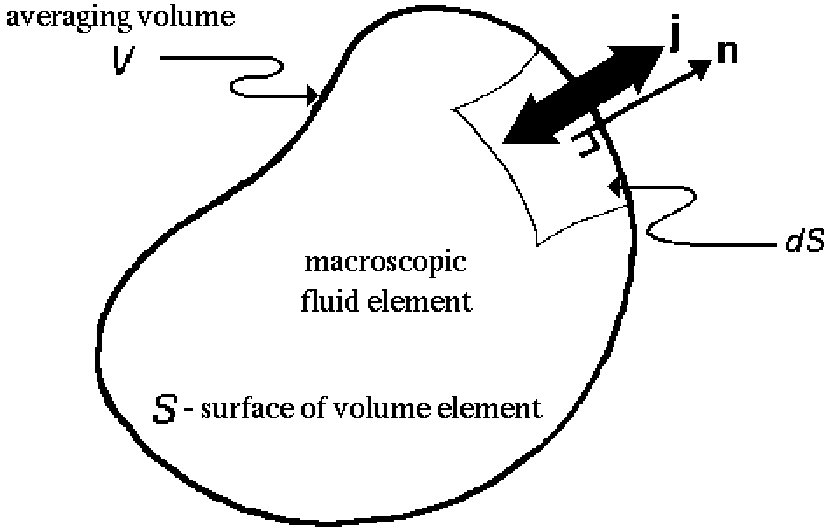

Figure 3: Averaging volume element V of a single component fluid.

This can be expressed formally in the following way. The total mass flux moving through the surface of the volume element V is given by the double integral,

$$\left(\frac{\partial m}{\partial t}\right)_S = \iint_S \mathbf{j}\cdot\mathbf{n}dS = \iint_S \rho\mathbf{v}\cdot\mathbf{n}dS \tag{17}$$

where $\mathbf{j} = \rho\mathbf{v}$ is the *mass flux density*, $\mathbf{n}$ is the outward unit vector normal to the surface element dS. $\mathbf{n}dS$ is taken along the outward normal by convention, such that mass flowing out of the volume is positive flow and mass flowing in is negative. The mass within the volume is given by the volume integral,

$$m(t) = \iiint_V \rho(x,y,z,t)\mathrm{d}V \tag{18}$$

Over time mass within the volume decreases at the rate,

$$\left(\frac{\partial m}{\partial t}\right)_V = -\frac{\partial}{\partial t}\iiint_V \rho\mathrm{d}V \tag{19}$$

This rate is negative since mass is moving out of the volume element. Since mass within the entire system must be conserved,

$$\frac{\mathrm{d}m}{\mathrm{d}t} = 0 \tag{20}$$

Then the mass flowing out of the surface must be equal the rate of mass lost within the volume, therefore, equation (17) and (19) must be equal, so,

$$\iint_S \rho \mathbf{v} \cdot \mathbf{n} dS = -\frac{\partial}{\partial t} \iiint_V \rho \mathrm{d}V \tag{21}$$

By the divergence theorem the surface integral can be re-written as a volume integral,

$$\iiint_V \nabla \cdot \rho \mathbf{v} \mathrm{d}V + \iiint_V \frac{\partial \rho}{\partial t} \mathrm{d}V = 0 \tag{22}$$

therefore,

$$\iiint_V \left(\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} \right) \mathrm{d}V = 0 \tag{23}$$

Since this must hold for any constant volume element then the volume integral can be dropped.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0 \tag{24}$$

This is referred to as the conservation of mass or continuity equation, and is the basis of Luikov's theory. Additionally, the continuity equation can be expanded to the transport of other conserved quantities such as conservation of internal energy, which will be examined later.

3.3.4 Multiple Component Mass Transport Equation

The derivation of the continuity equation is now applied to mass transport in a multi-component system such as a capillary-porous body. Consider a fixed frame in which a stationary porous media is at rest. The *bound substance* refers to all the mass that is moving with respect to the fixed frame. The *dry body* refers to the non-moving portion of the mass with respect to a fixed frame. The components of the system will be differentiated by the subscript i. The mixture composing the bound substance will be defined as, 1-vapour, 2-liquid, 3-solid (ice), and 4-inert gas, and the dry body will be denoted as 0.

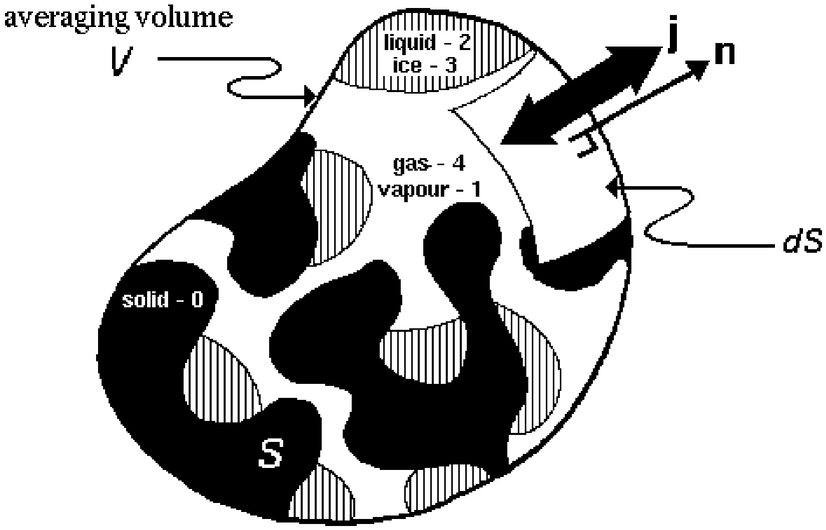

Figure 4: Averaging volume element V of a porous-capillary material.

For a single component fluid, the density or concentration, ρ is defined as the instantaneous mass m of some macroscopic element of fluid within a small enclosing volume V,

$$\rho = \frac{m}{V} \tag{25}$$

For the multi-component system, ρ can be defined as the sum of the densities of the each of the components of the mixture.

$$\rho = \sum_i \rho_i = \sum_i \frac{m_i}{V} \tag{26}$$

where m_i is the instantaneous mass of the i component in the volume element V, and ρ_i is the density or concentration of the i component of the mixture. In the case of mass transport in a porous-capillary body, the definition of density given by ρ_i will be insufficient for the final form of equation (26), but it will be left as is for now in an effort to keep the derivation less cluttered. Additionally, the velocity of the multi-component mixture $\mathbf{v}$ expands to,

$$\mathbf{v} = \frac{1}{\rho} \sum_i \rho_i \mathbf{v}_i \tag{27}$$

therefore, equation (24) becomes,

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \rho_i \mathbf{v_i} = I_i \tag{28}$$

where the added term I_i represents a source or sink as a result of chemical reactions and/or phase changes of the i component. Since mass must be conserved in the system during chemical and/or phase changes, it is also useful to note,

$$\sum_{i=1}^{4} I_i = 0 \tag{29}$$

The velocity of the mixture, $\mathbf{v}$ will now be examined. The process of diffusion implies that a larger number of molecules is passing in the direction of the diffusion current than in the opposite direction. The result is an net displacement of one substance into another at some conventional linear rate $\mathbf{v}_{diff}$. The diffusion velocity is not a velocity in the ordinary sense of the word, but it can be described as the difference between the molar velocity of the i component and the overall velocity of the mixture relative to the center of mass,

$$\mathbf{v}_{diff} = \mathbf{v}_i - \mathbf{v} \tag{30}$$

therefore the mass flux moving through the volume as a result of diffusion (molecular transport) of the individual components is given by,

$$\mathbf{j}_{iM} = \rho_i(\mathbf{v}_i - \mathbf{v}) \tag{31}$$

where the subscript M is to denote molecular transport. The $\rho_i \mathbf{v_i}$ term in equation (28) can therefore be given as,

$$\rho_i \mathbf{v}_i = \rho_i \mathbf{v} + \mathbf{j}_{iM} \tag{32}$$

as a result equation (28) becomes,

$$\frac{\partial \rho_i}{\partial t} = -\nabla \cdot (\rho_i \mathbf{v} + \mathbf{j}_{iM}) + I_i \tag{33}$$

where j_{iM} is the molecular transport flux and the $\rho_i \mathbf{v}$ term represents the molar flux.

As mentioned earlier the current definition of density,

$$\rho_i = \frac{m_i}{V} \tag{34}$$

is insufficient to describe molar transport in porous bodies, therefore a more suitable definition of density is required. The reason it is unsuitable is because if we consider a small volume element V of a capillary-porous body, the volume being occupied by component i is smaller then the volume element V itself. The result is that each component will have a different sized volume element associated with it within the larger enclosing volume element V. The reason for this is the pore volume may not be totally filled with the component being considered and the wood structure is taking up part of the volume within the enclosing volume element. The problem with this approach is that equation (28) is no longer valid if the volume element being considered in equation (18) is not the same for each component i. This problem can be addressed by *normalizing* the density of each component over the enclosing volume element V. This can be accomplished by multiplying the density ρ_i, by some factor which depends on the pore volume and the fill fraction of the pores, the result is know as the *volumetric concentration*,

$$\omega_i = \rho_i \Pi b_i = \frac{m_i}{V} \Pi b_i \tag{35}$$

where b_i is the pore fill fraction, Π is the density of the dry body, and V is the enclosing volume element used in the derivation of equation (28). For example, on average, the pores may be filled with three quarters liquid water and one quarter inert gas/vapour mix, in this case, $b_{\text{water}} = \frac{3}{4}$ and $b_{\text{inert gas/vapour}} = \frac{1}{4}$. The fill fraction, b_i will can be approximated for each component through thermodynamic arguments. It is reasonable to say that the pore volume Π is a function of porosity of the dry body, the porosity of most wood species is know, and therefore this value will be easily obtainable. By using this definition for the density or concentration, the derivation of equation (28) still holds true since the volume element being considered stays constant.

One more clarification is needed here before moving on. Since the molar transport must occur by filtering through the pores, then the pore fill fraction, b_i in the molar transport term must be equal to one, therefore,

$$\omega_{iK} = \rho_i \Pi \tag{36}$$

So equation (33) becomes,

$$\frac{\partial \omega_i}{\partial t} = -\nabla \cdot (\omega_{iK} \mathbf{v} + \mathbf{j}_{iM}) + I_i \tag{37}$$

In order to make the quantities in the model comparable to experimentally obtainable quantities, a few more definitions must be given,

$$\gamma_0 = \frac{m_0}{V_0} \tag{38}$$

which is the dry body density, where m_0 is the mass of the dry body, and the volume V_0 of the dry body. Now let,

$$u_i = \frac{m_i}{m_o} \tag{39}$$

which is the ratio of the dry body to the bound substance. Then the volumetric concentration can be written as,

$$\omega_i = \gamma_0 u_i = \gamma_0 \frac{m_i}{m_o} \tag{40}$$

and if we denote the molar transport term as a flux $\mathbf{j}_{iK}$, then equation (37) becomes,

$$\frac{\partial(\gamma_0 u_i)}{\partial t} = -\nabla \cdot (\mathbf{j}_{iK} + \mathbf{j}_{iM}) + I_i \tag{41}$$

The result is the mass transport equation for each component of the bound substance without the gradients of the appropriate potentials. Luikov reasons that both the molar and molecular fluxes are in the same direction therefore,

$$\mathbf{j}_i = \mathbf{j}_{iK} + \mathbf{j}_{iM} \tag{42}$$

and as a result,

$$\frac{\partial(\gamma_0 u_i)}{\partial t} = -\nabla \cdot \mathbf{j}_i + I_i \tag{43}$$

We can then conclude that since the sum of the source and sink terms of the bound substance is zero, then the total mass transport equation for the bound substance is,

$$\frac{\partial(\gamma_0 \bar{u})}{\partial t} = -\nabla \cdot \sum_{i=4}^{4} \mathbf{j}_i \tag{44}$$

3.3.5 Heat Transport Equation

The heat transport equation is based on the equation of transfer of internal energy. At constant pressure and assuming the framework of the dry body is of constant volume, the time derivative of the volumetric concentration of enthalpy of the system is equal to the divergence of the flow of enthalpy, therefore,

$$\frac{\partial}{\partial t}(h_0\gamma_0 + \sum_{i=4}^{4} h_i\gamma_0 u_i) = -\nabla \cdot (\mathbf{j}_q + \sum_{i=1}^{4} h_i\mathbf{j}_i) \tag{45}$$

where h_i is the specific enthalpy of the bound substance, h_0 is the specific enthalpy of the dry body, and γ_0 is the density of the dry body. The isobaric specific heat,

$$c_i = \left(\frac{dh_i}{dT}\right)_p \tag{46}$$

is substituted into equation (45) then,

$$(c_o\gamma_0 + \sum_{i=1}^{4} c_i\gamma_0 u_i)\frac{\partial T}{\partial t} + \sum_{i=1}^{4} h_i \frac{\partial(\gamma_0 u_i)}{\partial t} = -\nabla\cdot\mathbf{j}_q - \sum_{i=1}^{4}\mathbf{j}_i c_i \nabla T - \sum_{i=1}^{4} h_i \nabla\cdot\mathbf{j}_i \tag{47}$$

Now if equation (43) is multiplied by the enthalpy h_i and summing for i from 1 to 4,

$$\sum_{i=1}^{4} h_i \frac{\partial(\gamma u_i)}{\partial t} = -\sum_{i=1}^{4} h_i(\nabla \cdot \mathbf{j}_i) + \sum_{i=1}^{4} h_i I_i \tag{48}$$

Now this can be substituted into equation (47) resulting in,

$$c\gamma_0 \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{j}_q - \sum_{i=1}^{3} h_i I_i - \sum_{i=1}^{4} \mathbf{j}_i c_i \nabla T \tag{49}$$

where the specific heat of the bound substance is,

$$c = \sum_{i=1}^{4} c_i u_i \tag{50}$$

The third term in equation (49) is a convective term. Luikov reasons that in a capillary-porous body where filtration motion of the liquid is absent, the convective heat transport component is small and therefore the third term of equation (49) can be dropped.

3.3.6 Simplified Transport Equations

Some simplifications can made based on some approximations. Since there is no molar or molecular transport occurring in the solid state then, the flux $\mathbf{j}_3 = 0$, therefore the mass transport equation for the solid state would be,

$$\frac{\partial(\gamma u_3)}{\partial t} = I_3 \tag{51}$$

If the barometric pressure is equal to that of the humid air, then the mass of air (inert gases) and vapour are negligible, and therefore the following approximation can be made,

$$u = u_2 + u_3 \qquad \text{and} \qquad u_1 = u_4 \approx 0 \tag{52}$$

In the case where the ice phase is not present ($T > 0$), then a further simplification can be made, $u_3 = 0$ and therefore $u = u_2$. In our case sub-zero temperatures will be considered therefore keeping the ice phase term is important. As a result the following is true,

$$\frac{\partial(\gamma u_1)}{\partial t} = -\nabla \cdot \mathbf{j}_1 + I_1 = 0 \tag{53}$$

and,

$$\nabla \cdot \mathbf{j}_1 = I_1 \tag{54}$$

It is also safe to assume that the inert gas will not undergo any phase changes or chemical reactions, as a result $I_4 = 0$ therefore,

$$\nabla \cdot \mathbf{j}_4 = 0 \tag{55}$$

Now since the only phase change that the vapour component can undergo is vapour to liquid, then $I_2 = I_1$, hence,

$$I_2 = -I_1 = -\nabla \cdot \mathbf{j}_1 \tag{56}$$

therefore,

$$I_2 = -\nabla \cdot \mathbf{j}_1 \tag{57}$$

and the transport equation for the liquid state can be written as,

$$\frac{\partial(\gamma u_2)}{\partial t} = -\nabla \cdot \mathbf{j}_2 + I_2 = \nabla \cdot (\mathbf{j}_1 + \mathbf{j}_2) \tag{58}$$

The equations describing heat and mass transport in capillary-porous bodies can be stated without the gradients of the appropriate potentials.

$$c\gamma_0 \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{j}_q - \sum_{i=1}^{3} h_i I_i \tag{59}$$

$$\frac{\partial(\gamma u_2)}{\partial t} = -\nabla \cdot (\mathbf{j}_1 + \mathbf{j}_2) \tag{60}$$

$$\frac{\partial(\gamma u_3)}{\partial t} = I_3 \tag{61}$$

3.3.7 Heat and Mass Transport Potentials

The derivations of the heat and mass transport potentials for capillary-porous bodies are summarized by Luikov. The transport potentials given by Luikov are for capillary-porous bodies in general, but may differ slightly for wood. For this this reason the discussion of the potentials will be kept brief, and part of the focus of the thesis research will be to understand and tailor the transport potentials to wood. Luikov concludes that the vapour flow potential depends on temperature and moisture gradients in the following way,

$$\mathbf{j}_1 = a_{m1}\gamma_0[\nabla u + \delta_1 \nabla T] \tag{62}$$

where a_{m1} is the coefficient of vapour transport, δ_1 is the thermal gradient coefficient for vapour transport, and $u = u_2 + u_3$ or total moisture content. Luikov proposes the liquid flow potential as a result of capillary transport as,

$$\mathbf{j}_2 = k\nabla\psi \tag{63}$$

where ψ is the capillary potential, and k is the capillary conductance. It can be shown that the resulting gradient of liquid moisture due to capillary transport is given by,

$$\mathbf{j}_2 = a_{m2}\gamma_0[\nabla u + \delta_2 \nabla T] \tag{64}$$

where a_{m2} is the liquid diffusion coefficient in the body, and δ_2 is the thermal gradient coefficient. Theoretical models of the coefficients, a_{m1}, a_{m2}, δ_1, and δ_2 are given by Luikov, but further study is required to justify the applicability to wood. Luikov reasons that since both vapour and liquid transport takes place under the influence of the same two gradients, ∇u and ∇T, then they can be combined,

$$\mathbf{j}_m = \mathbf{j}_1 + \mathbf{j}_2 = -a_m\gamma_0[\nabla u + \delta \nabla T] \tag{65}$$

where the coefficients are equal to,

$$a_m = a_{m1} + a_{m2} \tag{66}$$

$$\delta = \frac{a_{m1}\delta_1 + a_{m2}\delta_2}{a_{m1} + a_{m2}} \tag{67}$$

Conductive heat transport in capillary-porous bodies is given by Fourier's law of heat conduction,

$$j_q = -\lambda \nabla T \tag{68}$$

where λ is the total coefficient of thermal conductivity of the entire material.

3.3.8 Final Transport Equations

Now if all the transport potentials are substituted into equations (59 - 61), then the heat and mass transport equations, without ice being present, become,

$$\frac{\partial u}{\partial t} = \nabla \cdot [a_m \nabla u + \delta a_m \nabla T] \tag{69}$$

$$c\gamma_0 \frac{\partial T}{\partial t} = \nabla \cdot \lambda \nabla T + r_{12} \nabla \cdot [a_{m1}\gamma_0(\nabla u + \delta_1 \nabla T)] \tag{70}$$

where $r_{12} = h_1 - h_2$ is the latent heat of evaporation. As mentioned earlier, a more in depth study of the transport potentials will be required before they can be applied to wood. Some of the concerns with this model is the ability to collect the required experimental coefficients used in the transport equations.

3.4 Whitaker's Model

In the review by Whitaker [2], previous models such as the diffusive models and Siau's model, are concluded to be unsatisfactory in accurately describing heat and mass transport in wood below FSP and even less accurate above FSP. Whitaker mentions that diffusive models are only accurate in late stages of drying, as in the study done by Avramidis [4], when capillary forces can be safely ignored. As mentioned earlier capillary transport cannot be ignored in this case and therefore Whitaker's model will need to be considered.

Whitaker gives a rigorous derivation of a model of heat, mass and momentum transport in porous media from first principals. The approach is similar to Luikovs in that it is based on the continuity equation and what Whitaker calls the ith species continuity equation.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0 \tag{71}$$

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \rho_i \mathbf{v_i} + \Phi_i = 0 \tag{72}$$

as well as the thermal energy equation,

$$\rho \frac{Dh}{Dt} = -\nabla \cdot \mathbf{q} + \Phi \tag{73}$$

where it is assumed that compressional work and viscous dissipation are negligible in all phases, and Φ is the sink source term. Another difference is that Whitaker does not consider the ice phase in the derivation of the transport equations, although the addition of the ice term is possible. In the derivation, Whitaker denotes the gas/vapour phase as γ, the liquid phase as β and the solid framework or dry body as σ. One of the biggest differences between the two models is that Luikov uses the idea of *volumetric concentration* and uses empirical arguments to justify it. Whitaker handles this using an averaging approach. Whitaker derives the continuity equation, species continuity equation, and thermal energy equation for each phase. The result is a set of point equations describing the three separate phases separately. The individual equations can then be volume averaged resulting in a set volume averaged equations valid anywhere throughout the porous body.

3.4.1 Solid Phase

Whitaker's approach will be summarized starting with the development of a set of point equations for each phase. We can start with the solid phase, or fixed rigid matrix denoted by the σ subscript. Since the solid phase is fixed, then we know that $v_\sigma = 0$, therefore equations (72) and (72) inconsequential, and equation (73) simplifies to,

$$\rho_\sigma \left(\frac{\partial h_\sigma}{\partial t} \right) = -\nabla \cdot \mathbf{q}_\sigma + \Phi_\sigma \tag{74}$$

A few reasonable assumptions can be made for all three phases. The first is that enthalpy given by h is independent of pressure and all heat capacities are constant. From this we can conclude that the enthalpy can be replaced by c_pT plus some arbitrary constant,

$$h = h(T) = c_p T + constant \tag{75}$$

therefore equation (74) can be written as,

$$\rho_\sigma (c_p)_\sigma \left(\frac{\partial T_\sigma}{\partial t} \right) = -\nabla \cdot \mathbf{q}_\sigma + \Phi_\sigma \tag{76}$$

This can be taken one step further by applying Fourier's law,

$$\rho_\sigma (c_p)_\sigma \left(\frac{\partial T_\sigma}{\partial t} \right) = -k_\sigma \nabla^2 T_\sigma + \Phi_\sigma \tag{77}$$

where k_σ is the thermal conductivity coefficient of the fixed solid matrix. It is interesting to note that the thermal conductivity coefficient is constant, unlike previous models.

3.4.2 Liquid Phase

The point equations describing the liquid phase are derived in a similar fashion. The continuity equation in the liquid phase would be,

$$\frac{\partial \rho_\beta}{\partial t} + \nabla \cdot (\rho_\beta \mathbf{v}_\beta) = 0 \tag{78}$$

and the thermal equation takes on the form,

$$\rho_\beta \frac{D h_\beta}{D t} = -\nabla \cdot \mathbf{q}_\beta + \Phi_\beta \tag{79}$$

The material derivative can be expanded to yield,

$$\rho_\beta \left(\frac{\partial h_\beta}{\partial t} + \mathbf{v}_\beta \cdot \nabla h_\beta \right) = -\nabla \cdot \mathbf{q}_\beta + \Phi_\beta \tag{80}$$

and making use of (75) and Fourier's law,

$$\rho_\beta (c_p)_\beta \left(\frac{\partial T_\beta}{\partial t} + \mathbf{v}_\beta \cdot \nabla T_\beta \right) = -k_\beta \nabla^2 T_\sigma + \Phi_\beta \tag{81}$$

3.4.3 Vapour/Inert Gas Phase

The gas phase is slightly more complex. The complexity lies in the fact that the gas is a mixture of vapour and any number of other gases. This can be dealt with using the species continuity equation as well as the regular continuity equation for the entire mixture along with some sums in the thermal energy equation. The continuity equation for the entire vapour/inert gas mixture is,

$$\frac{\partial \rho_\gamma}{\partial t} + \nabla \cdot (\rho_\gamma \mathbf{v}_\gamma) = 0 \tag{82}$$

and the species continuity equation is written as,

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = 0 \tag{83}$$

where the sink source term Φ is zero since we can assume there will be no chemical reactions between the vapour and the gases in the mixture. The density ρ_γ and velocity $\mathbf{v}_\gamma$ are defined identical to equation (26) and (27) in Luikov's model, therefore,

$$\rho_\gamma = \sum_{i=1}^{i=N} \rho_i \tag{84}$$

and,

$$\rho_\gamma \mathbf{v}_\gamma = \sum_{i=1}^{i=N} \rho_i \mathbf{v}_i \tag{85}$$

Now the species velocity $\mathbf{v}_i$ can be written in terms of the mass average velocity $\mathbf{v}_\gamma$ and the diffusion velocity $\mathbf{u}_i$, such as in equation (30),

$$\mathbf{v}_i = \mathbf{v}_\gamma + \mathbf{u}_i \tag{86}$$

Then equation (83) can be split up into a *molar* and *molecular* flux terms similar to equation (33) in Luikov's model, therefore,

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_\gamma) = -\nabla \cdot (\rho_i \mathbf{u_i}) \tag{87}$$

Whitaker expresses the diffusive flux as,

$$\rho_i \mathbf{u}_i = -\rho_\gamma \mathcal{D} \nabla \left(\frac{\rho_i}{\rho_\gamma} \right) \tag{88}$$

where $\mathcal{D}$ is the gas diffusion coefficient. The diffusive flux can now be substituted in equation (87) leading to,

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_\gamma) = \nabla \cdot \left[\rho_\gamma \mathcal{D} \nabla \left(\frac{\rho_i}{\rho_\gamma}\right)\right] \tag{89}$$

The thermal energy equation for a multicomponent vapour/inert gas phase, neglecting compressional work and viscous dissipation, is given by Whitaker as,

$$\frac{\partial}{\partial t}\left(\sum_{i=1}^{i=N} \rho_i \bar{h}_i\right) + \nabla \cdot \left(\sum_{i=1}^{i=N} \rho_i \mathbf{v}_i \bar{h}_i\right) = -\nabla \cdot \mathbf{q}_\gamma + \Phi_\gamma + \sum_{i=1}^{i=N} \rho_i \mathbf{u}_i \cdot \mathbf{f}_i - \rho_\gamma \frac{D}{Dt} \sum_{i-1}^{i=N} \frac{1}{2} \frac{\rho_i}{\rho_\gamma} u_i^2 \tag{90}$$

Whitaker assumes that the last two terms correspond to the diffusive body force rate of work and the time rate of change of the diffusive kinetic energy respectively and can be safely neglected, resulting in the simplified thermal heat equation for the vapour/inert gas phase,

$$\frac{\partial}{\partial t}\left(\sum_{i=1}^{i=N} \rho_i \bar{h}_i\right) + \nabla \cdot \left(\sum_{i=1}^{i=N} \rho_i \mathbf{v}_i \bar{h}_i\right) = -\nabla \cdot \mathbf{q}_\gamma + \Phi_\gamma \tag{91}$$

where $\bar{h}_i$ is the partial mass enthalpy and can be interpreted in a way similar to equation (75).

$$\bar{h}_i = \bar{h}_i(T) = (\bar{c}_p)_i T \tag{92}$$

using the definition of $\rho_\gamma h_\gamma$,

$$\rho_\gamma h_\gamma = \sum_{i=1}^{i=N} \rho_i \bar{h}_i \tag{93}$$

along with equation (83), (86), and (93) the thermal energy equation becomes,

$$\rho_\gamma \left(\frac{\partial h_\gamma}{\partial t} + \mathbf{v}_\gamma \cdot \nabla h_\gamma\right) = -\nabla \cdot \mathbf{q}_\gamma - \nabla \cdot \left(\sum_{i=1}^{i=N} \rho_i \mathbf{u}_i \bar{h}_i\right) + \Phi_\gamma \tag{94}$$

and finally applying Fourier's law and equation (92),

$$\rho_\gamma (c_p)_\gamma \left(\frac{\partial T_\gamma}{\partial t} + \mathbf{v}_\gamma \cdot \nabla T_\gamma\right) = k_\gamma \nabla^2 T - \nabla \cdot \left(\sum_{i=1}^{i=N} \rho_i \mathbf{u}_i \bar{h}_i\right) + \Phi_\gamma \tag{95}$$

3.4.4 Boundary Conditions

There are three interfaces areas within the porous body. The three interfaces are the solid-liquid, solid-vapour/inert gas, and liquid-vapour/inert gas. The surface areas of the interfaces will be denoted in the following way,

$$\begin{aligned}&A_{\sigma\beta} \text{ - solid-liquid interface area}\\&A_{\sigma\gamma} \text{ - solid-vapour/inert gas interface area}\\&A_{\beta\gamma} \text{ - liquid-vapour/inert gas interface area}\end{aligned} \tag{96}$$

and it follows that,

$$A_{\sigma\beta} = A_{\beta\sigma}, \qquad A_{\sigma\gamma} = A_{\gamma\sigma}, \qquad A_{\beta\gamma} = A_{\gamma\beta} \tag{97}$$

For the solid-liquid interface, the following boundary conditions apply,

$$\begin{aligned}&\mathbf{v}_\beta = 0 \quad \text{on} \quad A_{\sigma\beta}\\&\mathbf{q}_\sigma \cdot \mathbf{n}_{\sigma\beta} + \mathbf{q}_\beta \cdot \mathbf{n}_{\beta\sigma} = 0 \quad \text{where} \quad \mathbf{n}_{\sigma\beta} = -\mathbf{n}_{\beta\sigma} \quad \text{on} \quad A_{\sigma\beta}\\&T_\sigma = T_\beta \quad \text{on} \quad A_{\sigma\beta}\end{aligned} \tag{98}$$

where $\mathbf{n}$ is an outward normal vector to the surface of the particular interface. This first boundary condition, $\mathbf{v}_\beta = 0$, assumes that the velocity of the liquid at the interface is zero. The second statement says the thermal flux from the solid to the liquid and the liquid to the solid are equal and opposite. The last statement says that the liquid and solid at the interface surface are in thermal equilibrium.

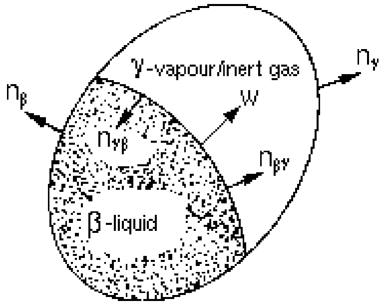

Figure 5: liquid-vapour/inert gas interface.

The solid-vapour/gas interface is identical to the solid-liquid interface,

$$\begin{aligned}
&\mathbf{v}_\gamma = 0 \quad \text{on} \quad A_{\sigma\gamma}\\
&\mathbf{q}_\sigma \cdot \mathbf{n}_{\sigma\gamma} + \mathbf{q}_\gamma \cdot \mathbf{n}_{\gamma\sigma} = 0 \quad \text{where} \quad \mathbf{n}_{\sigma\gamma} = -\mathbf{n}_{\gamma\sigma} \quad \text{on} \quad A_{\sigma\gamma}\\
&T_\sigma = T_\gamma \quad \text{on} \quad A_{\sigma\gamma}
\end{aligned} \tag{99}$$

The liquid-vapour/gas interface becomes more complicated because the interface $A_{\beta\gamma}$ is a moving singular surface. We can start by considering a *material volume* element $V_m(t)$ such as in figure 5. The volume element $V_m(t)$ contains both liquid and gas phases separated by the singular surface $A_{\beta\gamma} = A_{\gamma\beta}$. The velocity of the singular surface is denoted by w. The integral representation of the general thermal energy equation applies to any material volume regardless if it contains a singular surface where ρh or $\mathbf{q}$ suffer jump discontinuities, The volume averaged thermal energy equation over the material element $V_m(t)$ is,

$$\frac{D}{Dt}\iiint_{V_m(t)}(\rho h)dV = -\iint_{A_m(t)}\mathbf{q}\cdot\mathbf{n}dA + \iiint_{V_m(t)}\Phi dV \tag{100}$$

Now in order to accurately represent the liquid-vapour/inert gas interface equations (81) and (91) must be volume averaged over V_β and V_γ respectively before they can be combined to represent the entire material volume $V_m(t)$. By volume averaging both equations (81) and (91) any jump discontinuities at the singular surface in ρh or $\mathbf{q}$ can be represented without a problem since the derivatives move outside the the volume integrals.. Integrating the liquid phase thermal energy equation over the liquid volume $V_\beta(t)$ results in,

$$\begin{aligned}
\iiint_{V_\beta(t)}\frac{\partial}{\partial t}(\rho_\beta h_\gamma)dV + \iiint_{V_\beta(t)}\nabla\cdot(\rho_\beta h_\beta \mathbf{v}_\beta)dV = \\
-\iiint_{V_\beta(t)}\nabla\cdot\mathbf{q}_\beta dV + \iiint_{V_\beta(t)}\Phi_\beta dV
\end{aligned} \tag{101}$$

applying the general transport theorem and some algebra becomes,

$$\begin{aligned}
\frac{d}{dt}\iiint_{V_\beta(t)}(\rho_\beta h_\beta)dV + \iint_{A_{\beta\gamma}}\rho_\beta h_\beta(\mathbf{v}_\beta - \mathbf{w})\cdot\mathbf{n}_{\beta\gamma}dA = \\
-\iint_{A_\beta}\mathbf{q}_\beta\cdot\mathbf{n}_\beta dA - \\
-\iint_{A_{\beta\gamma}}\mathbf{q}_\beta\cdot\mathbf{n}_{\beta\gamma}dA + \iiint_{V_\beta(t)}\Phi_\beta dV
\end{aligned} \tag{102}$$

The vapour/inert gas thermal energy equation becomes,

$$\frac{d}{dt}\iiint_{V_\gamma(t)}(\rho_\gamma h_\gamma)dV + \iint_{A_{\gamma\beta}} \rho_\gamma h_\gamma(\mathbf{v}_\gamma - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta}dA = \\ -\iint_{A_\gamma}\left(\mathbf{q}_\gamma + \sum_{i=1}^{i=N}\rho_i\mathbf{u}_i\bar{h}_i\right)\cdot\mathbf{n}_\gamma dA - \\ -\iint_{A_{\gamma\beta}}\left(\mathbf{q}_\gamma + \sum_{i=1}^{i=N}\rho_i\mathbf{u}_i\bar{h}_i\right)\cdot\mathbf{n}_{\gamma\beta}dA + \iiint_{V_\gamma(t)}\Phi_\gamma dV \tag{103}$$

In adding equations (102) and (103) it is useful to note that,

$$\frac{D}{Dt}\iiint_{V_m(t)}\rho h dV = \frac{d}{dt}\iiint_{V_\beta(t)}\rho_\beta h_\beta dV + \frac{d}{dt}\iiint_{V_\gamma(t)}\rho_\gamma h_\gamma dV \tag{104}$$

and since $A_m(t) = A_\beta + A_\gamma$ and $A_\beta = A_\gamma$ then,

$$\iint_{A_m(t)}(\mathbf{q}\cdot\mathbf{n})dA = \iint_{A_\beta}(\mathbf{q}_\beta\cdot\mathbf{n}_\beta)dA + \iint_{A_\gamma}\mathbf{q}_\gamma\cdot\mathbf{n}_\gamma dA \tag{105}$$

Now we can add equations (102) and (103) and apply equations (104) and (105), resulting in,

$$\frac{D}{Dt}\iiint_{V_m(t)}(\rho h)dV + \iint_{A_{\beta\gamma}}[\rho_\beta h_\beta(\mathbf{v}_\beta - \mathbf{w})\cdot\mathbf{n}_{\beta\gamma} + \rho_\gamma h_\gamma(\mathbf{v}_\gamma - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta}]dA = \\ - \iint_{A_m(t)}(\mathbf{q}\cdot\mathbf{n})dA + \iiint_{V_m(t)}\Phi dV \\ - \iint_{A_{\beta\gamma}}\left[\mathbf{q}_\beta\cdot\mathbf{n}_{\beta\gamma} + \left(\mathbf{q}_\gamma + \sum_{i=1}^{i=N}\rho_i\mathbf{u}_i\bar{h}_i\right)\cdot\mathbf{n}_{\gamma\beta}\right]dA \tag{106}$$

Then applying equation (100), the thermal energy jump condition at the $\beta - \gamma$ interface reduces to,

$$[\rho_\beta h_\beta(\mathbf{v}_\beta - \mathbf{w})\cdot\mathbf{n}_{\beta\gamma} + \rho_\gamma h_\gamma(\mathbf{v}_\gamma - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta}] = \\ -\left[\mathbf{q}_\beta\cdot\mathbf{n}_{\beta\gamma} + \left(\mathbf{q}_\gamma + \sum_{i=1}^{i=N}\rho_i\mathbf{u}_i\bar{h}_i\right)\cdot\mathbf{n}_{\gamma\beta}\right] \tag{107}$$

Additionally, the mass average jump condition can be shown to be,

$$\rho_\beta(\mathbf{v}_\beta - \mathbf{w})\cdot\mathbf{n}_{\beta\gamma} + \rho_\gamma(\mathbf{v}_\gamma - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta} = 0 \tag{108}$$

The tangential component of the mass average velocities is considered to be continuous, therefore,

$$\mathbf{v}_\beta \cdot \lambda_{\beta\gamma} = \mathbf{v}_\gamma \cdot \lambda_{\beta\gamma} \tag{109}$$

where $\lambda_{\beta\gamma}$ is any tangent vector to the surface $A_{\beta\gamma}$. The species jump condition can be derived resulting in,

$$\begin{aligned} \rho_1(\mathbf{v}_1 - \mathbf{w}) \cdot \mathbf{n}_{\gamma\beta} + \rho_\beta(\mathbf{v}_\beta - \mathbf{w}) \cdot \mathbf{n}_{\beta\gamma} &= 0 \\ \rho_i(\mathbf{v}_i - \mathbf{w}) \cdot \mathbf{n}_{\gamma\beta} &= 0 \text{ for } i = 2, 3, ... \end{aligned} \tag{110}$$

3.4.5 Volume Average

The next step is to develop a set of equations which apply to all points within the volume element V, such as in figure 4. For instance, the temperature of the solid phase averaged over some small volume element V would be denoted by Whitaker as,

$$\langle T_\sigma \rangle = \frac{1}{V} \iiint_V T_\sigma dV \tag{111}$$

where T_σ is only defined in the σ phase and is zero in all other phases. This is called the *phase average* by Whitaker. The *phase average* is essentially the same idea as the *volumetric concentration* proposed by Luikov. It is useful to note that the phase average doesn't reflect the true temperature of the solid, it is the temperature of a particular phase averaged over the volume V instead of the actual volume occupied by that particular phase. A more accurate representation of the temperature of, for instance, the solid would be, what Whitaker calls, the *intrinsic phase average* defined as,

$$\langle T_\sigma \rangle^\sigma = \frac{1}{V_\sigma} \iiint_V T_\sigma dV \tag{112}$$

A useful quantity referred to as the *volume fraction* would be the following for each of the three phases,

$$\epsilon_\sigma = \frac{V_\sigma}{V}, \qquad \epsilon_\beta = \frac{V_\beta}{V}, \qquad \epsilon_\gamma = \frac{V_\gamma}{V} \tag{113}$$

then it follows that,

$$\epsilon_\sigma = \epsilon_\beta(t) = \epsilon_\gamma(t) = 1 \tag{114}$$

and the *phase average* and *intrinsic phase average* are related by,

$$\epsilon_\sigma \langle T_\sigma \rangle^\sigma = \langle T_\sigma \rangle \tag{115}$$

The process of volume averaging all the point equations can start with the thermal energy equation for the solid phase. The thermal energy equation for the solid phase can be volume averaged by integrating over V_σ and dividing by the total volume V,

$$\frac{1}{V}\iiint_{V_\sigma}\frac{\partial}{\partial t}(\rho_\sigma h_\sigma)dV = -\frac{1}{V}\iiint_{V_\sigma}\nabla\cdot\mathbf{q}_\sigma dV + \frac{1}{V}\iiint_{V_\sigma}\Phi_\sigma dV \tag{116}$$

and therefore becomes,

$$\frac{\partial}{\partial t}\langle\rho_\sigma h_\sigma\rangle = -\langle\nabla\cdot\mathbf{q}_\sigma\rangle + \langle\Phi_\sigma\rangle \tag{117}$$

Now, what Whitaker refers to as the *averaging theorem*, can be applied. The averaging theorem can be understood by considering the arbitrary function ψ. The volume average of the divergence of the arbitrary function ψ can be expanded using the so called *averaging theorem* in the following fashion,

$$\langle\nabla\cdot\psi_\beta\rangle = \nabla\cdot\langle\psi_\beta\rangle + \frac{1}{V}\iint_{A_{\beta\sigma}}\psi_\beta\cdot\mathbf{n}_{\beta\sigma}\mathrm{d}A + \frac{1}{V}\iint_{A_{\beta\gamma}}\mathbf{q}_\beta\cdot\mathbf{n}_{\beta\gamma}\mathrm{d}A \tag{118}$$

The *averaging theorem* can be applied to the solid phase thermal energy equation in the following way,

$$\frac{\partial}{\partial t}\langle\rho_\sigma h_\sigma\rangle = -\nabla\cdot\langle\mathbf{q}_\sigma\rangle - \frac{1}{V}\iint_{A_{\sigma\beta}}\mathbf{q}_\sigma\cdot\mathbf{n}_{\sigma\beta}\mathrm{d}A - \frac{1}{V}\iint_{A_{\sigma\gamma}}\mathbf{q}_\sigma\cdot\mathbf{n}_{\sigma\gamma}\mathrm{d}A + \langle\Phi_\sigma\rangle \tag{119}$$

Now the *averaging theorem* can be applied to the volume averaged version of Fourier's law and can be substituted into the solid phase thermal energy equation,

$$\langle\mathbf{q}_\sigma\rangle = -k_\sigma\langle\nabla T_\sigma\rangle = -k_\sigma\left[\nabla\langle T_\sigma\rangle + \frac{1}{V}\iint_{A_{\sigma\beta}}T_\sigma\mathbf{n}_{\sigma\beta}dA + \frac{1}{V}\iint_{A_{\sigma\gamma}}T_\sigma\mathbf{n}_{\sigma\gamma}dA\right] \tag{120}$$

The final volume averaged version of the solid phase thermal energy equation is,

$$\begin{aligned}\epsilon_\sigma\rho_\sigma(c_p)_\sigma\frac{\partial\langle T_\sigma\rangle^\sigma}{\partial t} = \nabla\cdot\Bigg[k_\sigma\Bigg[\nabla(\epsilon_\sigma\langle T_\sigma\rangle^\sigma) + \frac{1}{V}\iint_{A_{\sigma\beta}}T_\sigma\mathbf{n}_{\sigma\beta}dA \\ + \frac{1}{V}\iint_{A_{\sigma\gamma}}T_\sigma\mathbf{n}_{\sigma\gamma}dA\Bigg]\Bigg] \\ -\frac{1}{V}\iint_{A_{\sigma\beta}}\mathbf{q}_\sigma\cdot\mathbf{n}_{\sigma\beta}\mathrm{d}A - \frac{1}{V}\iint_{A_{\sigma\gamma}}\mathbf{q}_\sigma\cdot\mathbf{n}_{\sigma\gamma}\mathrm{d}A + \langle\Phi_\sigma\rangle\end{aligned} \tag{121}$$

The volume averaged continuity equation for the liquid phase would be,

$$\frac{1}{V}\iiint_{V_\beta(t)}\left(\frac{\partial\rho_\beta}{\partial t}\right)dV+\frac{1}{V}\iiint_{V_\beta(t)}\nabla\cdot(\rho_\beta\mathbf{v}_\beta)dV=0 \tag{122}$$

A few assumptions and restrictions can now be made. It can be assumed that the density of the liquid phase is constant, therefore ρ_β = constant. It then follows that,

$$\langle\rho_\beta\mathbf{v}_\beta\rangle = \rho_\beta\langle\mathbf{v}_\beta\rangle \tag{123}$$

$$\langle\rho_\beta\rangle = \epsilon_\beta\rho_\beta \tag{124}$$

Now if we apply the above assumptions along with the general transport theorem and the averaging theorem then the volume averaged liquid phase continuity equation becomes,

$$\frac{\partial\epsilon_\beta}{\partial t}+\nabla\cdot\langle\mathbf{v}_\beta\rangle+\frac{1}{V}\iint_{A_{\beta\gamma}}(\mathbf{v}_\beta-\mathbf{w})\cdot\mathbf{n}_{\beta\gamma}dA=0 \tag{125}$$

The volume averaged liquid phase thermal energy equation can be obtained in way similar to the solid phase, by integrating over the volume $V_\beta(t)$ and divide by the the volume element V.

$$\begin{aligned}\frac{1}{V}\iiint_{V_\beta(t)}\frac{\partial}{\partial t}(\rho_\beta h_\beta)dV &+ \frac{1}{V}\iiint_{V_\beta(t)}\nabla\cdot(\rho_\beta h_\beta\mathbf{v}_\beta) \\ &= -\frac{1}{V}\iiint_{V_\beta(t)}\nabla\cdot\mathbf{q}_\beta dV+\frac{1}{V}\iiint_{V_\beta(t)}\Phi_\beta dV\end{aligned} \tag{126}$$

Now, as usual, we can apply the general transport theorem and averaging theorem to equation (127). The result is,

$$\frac{\partial}{\partial t}\langle\rho_\beta h_\beta\rangle+\nabla\cdot\langle\rho_\beta h_\beta\mathbf{v}_\beta\rangle+\frac{1}{V}\iint_{A_{\beta\gamma}}\rho_\beta h_\beta(\mathbf{v}_\beta-\mathbf{w})\cdot\mathbf{n}_{\beta\gamma}dA \tag{127}$$

$$=-\nabla\cdot\langle\mathbf{q}_\beta\rangle-\frac{1}{V}\iint_{A_{\beta\gamma}}\mathbf{q}_\beta\cdot\mathbf{n}_{\beta\gamma}dA-\frac{1}{V}\iint_{A_{\beta\sigma}}\mathbf{q}_\beta\cdot\mathbf{n}_{\beta\sigma}dA+\langle\Phi_\beta\rangle \tag{128}$$

The enthalpy can be expressed in terms of the temperature as in equation (75),

$$h_\beta=h_\beta^\circ+(c_p)_\beta(T_\beta-T_\beta^\circ) \tag{129}$$

where h°_β is the enthalpy at the reference temperature T°_β. After making this substitution into equation (128), the term, $\nabla \cdot \langle T_\beta \mathbf{v}_\beta \rangle$ emerges. It is required that this term be represented in terms of $\langle T_\beta \rangle$ and $\langle \mathbf{v}_\beta \rangle$. The way Whitaker does this is to represent T_β and $\mathbf{v}_\beta$ in terms of average values and deviations from these average values. The point functions T_β and $\mathbf{v}_\beta$ can therefore be represented in the following way,

$$\begin{aligned} T_\beta &= \langle T_\beta \rangle^\beta + \delta T_\beta \qquad \text{in the } \beta \text{ phase} && (130)\\ T_\beta &= \delta T_\beta = 0 \qquad \text{in the } \sigma \text{ and } \gamma \text{ phases} && (131)\\ \mathbf{v}_\beta &= \langle \mathbf{v}_\beta \rangle + \delta \mathbf{v}_\beta \qquad \text{in the } \beta \text{ phase} && (132)\\ \mathbf{v}_\beta &= \delta \mathbf{v}_\beta = 0 \qquad \text{in the } \sigma \text{ and } \gamma \text{ phases} && (133) \end{aligned}$$

where $\langle T_\beta \rangle^\beta$ is the *intrinsic phase average* as defined earlier. We can now write $\langle T_\beta \mathbf{v}_\beta \rangle$ in terms of the dispersion vector in the following way,

$$\langle T_\beta \mathbf{v}_\beta \rangle = \langle T_\beta \rangle^\beta \langle \mathbf{v}_\beta \rangle + \langle \delta T_\beta \delta \mathbf{v}_\beta \rangle \tag{134}$$

Through some algebra and the above expression, the final form of the liquid phase thermal energy equation can be written as,

$$\begin{aligned} \epsilon_\beta \rho_\beta (c_p)_\beta \frac{\partial \langle T_\beta \rangle^\beta}{\partial t} + \rho_\beta (c_p)_\beta \langle \mathbf{v}_\beta \rangle \cdot \nabla \langle T_\beta \rangle^\beta + \rho_\beta (c_p)_\beta \nabla \cdot \langle \delta T_\beta \delta \mathbf{v}_\beta \rangle \\ + \iint_{A_{\beta\gamma}} \rho_\beta (c_p)_\beta \delta T_\beta (\mathbf{v}_\beta - \mathbf{w}) \cdot \mathbf{n}_{\beta\gamma} dA \\ = \nabla \cdot \left[k_\beta \left[\nabla (\epsilon_\beta \langle T_\beta \rangle^\beta) + \frac{1}{V} \iint_{A_{\beta\sigma}} T_\beta \mathbf{n}_{\beta\sigma} dA + \frac{1}{V} \iint_{A_{\beta\gamma}} T_\beta \mathbf{n}_{\beta\gamma} dA \right] \right] \\ - \frac{1}{V} \iint_{A_{\beta\sigma}} \mathbf{q}_\sigma \cdot \mathbf{n}_{\beta\sigma} \mathrm{d}A - \frac{1}{V} \iint_{A_{\beta\gamma}} \mathbf{q}_\beta \cdot \mathbf{n}_{\beta\gamma} \mathrm{d}A + \langle \Phi_\beta \rangle \end{aligned} \tag{135}$$

Fourier's law has been applied to express $\nabla \cdot \mathbf{q}_\beta$ in terms of T_β, as was done in the case of the thermal energy equation for the solid phase.

The vapour/inert gas phase can now be examined. We can start with the continuity equation for the vapour/inert gas phase.

$$\frac{\partial \rho_\gamma}{\partial t} + \nabla \cdot (\rho_\gamma \mathbf{v}_\gamma) = 0 \tag{136}$$

In the same manner as the liquid phase continuity equation, the vapour/inert gas phase continuity equation can be volume averaged and after some algebra results in,

$$\frac{\partial \rho_\gamma}{\partial t} + \nabla \cdot \langle \rho_\gamma \mathbf{v}_\gamma \rangle + \frac{1}{V} \iint_{A_{\gamma\beta}} \rho_\gamma (\mathbf{v}_\gamma - \mathbf{w}) \cdot \mathbf{n}_{\gamma\beta} dA = 0 \tag{137}$$

Unlike the liquid phase equation the gas phase continuity equation cannot be further simplified since ρ_γ depends on both temperature and composition of the gas phase. Similar to the liquid phase thermal energy equation, the term $\langle \rho_\gamma \mathbf{v}_\gamma \rangle$ can be separated be expressing the point functions ρ_γ and $\mathbf{v}_\gamma$ in terms of the intrinsic phase averages and a deviation.

$$\begin{aligned}
\rho_\gamma &= \langle \rho_\gamma \rangle^\gamma + \delta\rho_\gamma \qquad \text{in the } \gamma \text{ phase} && (138)\\
\rho_\gamma &= \delta\rho_\gamma = 0 \qquad \text{in the } \sigma \text{ and } \beta \text{ phases} && (139)\\
\mathbf{v}_\gamma &= \langle \mathbf{v}_\gamma \rangle + \delta\mathbf{v}_\gamma \qquad \text{in the } \gamma \text{ phase} && (140)\\
\mathbf{v}_\gamma &= \delta\mathbf{v}_\gamma = 0 \qquad \text{in the } \sigma \text{ and } \beta \text{ phases} && (141)
\end{aligned}$$

The intrinsic phase average will again be utilized here, which is defined for the γ phase density as,

$$\langle \rho_\gamma \rangle = \epsilon_\gamma \langle \rho_\gamma \rangle^\gamma \tag{142}$$

Using the above expressions, equation (137) can be written as,

$$\begin{aligned}
\frac{\partial(\epsilon_\gamma \langle \rho_\gamma \rangle^\gamma)}{\partial t} + \nabla \cdot (\langle \rho_\gamma \rangle^\gamma \langle \mathbf{v}_\gamma \rangle) &+ \nabla \cdot \langle \delta\rho_\gamma \delta\mathbf{v}_\gamma \rangle \\
&+ \frac{1}{V} \iint_{A_{\gamma\beta}} \rho_\gamma (\mathbf{v}_\gamma - \mathbf{w}) \cdot \mathbf{n}_{\gamma\beta} dA = 0 \qquad (143)
\end{aligned}$$

Whitaker reasons that deviations from the phase averages and intrinsic phase averages are much smaller then the phase averages and intrinsic phase averages themselves, therefore it is assumed that $\delta\psi_\omega \ll \langle \psi_\omega \rangle^\omega$ and $\delta\Psi_\omega \ll \langle \Psi_\omega \rangle^\omega$, where ω represents any of the phases σ, β, γ. This statement allows (143) to be written without the deviation term,

$$\frac{\partial(\epsilon_\gamma \langle \rho_\gamma \rangle^\gamma)}{\partial t} + \nabla \cdot (\langle \rho_\gamma \rangle^\gamma \langle \mathbf{v}_\gamma \rangle) + \frac{1}{V} \iint_{A_{\gamma\beta}} \rho_\gamma (\mathbf{v}_\gamma - \mathbf{w}) \cdot \mathbf{n}_{\gamma\beta} dA = 0 \tag{144}$$

It is useful to note that the dispersion vector $\nabla \cdot \langle \delta T_\beta \delta \mathbf{v}_\beta \rangle$ was not considered negligible relative to $\langle \mathbf{v}_\beta \rangle \cdot \nabla \langle T_\beta \rangle^\beta$ in the liquid phase thermal energy equation. Whitaker reasons that the dispersion term $\nabla \cdot \langle \delta T_\beta \delta \mathbf{v}_\beta \rangle$ may be small compared to convection terms in the γ phase thermal energy equation but maybe large or on the order of the relatively smaller conduction terms. Through this reasoning the dispersion vector $\nabla \cdot \langle \delta T_\beta \delta \mathbf{v}_\beta \rangle$ should remain in the liquid phase thermal energy equation.

We can now examine the species continuity equation. If we recall the species continuity equation from earlier,

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v_i}) = 0 \tag{145}$$

In the usual manner the volume averaged version is found to be,

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \langle \rho_i \mathbf{v}_i \rangle + \frac{1}{V} \iint_{A_{\gamma\beta}} \rho_i (\mathbf{v}_i - \mathbf{w}) \cdot \mathbf{n}_{i\beta} dA = 0 \tag{146}$$

Now as previously done, the species continuity equation can be written in terms of the mass average velocity $\mathbf{v}_\gamma$ and the diffusion velocity $\mathbf{u}_\gamma$, and the resulting diffusive flux term can be substituted with Fick's law. As a result the vapour/inert gas species equation becomes,

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \langle \rho_i \mathbf{v}_\gamma \rangle + \frac{1}{V} \iint_{A_{\gamma\beta}} \rho_i (\mathbf{v}_i - \mathbf{w}) \cdot \mathbf{n}_{\gamma\beta} dA = \nabla \cdot \left\langle \rho_\gamma \mathcal{D} \nabla \left(\frac{\rho_i}{\rho_\gamma} \right) \right\rangle \tag{147}$$

As before, an expression terms of the intrinsic phase average $\langle \rho_i \rangle^\gamma$ as well as eliminating the terms containing products with ρ_i in them. The first two terms can be handle just as the gas phase continuity equation, therefore,

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot \langle \rho_i \mathbf{v}_\gamma \rangle = \frac{\partial (\epsilon_\gamma \langle \rho_i \rangle^\gamma)}{\partial t} + \nabla \cdot (\langle \rho_i \rangle^\gamma \langle \mathbf{v}_\gamma \rangle) \tag{148}$$

The diffusion term on the left hand size is approximated by Whitaker as,

$$\langle \rho_\gamma \nabla (\rho_i / \rho_\gamma) \rangle = \langle \rho_\gamma \rangle^\gamma [\nabla (\langle \rho_i \rangle / \langle \rho_\gamma \rangle^\gamma) + \delta \Omega_i] \tag{149}$$

Where $\delta\Omega_i$ is the deviation term given as,

$$\begin{aligned} \delta\Omega_i = \frac{1}{V} \iint_{A_{\gamma\beta}} \left(\frac{\rho_i}{\langle \rho_\gamma \rangle^\gamma} \right) \mathbf{n}_{\gamma\beta} dA \quad &+ \quad \frac{1}{V} \iint_{A_{\gamma\sigma}} \left(\frac{\rho_i}{\langle \rho_\gamma \rangle^\gamma} \right) \mathbf{n}_{\gamma\sigma} dA \\ - \left\langle \nabla \left[\frac{\rho_i}{\langle \rho_\gamma \rangle^\gamma} \left(\frac{\delta \rho_\gamma}{\langle \rho_\gamma \rangle^\gamma} \right) \right] \right\rangle \quad &+ \quad \frac{\left\langle \delta \rho_\gamma \nabla \left(\frac{\rho_i}{\langle \rho_\gamma \rangle^\gamma} \right) \right\rangle}{\langle \rho_\gamma \rangle^\gamma} \end{aligned} \tag{150}$$

The final form of the volume averaged vapour/inert gas phase species continuity equation would be,

$$\frac{\partial(\epsilon_\gamma\langle\rho_i\rangle^\gamma)}{\partial t} + \nabla\cdot(\langle\rho_i\rangle^\gamma\langle\mathbf{v}_\gamma\rangle) + \frac{1}{V}\iint_{A_{\gamma\beta}} \rho_i(\mathbf{v}_i - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta}dA$$
$$= \nabla\cdot\left[\langle\rho_\gamma\rangle^\gamma\mathcal{D}[\nabla(\langle\rho_i\rangle/\langle\rho_\gamma\rangle^\gamma) + \delta\Omega_i] - \langle\delta\rho_i\delta\mathbf{v}_\gamma\rangle\right] \quad (151)$$

The thermal energy equation for the vapour/inert gas phase can now be examined. If we recall the thermal energy equation for the vapour/inert gas phase,

$$\frac{\partial}{\partial t}\left(\sum_{i=1}^{i=N}\rho_i\bar{h}_i\right) + \nabla\cdot\left(\sum_{i=1}^{i=N}\rho_i\mathbf{v}_i\bar{h}_i\right) = -\nabla\cdot\mathbf{q}_\gamma + \Phi_\gamma \quad (152)$$

it can be seen that it has the nearly identical form the liquid phase thermal energy equation. So if we integrate each term over the volume V_γ and divide by V we can then apply the general transport theorem and averaging theorem in the usual manner which results int he following,

$$\frac{\partial}{\partial t}\left\langle\sum_{i=1}^{i=N}\rho_i\bar{h}_i\right\rangle + \nabla\cdot\left\langle\sum_{i=1}^{i=N}\rho_i\mathbf{v}_i\bar{h}_i\right\rangle + \frac{1}{V}\iint_{A_{\gamma\beta}}\left(\sum_{i=1}^{i=N}\rho_i\bar{h}_i(\mathbf{v}_i - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta}\right)dA$$
$$= -\nabla\cdot\langle\mathbf{q}_\gamma\rangle - \frac{1}{V}\iint_{A_{\gamma\beta}}\mathbf{q}_\gamma\cdot\mathbf{n}_{\gamma\beta}dA - \frac{1}{V}\iint_{A_{\gamma\sigma}}\mathbf{q}_\gamma\cdot\mathbf{n}_{\gamma\sigma}dA + \langle\Phi_\gamma\rangle \quad (153)$$

The partial mass enthalpy can be represented nearly identically to the liquid phase partial mass enthalpy,

$$h_i = h_i^\circ + (c_p)_i(T_\gamma - T_\gamma^\circ) \quad (154)$$

Since we are expressing the partial mass enthalpy in terms of the pure component capacity $(c_p)_i$ instead of the partial mass heat capacity $\bar{c_{p_i}}$ then we are assuming that the gas phase is ideal. After substituting equation (154) into equation (153), as before, the terms containing products with $\rho_i\mathbf{v}_i$ in them must be eliminated. Similar to the liquid phase, averages and deviations from the averages must be used to separate the products. Unlike the liquid phase ρ_i cannot be treated as a constant. We can therefore use the following representation,

$$T_\gamma = \langle T_\gamma\rangle^\gamma + \delta T_\gamma \qquad \text{in the } \gamma \text{ phase} \quad (155)$$
$$T_\gamma = \delta T_\gamma = 0 \qquad \text{in the } \sigma \text{ and } \beta \text{ phases} \quad (156)$$
$$\rho_i\mathbf{v}_i = \langle\rho_i\mathbf{v}_i\rangle + \delta(\rho_i\mathbf{v}_i) \qquad \text{in the } \gamma \text{ phase} \quad (157)$$
$$\rho_i\mathbf{v}_i = \delta(\rho_i\mathbf{v}_i) = 0 \qquad \text{in the } \sigma \text{ and } \beta \text{ phases} \quad (158)$$

The first two terms of equation (153) therefore become,

$$\begin{aligned}
&\frac{\partial}{\partial t}\left\langle\sum_{i=1}^{i=N}\rho_i\bar{h}_i\right\rangle+\nabla\cdot\left\langle\sum_{i=1}^{i=N}\rho_i\mathbf{v}_i\bar{h}_i\right\rangle\\
&=\left(\sum_{i=1}^{i=N}\langle\rho_i\rangle(c_p)_i\right)\frac{\partial\langle T_\gamma\rangle^\gamma}{\partial t}+\left(\sum_{i=1}^{i=N}(c_p)_i\langle\rho_i\mathbf{v}_i\rangle\right)\cdot\nabla\langle T_\gamma\rangle^\gamma\\
&+\sum_{i=1}^{i=N}\left[h_i^\circ+(c_p)_i(\langle T_\gamma\rangle^\gamma-T_\gamma^\circ)\right]\left(\frac{1}{V}\iint_{A_{\gamma\beta}}\rho_i(\mathbf{v}_i-\mathbf{w})\cdot\mathbf{n}_{\gamma\beta}dA\right)\\
&+\frac{\partial}{\partial t}\sum_{i=1}^{i=N}(c_p)_i\langle\delta\rho_i\delta T_\gamma\rangle+\nabla\cdot\sum_{i=1}^{i=N}(c_p)_i\langle\delta(\rho_i\mathbf{v}_i)\delta T_\gamma\rangle \qquad (159)
\end{aligned}$$

Now if we substitute equation (159) into equation (153) and follow the derivation of the liquid thermal energy equation, we get,

$$\begin{aligned}
&\left(\sum_{i=1}^{i=N}\langle\rho_i\rangle(c_p)_i\right)\frac{\partial\langle T_\gamma\rangle^\gamma}{\partial t}+\left(\sum_{i=1}^{i=N}(c_p)_i\langle\rho_i\mathbf{v}_i\rangle\right)\cdot\nabla\langle T_\gamma\rangle^\gamma\\
&+\frac{1}{V}\iint_{A_{\gamma\beta}}\sum_{i=1}^{i=N}\rho_i(c_p)_i\delta T_\gamma(\mathbf{v}_i-\mathbf{w})\cdot\mathbf{n}_{\gamma\beta}dA\\
&+\frac{\partial}{\partial t}\sum_{i=1}^{i=N}(c_p)_i\langle\delta\rho_i\delta T_\gamma\rangle+\nabla\cdot\sum_{i=1}^{i=N}(c_p)_i\langle\delta(\rho_i\mathbf{v}_i)\delta T_\gamma\rangle\\
&=\nabla\cdot\left[k_\gamma\left[\nabla(\epsilon_\gamma\langle T_\gamma\rangle^\gamma)+\frac{1}{V}\iint_{A_{\gamma\sigma}}T_\gamma\mathbf{n}_{\gamma\sigma}dA+\frac{1}{V}\iint_{A_{\gamma\beta}}T_\gamma\mathbf{n}_{\gamma\beta}dA\right]\right]\\
&-\frac{1}{V}\iint_{A_{\gamma\sigma}}\mathbf{q}_\gamma\cdot\mathbf{n}_{\gamma\sigma}\mathrm{d}A-\frac{1}{V}\iint_{A_{\gamma\beta}}\mathbf{q}_\gamma\cdot\mathbf{n}_{\gamma\beta}\mathrm{d}A+\langle\Phi_\gamma\rangle \qquad (160)
\end{aligned}$$

3.4.6 Total Thermal Energy Equation

We can now make some reasonable assumptions to combine the thermal energy equations for each phase into one total thermal energy equation. A very reasonable assumption to make is that solid-liquid-gas system is in thermal equilibrium. This means that the intrinsic phase average temperatures are equal,

$$\langle T_\sigma\rangle^\sigma=\langle T_\beta\rangle^\beta=\langle T_\gamma\rangle^\gamma \qquad (161)$$

and since the spatial average temperature is defined as,

$$\langle T\rangle \equiv \epsilon_\sigma \langle T_\sigma\rangle^\sigma + \epsilon_\beta \langle T_\beta\rangle^\beta + \epsilon_\gamma \langle T_\gamma\rangle^\gamma \tag{162}$$

then it must be true that,

$$\langle T_\sigma\rangle^\sigma = \langle T_\beta\rangle^\beta = \langle T_\gamma\rangle^\gamma = \langle T\rangle \tag{163}$$

Now if we add equations (121), (135), and (160) after imposing the above assumption, the following results,

$$\begin{aligned}
&\left[\epsilon_\sigma \rho_\sigma (c_p)_\sigma + \epsilon_\beta \rho_\beta (c_p)_\beta + \epsilon_\gamma \left(\sum_{i=1}^{i=N} \langle \rho_i^\gamma \rangle (c_p)_i\right)\right] \frac{\partial \langle T\rangle}{\partial t} \\
&+\left[+ \rho_\beta (c_p)_\beta \langle \mathbf{v}_\beta\rangle + \sum_{i=1}^{i=N} (c_p)_i \langle \rho_i \mathbf{v}_i\rangle\right] \cdot \nabla\langle T\rangle \\
&+\frac{1}{V}\iint_{A_{\beta\gamma}} \rho_\beta (c_p)_\beta \delta T_\beta (\mathbf{v}_\beta - \mathbf{w}) \cdot \mathbf{n}_{\beta\gamma} dA + \\
&\frac{1}{V}\iint_{A_{\gamma\beta}} \sum_{i=1}^{i=N} \rho_i (c_p)_i \delta T_\gamma (\mathbf{v}_i - \mathbf{w}) \cdot \mathbf{n}_{\gamma\beta} dA \\
= {}& \nabla \cdot \Bigg(\nabla[(k_\sigma \epsilon_\sigma + k_\beta \epsilon_\beta + k_\gamma \epsilon_\gamma)\langle T\rangle] + (k_\sigma - k_\beta)\frac{1}{V}\iint_{A_{\sigma\beta}} T_\sigma \mathbf{n}_{\gamma\sigma} dA \\
&+(k_\beta - k_\gamma)\frac{1}{V}\iint_{A_{\beta\gamma}} T_\beta \mathbf{n}_{\beta\gamma} dA + (k_\gamma - k_\sigma)\frac{1}{V}\iint_{A_{\gamma\sigma}} T_\gamma \mathbf{n}_{\gamma\sigma} dA\Bigg) \\
&-\frac{1}{V}\iint_{A_{\sigma\beta}} (\mathbf{q}_\sigma - \mathbf{q}_\beta) \cdot \mathbf{n}_{\sigma\beta} dA - \frac{1}{V}\iint_{A_{\beta\gamma}} (\mathbf{q}_\beta - \mathbf{q}_\gamma) \cdot \mathbf{n}_{\beta\gamma} dA \\
&-\frac{1}{V}\iint_{A_{\gamma\sigma}} (\mathbf{q}_\gamma - \mathbf{q}_\sigma) \cdot \mathbf{n}_{\gamma\sigma} dA - \nabla \cdot \langle \xi\rangle + \langle \Phi\rangle
\end{aligned} \tag{164}$$

where $\langle \Phi\rangle$ represents the source terms,

$$\langle \Phi\rangle = \langle \Phi_\sigma\rangle + \langle \Phi_\beta\rangle + \langle \Phi_\gamma\rangle + \frac{\partial}{\partial t}\sum_{i=1}^{i=N} (c_p)_i \langle \delta\rho_i \delta T_\gamma\rangle \tag{165}$$

and $\nabla \cdot \langle \xi\rangle$ represents the liquid and gas phase dispersion,

$$\nabla \cdot \langle \xi\rangle = \rho_\beta (c_p)_\beta \nabla \cdot \langle \delta T_\beta \delta \mathbf{v}_\beta\rangle + \nabla \cdot \sum_{i=1}^{i=N} (c_p)_i \langle \delta(\rho_i \mathbf{v}_i)\delta T_\gamma\rangle \tag{166}$$

The following is also utilized,

$$\mathbf{n}_{\sigma\beta} = -\mathbf{n}_{\sigma\beta}, \qquad \mathbf{n}_{\beta\gamma} = -\mathbf{n}_{\gamma\beta}, \qquad \mathbf{n}_{\gamma\sigma} = -\mathbf{n}_{\sigma\gamma} \tag{167}$$

and,

$$T_\beta = T_\sigma \qquad \text{over} \qquad A_{\beta\sigma} = T_{\sigma\beta} \tag{168}$$

$$T_\gamma = T_\beta \qquad \text{over} \qquad A_{\gamma\beta} = T_{\beta\sigma} \tag{169}$$

$$T_\sigma = T_\gamma \qquad \text{over} \qquad A_{\sigma\gamma} = T_{\gamma\sigma} \tag{170}$$

and it should also be noted that the thermal conductivities are constant. Equation (164) can be simplified by substituting some more measurable quantities. We can start by substituting the *spatial average density* which is a measurable quantity given by,

$$\langle\rho\rangle = \epsilon_\sigma\langle\rho_\sigma\rangle^\sigma + \epsilon\beta\langle\rho_\beta\rangle^\beta + \epsilon_\gamma\sum_{i=1}^{i=N}\langle\rho_i\rangle^\gamma \tag{171}$$

The *spatial average density* can then be used to define a *mass fraction weighted average heat capacity* as,

$$C_p = \frac{\epsilon_\sigma\rho_\sigma(c_p)_\sigma + \epsilon_\beta\rho_\beta(c_p)_\beta + \epsilon_\gamma\sum_{i=1}^{i=N}\langle\rho_i\rangle^\gamma(c_p)_i}{\langle\rho\rangle} \tag{172}$$

The first term on the left hand side now simplifies to $\langle\rho\rangle C_p\frac{\partial\langle T\rangle}{\partial t}$. Referring back to the solid-liquid and solid-vapour/inert gas boundary conditions it is apparent that two of the inter-phase flux terms on the right hand side of equation (164), terms 2 and 4, are zero. After some algebra the jump condition in equation (107) can be manipulated into the following form,

$$\rho_\beta h_\beta(\mathbf{v}_\beta - \mathbf{w})\cdot\mathbf{n}_{\beta\gamma} + \sum_{i=1}^{i=N}\rho_i\bar{h}_i(\mathbf{v}_i - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta} = -(\mathbf{q}_\beta - \mathbf{q}_\gamma)\cdot\mathbf{n}_{\beta\gamma} \tag{173}$$

This can now be substituted into the third term of equation (164),

$$\begin{aligned}&\frac{1}{V}\iint_{A_{\beta\gamma}}(\mathbf{q}_\beta - \mathbf{q}_\gamma)\cdot\mathbf{n}_{\beta\gamma}dA\\ &\qquad = \frac{1}{V}\iint_{A_{\beta\gamma}}\rho_\beta h_\beta(\mathbf{v}_\beta - \mathbf{w})\cdot\mathbf{n}_{\beta\gamma} + \sum_{i=1}^{i=N}\rho_i\bar{h}_i(\mathbf{v}_i - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta}dA\end{aligned} \tag{174}$$

Now if we substitute all the above simplifications, equation (164) becomes,

$$\begin{aligned}
&\langle\rho\rangle C_p\frac{\partial\langle T\rangle}{\partial t}+\left[\rho_\beta(c_p)_\beta\langle\mathbf{v}_\beta\rangle+\sum_{i=1}^{i=N}(c_p)_i\langle\rho_i\mathbf{v}_i\rangle\right]\cdot\nabla\langle T\rangle\\
&-\frac{1}{V}\iint_{A_{\beta\gamma}}\left[\rho_\beta(\mathbf{v}_\beta-\mathbf{w})((c_p)_\beta\delta T_\beta-h_\beta)\cdot\mathbf{n}_{\beta\gamma}\right.\\
&\left.+\sum_{i=1}^{i=N}\rho_i(\mathbf{v}_i-\mathbf{w})((c_p)_i\delta T_\gamma-h_i)\cdot\mathbf{n}_{\gamma\beta}\right]dA\\
&=\nabla\cdot\left[\nabla[(k_\sigma\epsilon_\sigma+k_\beta\epsilon_\beta+k_\gamma\epsilon_\gamma)\langle T\rangle]+(k_\sigma-k_\beta)\frac{1}{V}\iint_{A_{\sigma\beta}}T_\sigma\mathbf{n}_{\gamma\sigma}dA\right.\\
&\left.+(k_\beta-k_\gamma)\frac{1}{V}\iint_{A_{\beta\gamma}}T_\beta\mathbf{n}_{\beta\gamma}dA+(k_\gamma-k_\sigma)\frac{1}{V}\iint_{A_{\gamma\sigma}}T_\gamma\mathbf{n}_{\gamma\sigma}dA\right]\\
&-\nabla\cdot\langle\xi\rangle+\langle\Phi\rangle \qquad (175)
\end{aligned}$$

Now if the enthalpies are expressed as,

$$\begin{aligned}
h_\beta &= h_\beta^\circ+(c_p)_\beta(T_\beta-T_\beta^\circ) \qquad (176)\\
\bar{h}_i &= h_i^\circ+(c_p)_i(T_\gamma-T_\gamma^\circ \qquad (177)
\end{aligned}$$

the integrand on the left side of equation (175) becomes,

$$\begin{aligned}
&\rho_\beta(\mathbf{v}_\beta-\mathbf{w})((c_p)_\beta\delta T_\beta-h_\beta)\cdot\mathbf{n}_{\beta\gamma}+\sum_{i=1}^{i=N}\rho_i(\mathbf{v}_i-\mathbf{w})((c_p)_i\delta T_\gamma-h_i)\cdot\mathbf{n}_{\gamma\beta}\\
&=[h_\beta^\circ+(c_p)_\beta(\langle T_\beta\rangle^\beta-T_\beta^\circ)]\rho_\beta(\mathbf{v}_\beta-\mathbf{w})\cdot\mathbf{n}_{\beta\gamma}+\\
&+\sum_{i=1}^{i=N}[h_i^\circ+(c_p)_i(\langle T_\gamma\rangle^\gamma-T_\gamma^\circ)]\rho_i(\mathbf{v}_i-\mathbf{w})\cdot\mathbf{n}_{\gamma\beta} \qquad (178)
\end{aligned}$$

The species jump condition, equations (110), can now be substituted to simplify the right side of the above expression resulting in,

$$\begin{aligned}
&\rho_\beta(\mathbf{v}_\beta-\mathbf{w})((c_p)_\beta\delta T_\beta-h_\beta)\cdot\mathbf{n}_{\beta\gamma}+\sum_{i=1}^{i=N}\rho_i(\mathbf{v}_i-\mathbf{w})((c_p)_i\delta T_\gamma-h_i)\cdot\mathbf{n}_{\gamma\beta}\\
&=[h_\beta^\circ-h_1^\circ+(c_p)_\beta(\langle T\rangle-T_\beta^\circ)-(c_p)_1(\langle T\rangle-T_\gamma^\circ)]\rho_\beta(\mathbf{v}_\beta-\mathbf{w})\cdot\mathbf{n}_{\beta\gamma} \qquad (179)
\end{aligned}$$

Whitaker identifies the *enthalpy of vapourization* per unit mass at the temperature $\langle T\rangle$ as,

$$\Delta h_{vap} = [h_\beta^\circ - h_1^\circ + (c_p)_\beta(\langle T\rangle - T_\beta^\circ) - (c_p)_1(\langle T\rangle - T_\gamma^\circ)] \tag{180}$$

and the *mass rate of vapourization* per unit volume as,

$$\langle\dot{m}\rangle = \frac{1}{V}\iint_{A_{\beta\gamma}} \rho_\beta(\mathbf{v}_\beta - \mathbf{w})\cdot\mathbf{n}_{\beta\gamma}dA \tag{181}$$

with above two definitions, the total thermal energy equation reduces to an even simpler form,

$$\begin{aligned}
&\langle\rho\rangle C_p\frac{\partial\langle T\rangle}{\partial t} + \left[\rho_\beta(c_p)_\beta\langle\mathbf{v}_\beta\rangle + \sum_{i=1}^{i=N}(c_p)_i\langle\rho_i\mathbf{v}_i\rangle\right]\cdot\nabla\langle T\rangle + \Delta h_{vap}\langle\dot{m}\rangle \\
&= \nabla\cdot\left[\nabla[(k_\sigma\epsilon_\sigma + k_\beta\epsilon_\beta + k_\gamma\epsilon_\gamma)\langle T\rangle] + (k_\sigma - k_\beta)\frac{1}{V}\iint_{A_{\sigma\beta}} T_\sigma\mathbf{n}_{\gamma\sigma}dA\right. \\
&\left.+(k_\beta - k_\gamma)\frac{1}{V}\iint_{A_{\beta\gamma}} T_\beta\mathbf{n}_{\beta\gamma}dA + (k_\gamma - k_\sigma)\frac{1}{V}\iint_{A_{\gamma\sigma}} T_\gamma\mathbf{n}_{\gamma\sigma}dA\right] \\
&-\nabla\cdot\langle\xi\rangle + \langle\Phi\rangle
\end{aligned} \tag{182}$$

At this point there are still some terms with T_σ, T_β, and T_γ in them. Whitaker suggests a theorem developed by Gray [14],

$$\langle\phi_\sigma\rangle^\sigma\nabla\epsilon_\sigma = \frac{1}{V}\iint_{A_{\sigma\beta}}(\delta\phi_\sigma - \phi_\sigma)\mathbf{n}_{\sigma\beta}dA + \frac{1}{V}\iint_{A_{\sigma\gamma}}(\delta\phi_\sigma - \phi_\sigma)\mathbf{n}_{\sigma\gamma}dA \tag{183}$$

can be applied in a round about way to eliminate the point functions and replace them by the deviations δT_σ, δT_β, and δT_γ. Therefore equation (183) can be applied to equation (182, and after a considerable amount of algebra reduces to,

$$\begin{aligned}
&\langle\rho\rangle C_p\frac{\partial\langle T\rangle}{\partial t} + \left[\rho_\beta(c_p)_\beta\langle\mathbf{v}_\beta\rangle + \sum_{i=1}^{i=N}(c_p)_i\langle\rho_i\mathbf{v}_i\rangle\right]\cdot\nabla\langle T\rangle + \Delta h_{vap}\langle\dot{m}\rangle \\
&= \nabla\cdot\left[(k_\sigma\epsilon_\sigma + k_\beta\epsilon_\beta + k_\gamma\epsilon_\gamma)\nabla\langle T\rangle + (k_\sigma - k_\beta)\frac{1}{V}\iint_{A_{\sigma\beta}} \delta T_\sigma\mathbf{n}_{\gamma\sigma}dA\right. \\
&\left.+(k_\beta - k_\gamma)\frac{1}{V}\iint_{A_{\beta\gamma}} \delta T_\beta\mathbf{n}_{\beta\gamma}dA + (k_\gamma - k_\sigma)\frac{1}{V}\iint_{A_{\gamma\sigma}} \delta T_\gamma\mathbf{n}_{\gamma\sigma}dA\right] \\
&-\nabla\cdot\langle\xi\rangle + \langle\Phi\rangle
\end{aligned} \tag{184}$$

Although equation (184) has been substantially simplified from it's initial state, it still contains parts which are theoretical. The first term on the right hand side, or the conductive term, is still in a theoretical form and does not allow comparison to experiment at this point. Whitaker deals with the conductive term through a lengthy reasoning process. Whitaker conjectures that the deviations δT_σ, δT_β, and δT_γ can be expresses in the following functional form,

$$\delta T_\beta = \mathcal{F}\left(\frac{\partial \langle T_\beta \rangle^\beta}{\partial t}, \nabla \langle T_\beta \rangle^\beta\right) \tag{185}$$

Further reasons leads to the reasonable assumption that the deviations δT_σ, δT_β, and δT_γ can be approximated as linear functions. For instance,

$$\delta T_\beta = C_\beta \cdot \nabla \langle T_\beta \rangle^\beta \tag{186}$$

therefore it would follow that,

$$\frac{1}{V}\iint_{A_{\beta\gamma}} \delta T_\beta \mathbf{n}_{\beta\gamma} dA = \frac{1}{V}\iint_{A_{\beta\gamma}} C_\beta \cdot \nabla \langle T_\beta \rangle^\beta \mathbf{n}_{\beta\gamma} dA \tag{187}$$

Whitaker also assumes that the variations in $\langle T_\beta \rangle^\beta$ will be small compared to the spatial variations of δT_β, therefore,

$$\frac{1}{V}\iint_{A_{\beta\gamma}} \delta T_\beta \mathbf{n}_{\beta\gamma} dA = K_\beta \cdot \nabla \langle T_\beta \rangle^\beta \tag{188}$$

where K_β is a second order tensor. It should then follow that solid and gas phase terms also follow that same line of reasoning,

$$\frac{1}{V}\iint_{A_{\sigma\beta}} \delta T_\sigma \mathbf{n}_{\sigma\beta} dA = K_\sigma \cdot \nabla \langle T_\sigma \rangle^\sigma \frac{1}{V}\iint_{A_{\gamma\sigma}} \delta T_\gamma \mathbf{n}_{\gamma\sigma} dA = K_\gamma \cdot \nabla \langle T_\gamma \rangle^\gamma \tag{189}$$

Whitaker collects all of the thermal conductivities into an *effective thermal conductivity* give as,

$$K_{eff} = (\epsilon_\sigma k_\sigma + \epsilon_\beta k_\beta + \epsilon_\gamma k_\gamma)U + (k_\sigma - k_\beta)K_\sigma + (k_\beta - k_\gamma)K_\beta + (k_\gamma - k_\sigma)K_\gamma \tag{190}$$

where U is the unit tensor. One can also interpret the dispersion $\nabla \cdot \langle \xi \rangle$ in terms of a diffusion model as follows,

$$\langle \xi \rangle = -K_D \cdot \nabla \langle T \rangle \tag{191}$$

and therefore the total thermal energy equation simplifies even further to,

$$\langle\rho\rangle C_p\frac{\partial\langle T\rangle}{\partial t}+\left[\rho_\beta(c_p)_\beta\langle\mathbf{v}_\beta\rangle+\sum_{i=1}^{i=N}(c_p)_i\langle\rho_i\mathbf{v}_i\rangle\right]\cdot\nabla\langle T\rangle+\Delta h_{vap}\langle\dot{m}\rangle$$
$$=\nabla\cdot(K_{eff}\cdot\nabla\langle T\rangle)+\nabla\cdot(K_D\cdot\nabla\langle T\rangle)+\langle\Phi\rangle \qquad (192)$$

The effective thermal conductivity can be taken even one step further to give,

$$\langle\rho\rangle C_p\frac{\partial\langle T\rangle}{\partial t}+\left[+\rho_\beta(c_p)_\beta\langle\mathbf{v}_\beta\rangle+\sum_{i=1}^{i=N}(c_p)_i\langle\rho_i\mathbf{v}_i\rangle\right]\cdot\nabla\langle T\rangle+\Delta h_{vap}\langle\dot{m}\rangle$$
$$=\nabla\cdot(K^T_{eff}\cdot\nabla\langle T\rangle)+\langle\Phi\rangle \qquad (193)$$

where $K^T_{eff}=K_{eff}+K_D$. Further simplification to the total thermal energy equation above can be made by assuming diffusive thermal transport in the vapour/inert gas phase is negligible in the gas phase. This very reasonable assumption allows the following to be written,

$$\langle\rho_i\mathbf{v}_i\rangle=\langle\rho_i\mathbf{v}_\gamma\rangle \qquad (194)$$

it then follows,

$$\langle\rho_i\mathbf{v}_i\rangle=\langle\rho_i\rangle^\gamma\langle\mathbf{v}_\gamma\rangle+\langle\delta\rho_i\delta\mathbf{v}_\gamma\rangle \qquad (195)$$

If we assume that the product of the deviations will be considered negligible compared to the product of the averages then,

$$\langle\rho_i\mathbf{v}_i\rangle=\langle\rho_i^\gamma\rangle\langle\mathbf{v}_\gamma\rangle \qquad (196)$$

The traditional definition of the heat capacity of a gas mixture can be used,

$$\rho_\gamma(c_p)_\gamma=\sum_{i=1}^{i=N}\rho_i(c_p)_i \qquad (197)$$

together with the above assumption yields,

$$\sum_{i=1}^{i=N}\langle\rho_i\rangle^\gamma(c_p)_i\langle\mathbf{v}_\gamma\rangle=\langle\rho_\gamma\rangle^\gamma\langle(c_p)_\gamma\rangle^\gamma\langle\mathbf{v}_\gamma\rangle \qquad (198)$$

The total thermal energy equation can now be written as,

$$\langle\rho\rangle C_p\frac{\partial\langle T\rangle}{\partial t}+[\rho_\beta(c_p)_\beta\langle\mathbf{v}_\beta\rangle+\langle\rho_\gamma\rangle^\gamma\langle(c_p)_\gamma\rangle^\gamma\langle\mathbf{v}_\gamma\rangle]\cdot\nabla\langle T\rangle+\Delta h_{vap}\langle\dot{m}\rangle$$
$$=\nabla\cdot(K^T_{eff}\cdot\nabla\langle T\rangle)+\langle\Phi\rangle \qquad (199)$$

What is left is a simplified version of the original total thermal energy equation which is in a form which allows the comparison of theory and experiment. The only trouble seems to be with the *effective thermal conductivity*, K_{eff}^{T}. What Whitaker has done with this is to lump all the thermal conductivities into one, thereby effectively sweeping a lot of problems under the carpet. Realizing an experimental value for the *effective thermal conductivity* requires the collection of the nine components of the *effective thermal conductivity* tensor, which is not a trivial task.

3.4.7 Simplified Continuity Equations for Solid, Liquid and Vapour/Inert Gas Phases

The *mass rate of vapourization* give by equation (181) can be used to simplify the liquid continuity equation to,

$$\frac{\partial \epsilon_\beta}{\partial t} + \nabla \cdot \langle \mathbf{v}_\beta \rangle + \frac{\langle \dot{m} \rangle}{\rho_\beta} = 0 \tag{200}$$

Virtually the same reasoning can be applied to the vapour/inert gas continuity equation. By equation (108) and (181) the *mass rate of vapourization* can also be written as,

$$\langle \dot{m} \rangle = -\frac{1}{V} \iint_{A_{\gamma\beta}} \rho_\gamma (\mathbf{v}_\gamma - \mathbf{w}) \cdot \mathbf{n}_{\gamma\beta} dA \tag{201}$$

and therefore the vapour/inert gas phase continuity equation can be written as,

$$\frac{\partial (\epsilon_\gamma \langle \rho_\gamma \rangle^\gamma)}{\partial t} + \nabla \cdot (\langle \rho_\gamma \rangle^\gamma \langle \mathbf{v}_\gamma \rangle) = \langle \dot{m} \rangle \tag{202}$$

The only remaining theoretical parts of the transport equations are the deviation terms, $\delta\omega$ and $\langle \delta\rho_i \delta\mathbf{v}_\gamma \rangle$ in the vapour/inert gas species equation and the liquid and vapour/inert gas velocity fields $\langle v_\beta \rangle$ and $\langle v_\gamma \rangle$. The dispersion terms are handled by Whitaker in the following way. Similar to the thermal dispersion term, $\langle \delta\rho_i \delta\mathbf{v}_\gamma \rangle$ is approximated by a diffusion like expression,

$$\langle \delta\rho_i \delta\mathbf{v}_\gamma \rangle = -\langle \rho_\gamma \rangle^\gamma D_D^{(i)} \cdot \nabla (\langle \rho_i \rangle^\gamma / \langle \rho_\gamma \rangle^\gamma) \tag{203}$$

the argument Whitaker uses for using this expression is nearly identical to the thermal energy equation. Whitaker reasons that the $\delta\omega$ term can approximated relatively well by a diffusion expression such as the following,

$$\delta\omega = D_D^{(i)} \cdot \nabla (\langle \rho_i \rangle^\gamma / \langle \rho_\gamma \rangle^\gamma) \tag{204}$$

In this way the species continuity equation can be written in a more compacted form as,

$$\frac{\partial(\epsilon_\gamma \langle\rho_i\rangle^\gamma)}{\partial t} + \nabla\cdot(\langle\rho_i\rangle^\gamma\langle\mathbf{v}_\gamma\rangle) + \frac{1}{V}\iint_{A_{\gamma\beta}} \rho_i(\mathbf{v}_i - \mathbf{w})\cdot\mathbf{n}_{\gamma\beta}dA = \nabla\cdot\left[\langle\rho_\gamma\rangle^\gamma D^{(i)}_{eff}\cdot\nabla\left(\frac{\langle\rho_i\rangle^\gamma}{\langle\rho_\gamma\rangle^\gamma}\right)\right] \tag{205}$$

where, as in the case of the thermal energy equation a *total effective diffusivity*, $D^{(i)}_{eff}$ is used to represent the three separate diffusivities. As in the vapour/inert gas continuity equation the *mass rate of vaporization* can be substituted in for the area integral for for the vapour portion of the gas phase ($i = 1$), therefore,

$$\frac{\partial(\epsilon_\gamma \langle\rho_i\rangle^\gamma)}{\partial t} + \nabla\cdot(\langle\rho_i\rangle^\gamma\langle\mathbf{v}_\gamma\rangle) + \langle\dot{m}\rangle = \nabla\cdot\left[\langle\rho_\gamma\rangle^\gamma D^{(i)}_{eff}\cdot\nabla\left(\frac{\langle\rho_i\rangle^\gamma}{\langle\rho_\gamma\rangle^\gamma}\right)\right] \tag{206}$$

and for the remaining gases, $i = 2, 3, ...$ where phase changes do not occur, the continuity equation becomes,

$$\frac{\partial(\epsilon_\gamma \langle\rho_i\rangle^\gamma)}{\partial t} + \nabla\cdot(\langle\rho_i\rangle^\gamma\langle\mathbf{v}_\gamma\rangle) = \nabla\cdot\left[\langle\rho_\gamma\rangle^\gamma D^{(i)}_{eff}\cdot\nabla\left(\frac{\langle\rho_i\rangle^\gamma}{\langle\rho_\gamma\rangle^\gamma}\right)\right] \tag{207}$$

The *total effective diffusivity*, $D^{(i)}_{eff}$ presents the same problems as the *total effective thermal conductivity*, K^T_{eff}, in that defining an experimental interpretation is significant difficulty. Part of this thesis project is to figure out how to interpret these coefficients.

3.4.8 Liquid and Vapour/Inert Gas Velocity Fields

In order to close the transport equations, expressions for the liquid and vapour/inert gas phase velocity fields are required. Whitaker spends a significant amount of time discussing and deriving an appropriate expression to represent the liquid and gas phase velocity fields. A brief summary of Whitaker's discussion will be given here in order to give an idea of the origins of the liquid and velocity fields. Whitaker starts with the equation of linear momentum,

$$\rho_{\beta,\gamma}\frac{D\mathbf{v}_{\beta,\gamma}}{Dt} = \rho_{\beta,\gamma}\mathbf{g} + \nabla\cdot T_{\beta,\gamma} \tag{208}$$

to derive the volume average phase velocities as a form of Darcy's law. The difference in the derivation of the liquid and vapour/inert gas phases is that the gas phase is assumed to be continuous. In other words an imaginary line could be drawn through the vapor/inert gas phase without ever having to go through a phase boundary. This assumption can also apply to the liquid phase on some occasions. In the case of wood, the pores are tube like long narrow cells, connected at each end by small holes called pits. It is reasonable to assume that at relatively high moisture contents, the liquid water volume would be quiet large, on the order of the vapour/inert gas volume. This means the moisture within the long narrow tracheids would most like be filled with liquid with a vapour/inert gas bubble in the middle. It can therefore be reasoned that the liquid state, at high enough moisture contents, would be in the funicular, or continuous state. This assumption is used by Plumb et al. [1], to apply Whitaker's model to a one dimensional model of wood drying. For this reason the simpler continuous version of the volume averaged velocity field for the liquid state could be utilized. Whitaker finds that the volume averaged velocity field for the vapor/inert gas phase is,

$$\langle \mathbf{v}_\gamma \rangle = -\frac{1}{\mu_\gamma} K_\gamma \cdot \Big(\epsilon_\gamma [\nabla \langle p_\gamma - p_0 \rangle^\gamma - \rho_\gamma \mathbf{g}] \Big) \tag{209}$$

where K_γ is the permeability of the solid, μ_γ is the viscosity of the fluid, p_γ is the vapor/inert gas pressure, and p_0 is the reference pressure. A reasonable assumption is to treat the vapor/inert gas as an ideal gas and therefore the pressures can be calculated using the ideal gas law. The continuous state volume averaged liquid phase velocity field is derived by Whitaker as,

$$\langle \mathbf{v}_\beta \rangle = -\frac{1}{\mu_\beta} K_\beta \cdot \Big(\epsilon_\beta [\nabla \langle p_c \rangle - (\rho_\beta - \rho_\gamma) \mathbf{g}] \Big) \tag{210}$$

where p_c is the capillary pressure. Whitaker's theory provides a complete set of equations to describe heat and mass transport in porous media. Other then the *total effective diffusivity* and the *effective thermal conductivity* coefficients, the final set of equations provided by Whitaker are in a form that can be used to compare to experiment.

3.4.9 Definitions of Moisture Content

In most situations the information about the concentration of moving mass, such as water, within the porous solid is referred to as *moisture content.*

There is usually no way of distinguishing between liquid water and vapour water, therefore a quantity called *moisture content* is used to combine both into one measurable quantity. In Luikov's final transport equations *moisture content* was quantified as $u = u_2 + u_3$, where 2 denotes the liquid phase and 3 denotes the vapour phase. Luikov considers the ice and inert gas mass to be negligible. In this way moisture content is the mass of liquid and vapour per mass of dry wood structure, $u = (m_2 + m_3)/m_0$, and in Whitaker's notation would be equivalent to,

$$u = \frac{\rho_\beta \epsilon_\beta + \langle \rho_1 \rangle^\gamma \epsilon_\gamma}{\rho_\sigma \epsilon_\sigma} \tag{211}$$

The other form of expressing moisture content is in the form of *fractional* and *percentage moisture saturation.* In this form moisture content ranges from $0 - 1$ in the case of *fractional moisture saturation* and $0 - 100$ in the case of *percentage moisture saturation.* The *fractional moisture saturation* is given by,

$$S = \frac{\rho_\beta \epsilon_\beta + \langle \rho_1 \rangle^\gamma \epsilon_\gamma}{\rho_\beta (\epsilon_\beta + \epsilon_\gamma)} \tag{212}$$

where $(\epsilon_\beta + \epsilon_\gamma)$ translates to the *void volume fraction*, or the amount of empty space per unit volume of porous media. It then follows that the *percentage moisture saturation* is,

$$S = 100 \times \left[\frac{\rho_\beta \epsilon_\beta + \langle \rho_1 \rangle^\gamma \epsilon_\gamma}{\rho_\beta (\epsilon_\beta + \epsilon_\gamma)} \right] \tag{213}$$

The *percentage moisture saturation* is the preferred definition of moisture content in the study of porous media because the mass of the completely dry product is not always known. When the gas phase is considered negligible, moisture content can be simplified to the *fractional liquid saturation*,

$$S = \frac{\epsilon_\beta}{(\epsilon_\beta + \epsilon_\gamma)} \tag{214}$$

and the *percentage liquid saturation*,

$$S = 100 \times \left[\frac{\epsilon_\beta}{(\epsilon_\beta + \epsilon_\gamma)} \right] \tag{215}$$

3.4.10 Final Transport Equations

Whitaker's final closed set of transport equations can now be summarized. Starting with the *total thermal energy*,

$$\langle\rho\rangle C_p\frac{\partial\langle T\rangle}{\partial t}+[\rho_\beta(c_p)_\beta\langle\mathbf{v}_\beta\rangle+\langle\rho_\gamma\rangle^\gamma\langle(c_p)_\gamma\rangle^\gamma\langle\mathbf{v}_\gamma\rangle]\cdot\nabla\langle T\rangle+\Delta h_{vap}\langle\dot m\rangle$$
$$=\nabla\cdot(K^T_{eff}\cdot\nabla\langle T\rangle)+\langle\Phi\rangle \quad (216)$$

The *liquid continuity equation*

$$\frac{\partial\epsilon_\beta}{\partial t}+\nabla\cdot\langle\mathbf{v}_\beta\rangle+\frac{\langle\dot m\rangle}{\rho_\beta}=0 \quad (217)$$

The *vapour/inert gas continuity equation*

$$\frac{\partial(\epsilon_\gamma\langle\rho_\gamma\rangle^\gamma)}{\partial t}+\nabla\cdot(\langle\rho_\gamma\rangle^\gamma\langle\mathbf{v}_\gamma\rangle)=\langle\dot m\rangle \quad (218)$$

The *vapour/inert gas diffusion equation*, if we consider $i=1$ the vapour and $i=2$ the combined inert gas phase.

$$\frac{\partial(\epsilon_\gamma\langle\rho_2\rangle^\gamma)}{\partial t}+\nabla\cdot(\langle\rho_2\rangle^\gamma\langle\mathbf{v}_\gamma\rangle)=\nabla\cdot\left[\langle\rho_\gamma\rangle^\gamma D^{(2)}_{eff}\cdot\nabla\left(\frac{\langle\rho_2\rangle^\gamma}{\langle\rho_\gamma\rangle^\gamma}\right)\right] \quad (219)$$

The *liquid phase equation of motion*

$$\langle\mathbf{v}_\beta\rangle=-\frac{1}{\mu_\beta}K_\beta\cdot\Big(\epsilon_\beta[\nabla\langle p_c\rangle-(\rho_\beta-\rho_\gamma)\mathbf{g}]\Big) \quad (220)$$

The *vapour/inert gas phase equation of motion*

$$\langle\mathbf{v}_\gamma\rangle=-\frac{1}{\mu_\gamma}K_\gamma\cdot\Big(\epsilon_\gamma[\nabla\langle p_\gamma-p_0\rangle^\gamma-\rho_\gamma\mathbf{g}]\Big) \quad (221)$$

The *volume constraint*

$$\epsilon_\sigma=\epsilon_\beta(t)=\epsilon_\gamma(t)=1 \quad (222)$$

and the *thermodynamic relations*

$$\langle p_1\rangle^\gamma = \langle\rho_1\rangle^\gamma R_1\langle T\rangle \quad (223)$$
$$\langle p_2\rangle^\gamma = \langle\rho_2\rangle^\gamma R_2\langle T\rangle \quad (224)$$
$$\langle\rho_\gamma\rangle^\gamma = \langle\rho_1\rangle^\gamma+\langle\rho_2\rangle^\gamma \quad (225)$$
$$\langle p_\gamma\rangle^\gamma = \langle p_1\rangle^\gamma+\langle p_2\rangle^\gamma \quad (226)$$
$$\langle p_1\rangle^\gamma = p_1^0\exp-\left[(2\sigma_{\beta\gamma}/r\rho_\beta R_1\langle T\rangle)+\frac{\delta h_{vap}}{R_1}\left(\frac{1}{\langle T\rangle}-\frac{1}{T_0}\right)\right] \quad (227)$$

Were the last thermodynamic relation has been derived by Whitaker through a combination of the Kelvin equation and Clausius-Clapeyron equation. The Clausius-Clapeyron equation,

$$p_1 = p_1^0 \exp\left[-\frac{\delta h_{vap}}{R_1}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] \tag{228}$$

relates vapour pressure to temperature where the gas-liquid interface is flat, and p_1^0 is the vapour pressure at the reference temperature T_0. In the case of porous media where capillary forces hold liquid in place, then surface tension effects become important and one must take into account the effects of surface curvature therefore the Clausius-Clapeyron equation becomes inadequate. The Kelvin equation,

$$p_1 = p_1^0 \exp\left(\frac{-2\sigma_{\beta\gamma}}{r\rho_\beta R_1 T}\right) \tag{229}$$

relates vapour pressure to temperature in the region of capillary condensation. Where $\sigma_{\beta\gamma}$ is the inter-facial surface tension of the gas-liquid interface, and r is the characteristic length which must be determined experimentally as a function of ϵ_β. Whitaker combines the Clausius-Clapeyron and the Kelvin equation and takes the phase average where δT is assume to be negligible compared to $\langle T\rangle$, and therefore vanishes from the above equation.

Whitaker's derivation of heat and mass transport in porous media, results in a slightly cumbersome set of equations to solve, but can be simplified for a particular situation such as wood, to yield something more tractable such as was done in [1]. This was done by Plumb, Spolek, and Olmstead [1], who developed a one-dimensional model of wood drying based on Whitaker's theory. Although this thesis project will require the transport equations in three dimensions, [1] will be useful as a guide. In [1] it is pointed out that the transport coefficients in Luikov's model, are strong functions of temperature, moisture content, wood species, and drying conditions. It was reasoned that an extensive experimental program would be required in order to analyze the wide variety of wood species and drying methods. The diffusion model and Siau's model have very similar experimental requirements, therefore they also rely heavily on experimentally obtained transport coefficients, possibly to an even greater extent. This seems to be less apparent in Whitaker's model. Additionally, Whitaker's model provides a much more complete general theory of heat and mass transport in porous media from which a three-dimensional theory of log hydration can be derived with relative ease. Whitaker's theory will therefore be the basis of the wood hydration model to be used in this thesis project along with guidance from the other studies mentioned.

4 Numerical Method

4.1 Finite Difference

A numerical approach to the solution was chosen for essentially two reasons. First and for most, the complexity of the model will most likely make an analytical solution extremely difficult. Additionally, it will provide some experience in applying numerical methods for solving PDEs for real world problems. It is hoped that some experience with developing and using parallel multi-processor codes can be gained as part of the project.

Initially, the chosen numerical approach will be a finite difference scheme. If possible a finite element approach will also be applied and results compared. The code will be developed in C++. A scientific programming toolkit such as POOMA may be used to help facilitate the development of the C++ code. The advantages of such a toolkit is it will allow for faster development since most of the time intensive portions of the code have been already been developed and written in an optimized way. POOMA will also allow for automatic parallelization and scaling from PC to supercomputer if need be.

4.2 Cylindrical Coordinates

This thesis project involves solving the governing equations for full cylindrical timber segments. This means they will have to be transformed to cylindrical coordinates first. As a first approximation the system will be considered to be axially symmetric and isotropic. In doing this θ and z dependence can be ignored which will reduce the problem to one dimensional, or a disk shaped slice. As a second approximation, θ dependence can be included. This makes a good second approximation since the logs are stacked horizontally in the the conditioning tunnels and therefore assuming axial symmetry is not very reasonable. A final solution will be for a fully three dimensional cylindrical solution. This is will be required since wood is know to be anisotropic when comparing longitudinal to transverse transport. Another added refinement, which is relatively important in studying thermal and moisture content movement in wood is the difference in properties between the heartwood and the sapwood. This difference will also be considered, and the log will be split up into essentially two regions, a cylindrical outer shell and the cylindrical core. It isn't clear how this will be handled in the calculations yet.

5 Experimental Program

5.1 Model Verification

Verification of the model will require both thermal and possibly moisture content gradients to be measured for an intact log during conditioning. This will require temperature and moisture content sensors to be embedded directly into the log and the entire log placed into the conditioning tunnel. This will allow for the collection of time dependent thermal and moisture content gradients. In order to facilitate this, a large cradle has been built that can be loaded with logs and house all necessary data logging electronics. This will allow the experimental apparatus to be self contained and easily maneuvered by heavy equipment in and out of the conditioning tunnel. Experiments will be controlled as much as possible within the confines of an operating mill. The details as well as a few suggestions for over coming difficulties with controlling the experiments will be discussed below.

5.2 Controlling Experimental Setup

Producing controlled experiments in the conditioning tunnels of an operating mill will be one of the more difficult aspects of the projects. The log cradle has been built and tested to help in controlling and keeping the experiments relatively self contained. The cradle will allow reasonably accurate placement of the apparatus in different positions inside of the conditioning tunnel. Additionally, it will be possible to control and record the placement of logs in and around the test log. This is important since it is thought that the flow of water over the surface of the logs will be significantly different depending on the placement of the logs stacked on top. Placement within the experimental apparatus will be difficult to control due to the randomness in the log geometry. By knowing, how, or, if any logs are stacked on top or in and around the test subject effect on surface temperature and moisture distribution can be analyzed to a certain extent. The random surface and dimensions of the log segments results in contact between neighboring logs to effect heat and moisture distribution on the surface. It is possible, using the cradle, that one log could be isolated using some kind of scaffolding. By isolating a single log effects of contact with neighboring logs will be eliminated.

Another difficulty is the possible differences in flow rates of the different sprinklers. Along with this is the difference in ambient temperature and humidity throughout the conditioning tunnel, such as front to back differences. It is also thought that there is a significant temperature difference near to the outside walls of the conditioning tunnel during the winter months. These differences within the tunnel lead to possible differences in surface temperature and moisture content on the conditioning log. It is reasonable to assume that the surface conditions that the log is exposed to while conditioning will have a large bearing on conditioning times. One solution to this is to take data for all the different possible permutations of position and species. These would all have to be done 3 to 4 times to make sure the data is reproducible. This solution is probably not the best since the number of runs would be in the hundreds, and this is not practical. Additionally, even though the cradle is relatively portable and self contained, it's size and maneuverability would make it difficult to precisely place it in a known position within the tunnel. One possible solution to this problem is to collect surface data separate from the internal gradient data. Since the placement inside the tunnel only effects surface conditions, then all that is required is to record surface conditions in different parts of the tunnel. This can be accomplished by creating a very portable totally self contained single log shaped tube with sensors lining it's surface. All the required electronics can be enclosed inside of the cylinder protected from the harsh environment. This portable device can be easily loaded and placed in specific positions inside the conditioning chest. By doing this the number of runs with the cradle can be minimized by placing it in the same position within the chest for every run, and surface data can be collected separately. Temperature data could be collected by electrically and thermally isolating thermocouple in-bedded in the surface of the pipe. Water flow over the surface could be monitored using closely spaced electrical contacts that would close the circuit when conductive flow was sensed moving over the contacts. The flow of a certain region of the surface may fluctuate over time therefore the binary output of the flow sensors could be time averaged to gain an idea of flow over a certain time range.

5.3 Thermal Data

Thermal data is collected by way of thermocouples and thermistors. Due to the price of the thermistors, thermocouples are the sensor of choice in this study. For the internal data, holes are drilled in the log of order of the size of the thermocouple wire and the thermocouple is inserted. The space around the hole is filled with wood splinters and then the surface is plugged with rubber sealant. The thermocouple wire is run from the log to the data loggers located in a water proof box near the bottom of the cradle. The thermocouples used are type-T (? maybe type-K ?). Thermal data is logged via 32KB data loggers. The data logger/thermocouple setup has a resolution of 1.7^oC. The error associated with the thermocouple readings is relatively large but should be sufficient for model verification.

$$\pm 1^oC\% \text{ span} + 1.7^oC \text{ resolution} = \pm 2.7^oC. \tag{230}$$

5.4 Moisture Content Data

It may only be possible to collect initial and final moisture content gradients. There are a number of ways to collect moisture content data in wood, and only a few are portable enough to use in the conditioning tunnel. Out of the few that are reasonably portable it will still prove very difficult setup in a reliable way in the harsh conditioning tunnel environment. For this reason a coring device will be used to collect initial and final moisture content data. The initial and final moisture content data can be used to decide if real-time moisture content data will be required to verify the model. The coring device will allow rough moisture content gradients to be measured, as well as average moisture content.

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