Modelling height development of mature *Nothofagus obliqua*

Christian Salas [a,*], Oscar García [b]

[a]*Departamento de Ciencias Forestales, Universidad de La Frontera, Temuco, Chile*

[b]*University of Northern British Columbia, 3333 University Way, Prince George, B. C., Canada V2N 4Z9*

Abstract

We used stem-analysis data from 60 to 100 years-old second-growth stands, and single-point observations in 260–460 years-old trees, to study and model height development of *Nothofagus obliqua* ("roble"). The data covers a much wider range of ages than previous work on this species. A stochastic differential equation approach facilitated the estimation and comparison of alternative model variants based on the Richards equation. A number of methodological issues in the application of stem-analysis techniques were examined. It was found that breast-height age can improve the reliability of site quality assessment and growth projections, compared to the total or stump age currently used. Best results were obtained with a polymorphic model fitted to the stem-analysis observations, conditioning the asymptote to be compatible with the old-growth tree measurements.

Key words: Growth models, Breast-height age, Stem analysis, Site index, Stochastic differential equations, Roble, Chile

1 Introduction

The genus *Nothofagus*, southern beeches, includes some 40 species in Chile, Argentina, New Zealand, Australia, New Guinea, and New Caledonia, mostly

---

* Corresponding author. Current address: School of Forestry and Environmental Studies, Yale University, 360 Prospect Street, New Haven, CT 06511-2189, USA. Tel. +1(203)432-9398. Fax. +1(203)432-3809.

Email address: christian.salas@yale.edu (Christian Salas).

evergreen trees (Tuley, 1980). In Chile, the native species with the best potential for production forestry include *Nothofagus obliqua* (roble), *N. alpina* (= *procera* = *nervosa*, raulí), and further south, *N. pumilio* (lenga); all three are deciduous. *N. obliqua* is found between 33° and 41.5° of latitude South, with *N. alpina* spanning a slightly narrower latitudinal range (Donoso, 1995; Veblen et al., 1996). These two species are pioneers, with extensive young second-growth forests in south-central Chile, that could be easily managed due to their even-aged structure, high growth rates, and good quality wood (Grosse, 1989; Grosse and Quiroz, 1999; Rothermel, 2002). Actual management experience with adult stands is limited. Both *N. obliqua* and *N. alpina* have also generated much interest in the U.K., where growth rates surpass those of most other broadleaves and approach those of the fastest growing conifers (Pearce, 1977; Tuley, 1980; Danby, 1991). Less extensive trial plantings have been done in France and other countries.

As pointed out by Trincado et al. (2002), there is much ecological knowledge and scattered growth information about the second-growth roble (*N. obliqua*) forests, but few practical mensurational tools for forest management. Among these are size-density relationships from Puente et al. (1979), tree volume tables in Grosse and Cubillos (1991) and a delineation of growth zones by Donoso et al. (1993). Yield tables for plantations of roble and raulí in the UK were developed by Christie et al. (1974). An important element is site-index models, used to assess site productivity and for predicting height growth. Following some more localized studies, Trincado et al. (2002) developed models for *N. obliqua* and *N. alpina* based on stem analysis data collected by Donoso et al. (1993) over the whole distribution area. Old-growth roble forests are scarce, and all these studies have been based on data from young second-growth stands or plantations. Considerable uncertainty remains about later growth.

We used data from mature *N. obliqua* forest remnants in Rucamanque, near the city of Temuco, to supplement the existing knowledge about the height development of this species. Height-age curves based on detailed stem analysis were conditioned to match the ultimate heights attained in older stands. We found advantageous to use breast-height age, instead of the total or stump-height age previously used.

A number of methodological issues had to be addressed. These include stem-analysis bias reduction approaches (Dyer and Bailey, 1987), the definition and determination of breast-height age, the proper modelling of growth around breast height, and the constraining of height-curve asymptotes (Shifley and Brand, 1984). Although our data's geographical range is restricted, the methods and results should be useful for future modelling, and add to our current knowledge of the growth of *N. obliqua*.

2 Data

2.1 Study area

Stem-analysis data were obtained from three stands of 2.4, 13.2, and 35.2 ha, in Rucamanque (38° 39' S, 72° 35' W), Temuco county, Cautín province, IX Region, Chile. These stands are even-aged fire-origin second growth (renoval) dominated by roble (67% in number of trees, and 75% in basal area), with averages of 1006 stems/ha and 55 m^2/ha of basal area. They belong to the second-growth forests sub-type of the Roble-Raulí-Coigüe forest type according to Donoso (1995). Sample tree ages ranged from 63 to 109 years.

Rucamanque also contains 230 ha of a later successional stage dominated by tolerant tree species, with scattered emergent 200–500 year-old roble. Current heights and ages were obtained from some of these trees for information on height asymptotes. These old-growth stands have approximately 700 trees/ha, 100 m^2/ha of basal area, and 1,100 m^3/ha total volume, the most abundant trees being *Aextoxicon punctatum* (olivillo). Roble constitutes about 20% of the volume and basal area, with some 20 trees/ha and a breast-height diameter of over 80 cm. These stands belong to the original remnants sub-type of the Roble-Raulí-Coigüe forest type (Donoso, 1995).

The forest has had practically no human intervention. It is located at an altitude of between 350 and 450 m, on deep rich soils derived from volcanic ash (Andisols, "trumaos"), where productivity is above average (Frank, 1998; Salas, 2001). Further details are given by Salas (2001, 2002).

2.2 Measurements

In each second-growth stand, five two-plot clusters were selected on a 100 m grid. The plots were circular, of 200 m^2, separated by 25 m within a cluster. The largest-dbh roble in a plot was selected for stem analysis, provided it was healthy and of good form, of seed origin, and belonging to the upper canopy.

After measuring diameter at breast height (dbh) and total height (h), the selected trees were felled, and cross-sectional disks were obtained at stump height (30 cm), 80 cm, and breast-height (1.3 m), plus another 10 disks evenly spaced between breast-height and the crown base. Additional disks were extracted at 2 m intervals on the main stem between the crown base and a limiting diameter of 5 cm. Rings were counted and measured in the laboratory; only the counts were used in this study.

In the old growth forest, 30 healthy roble trees were selected from permanent sample plots. Heights were measured with a hypsometer, and ages estimated from breast-height increment cores. Old roble often acquires an extremely hard inner heartwood that cannot be penetrated by the increment borer. In those instances, a regression of breast-height age over dbh based on the second-growth stands was used to estimate missing rings in incomplete cores (Lorimer, 1980; Salas and Marinao, 2005).

Tree characteristics are summarized in Table 1 (see also Figures 3 and 5). Although less extensive, and originating from a narrow geographical area, our data goes well beyond the maximum stump age of 56 years in Trincado et al. (2002).

Table 1
Data summary. *Bh age* (breast-height age) is the number of growth rings at breast height, *Age* is estimated age from seed.

	Second-growth ($n = 30$)					Old-growth ($n = 30$)		
	DBH	Height	Bh age	Stump rings	Age	DBH	Height	Bh age
	(cm)	(m)	(years)	(number)	(years)	(cm)	(m)	(years)
Minimum	27.2	22.1	60	62	63	79.2	33.7	262
Maximum	57.8	37.0	95	103	109	126.1	53.5	461
Mean	43.9	30.6	76.5	83.5	85.7	97.0	43.6	325
SD	7.8	4.1	9.4	9.8	10.4	12.8	5.1	47.4

3 Methods

3.1 Age

Total age (years from seed) at the time of sampling was estimated by linear extrapolation of the 0.8 m and stump ring counts, down to ground level, rounding to the nearest integer. Differences between total age and stump count ranged from 1 to 6, averaging 2 years.

Nothofagus growth and site-index studies to date have used either total age (Moreno, 2001) or stump age (Donoso et al., 1993; Trincado et al., 2002). Obtaining these ages through ring counts near the base of the tree is inconvenient and inaccurate. Stump heights tend to vary among studies, making comparisons difficult. In addition, early growth is affected by establishment conditions and other factors unrelated to site productivity. Furthermore, a stand total age is difficult to define in many second-growth forests, where trees originate both from seed and from coppicing (Donoso, 1995). To address these difficulties, Husch (1956) proposed the use of *breast-height age*, the number of years

since reaching breast height, a measure that has since been extensively used for natural stands (Stage, 1963; Monserud, 1984; Hann, 1995; Smith et al., 1997). Of course, for some management purposes a total age is still needed, and this can be obtained by adding an independent estimate of years to breast height, appropriate to each particular situation.

There seems to be some ambiguity in exactly how breast-height age is defined and determined. Husch et al. (1972), for instance, say that it is the number of years since the tree reached breast height, but in applications it is usually calculated as the number of growth rings at breast height (Haddon, 1988). Nigh (1995) points out that these values are slightly different, for reasons analogous to those for section measurements discussed below, and proposes defining breast-height age as the breast-height ring count minus 0.5. We chose to maintain the simpler traditional definition of breast-height age as the number of rings, and to take into account the time difference in the modelling.

3.2 Section height-age estimates

Heights and ring counts from cross-section disks are used to calculate height-age pairs, reconstructing tree growth. A cross-section height is attained at an age equal to the (total or breast-height) age at the time of sampling, minus the section ring count, plus one. In general, at that age the tree has surpassed the section height, and Dyer and Bailey (1987) review methods of adjusting for this by estimating the length of the "hidden tip". They recommend the method of Carmean (1972), where the height at the section is increased by half of an annual height increment, estimated as the ratio of the difference in heights to the difference in ring counts between that cross-section and the one above it.

We used an alternative correction suggested by Dr. James Goudie, from the British Columbia Ministry of Forests (Nigh, 1995), that is simpler and perhaps more accurate. Instead of increasing the section height, the calculated section age is reduced by 0.5 years. That is, each cross-section height is assigned an age equal to the age at sampling, minus the ring count, plus 0.5 years. Figure 1 compares both methods on a typical tree.

By a similar argument, we assume that breast-height (1.3 m) is reached at 0.5 years breast-height age.

Following Dyer and Bailey (1987), it is now common to extend hidden tip adjustments to the generation of values for every year (e.g., Donoso et al., 1993; Nigh, 2000; Trincado et al., 2002). Apart from artificially inflating the sample size, however, this interpolation can distort the curves shape. As in Carmean (1972), we used a single height-age pair associated to each cross-
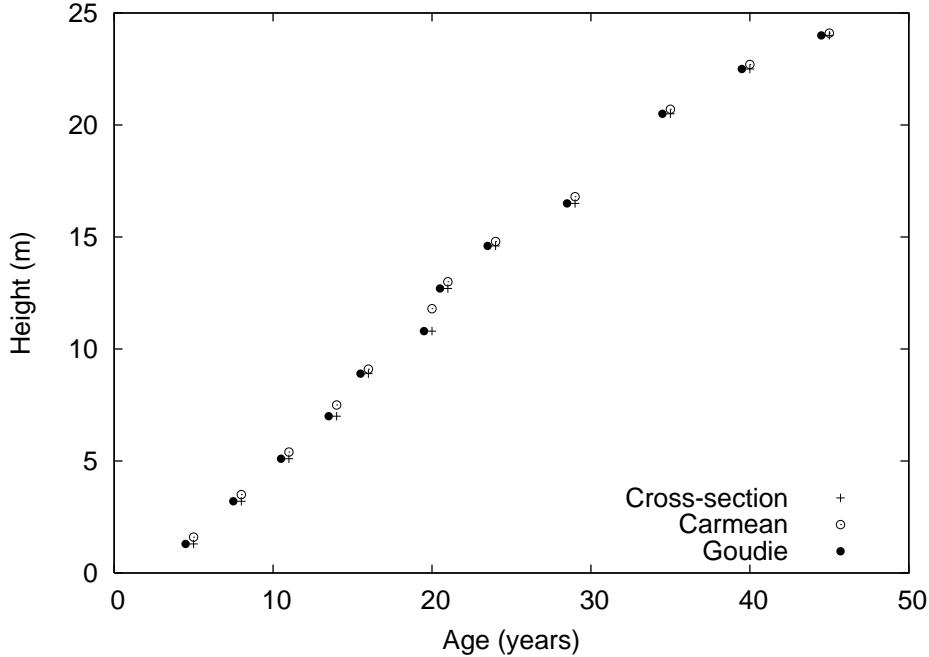
5

Fig. 1. Example of stem-analysis height-age corrections by the methods of Carmean (1972) and Goudie (Nigh, 1995). Tree of *Nothofagus obliqua*, 102 years-old, height 33 m, dbh 56 cm.

section.

3.3 Models and statistical inference

We used the Richards growth model (Richards, 1959), also known as Berta-lanffy or Chapman-Richards (Vanclay, 1994), one of the most commonly used height growth equations. Its differential form can be conveniently linearized as

$$\frac{\mathrm{d}H^c}{\mathrm{d}t} = b(a^c - H^c) \,, \tag{1}$$

which integrates to

$$H = a\{1 - [1 - (H_0/a)^c] \exp[-b(t - t_0)]\}^{1/c} \,, \tag{2}$$

where H and H_0 are heights (meters), t and t_0 are ages (years), and a, b and c are parameters to be estimated (García, 1983). One of the parameters (or a combination of them obtained through re-parameterization) is assumed to be site-quality dependent, being specific to each plot ("local"), while the others
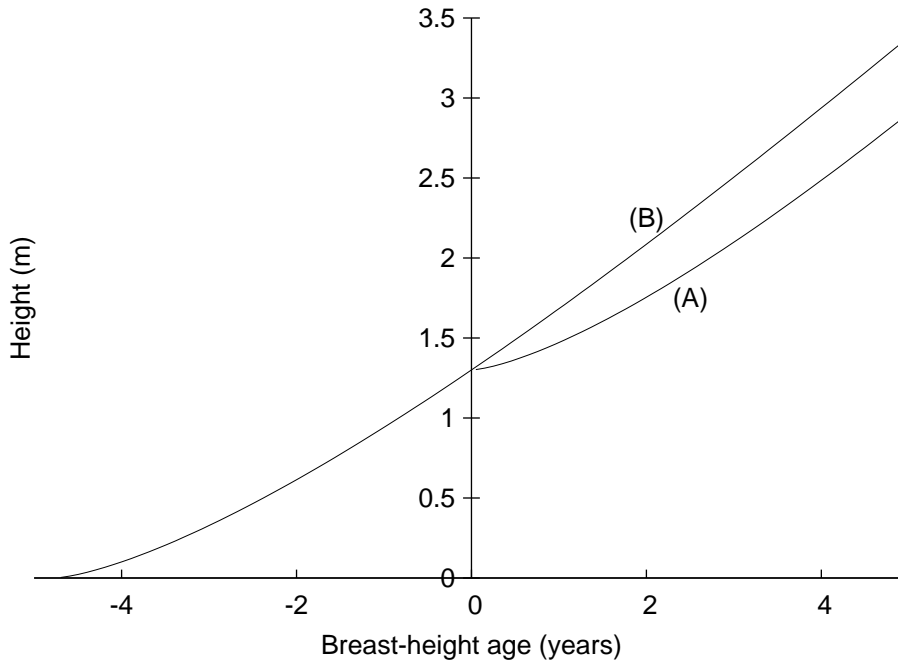
6

Fig. 2. Modelling growth near breast height. (A): Conventional vertical shifting of a growth curve $f(t)$, to accommodate the use of breast-height age, $H = 1.3 + f(t)$. Note zero growth rate at breast height. (B): Our approach corresponds to a horizontal shift, $H = f(t + k)$. In general, the years-to-breast-height k are site-dependent. Our definition of breast-height age differs by 0.5 years from the one implied in this figure (see text).

are common to all plots ("global"). With a fixed origin (t_0, H_0), equation (2) describes a family of height development curves. Each curve of the family is identified by a particular value of the local parameter, chosen as varying freely across sites. More conventionally, a site index S can be used, defined as the height reached at some base age t_b. The site index is a special case of local parameter, related to any other through the substitution of S and t_b for H and t in (2) (García, 1983, 2005).

We tested two model variants for the effect of site quality, one where the local parameter is the height asymptote a, and another where the local parameter is the time scale factor b. The first option produces anamorphic curves, the effect of site being a change in the vertical H scale. The second option gives so-called polymorphic curves, with the horizontal time scale dependent on site.

For models using total age, we made both t_0 and H_0 equal to zero in (2). Breast-height age models were obtained by setting $t_0 = 0.5$ and $H_0 = 1.3$. This is somewhat more complicated than the usual device of simply adding breast height to a zero-origin sigmoidal height-age equation (e. g., Monserud, 1984; Hann, 1995; Nigh, 1995; Smith et al., 1997). This last procedure, however, results in an unrealistic growth rate of zero at breast height (Figure 2).

Estimation used the approach of García (1983), implemented in the software package *EasySDE* (`http://forestgrowth.unbc.ca/sde`). Essentially, the method represents variability in the data by adding white noise to the right-hand-side of (1), simulating environmental fluctuation, and by including independent random measurement errors in the observed heights. The likelihood function for the resulting stochastic model is calculated, and the parameter values that maximize this function are found with a specialized optimization algorithm. That is, all the parameters, global and local, including the error variances, are estimated simultaneously by maximum likelihood. Extensive experience with the technique has proven it to be highly efficient and robust, even in instances of scarce or poor quality data.

The maximized log-likelihood can be used to compare model formulations, if necessary penalizing more complex models by subtracting between 1/2 and 2 log-likelihood units for each additional parameter (the popular AIC criterion implies 1 unit). With this adjustment, differences of about two units may be considered as "significant". Hypotheses can also be evaluated through a likelihood ratio test, based on the fact that twice the log-likelihood difference is asymptotically distributed as a χ^2 with degrees-of-freedom equal to the difference in the number of parameters (García, 2005).

4 Results

4.1 Total age

The total-age model with b as the local parameter gave a calculated log-likelihood 12.1 units higher than the anamorphic version. The equation is:

$$H = 41.89(1 - \mathrm{e}^{-bt})^{1/0.6454} \ . \tag{3}$$

The parameter b can be related to the conventional site index by substituting site index and base age for H and t in (3). The data and growth curves based on this model are shown in Figure 3.

It appears that some trees, although dominant and apparently normal at the time of sampling, had been severely suppressed in their early growth. Removing these from the dataset produced a small change in the model parameters. In applying the model to standing height-age observations, however, early growth patterns are normally unknown. It is clear that early suppression would cause serious prediction errors.
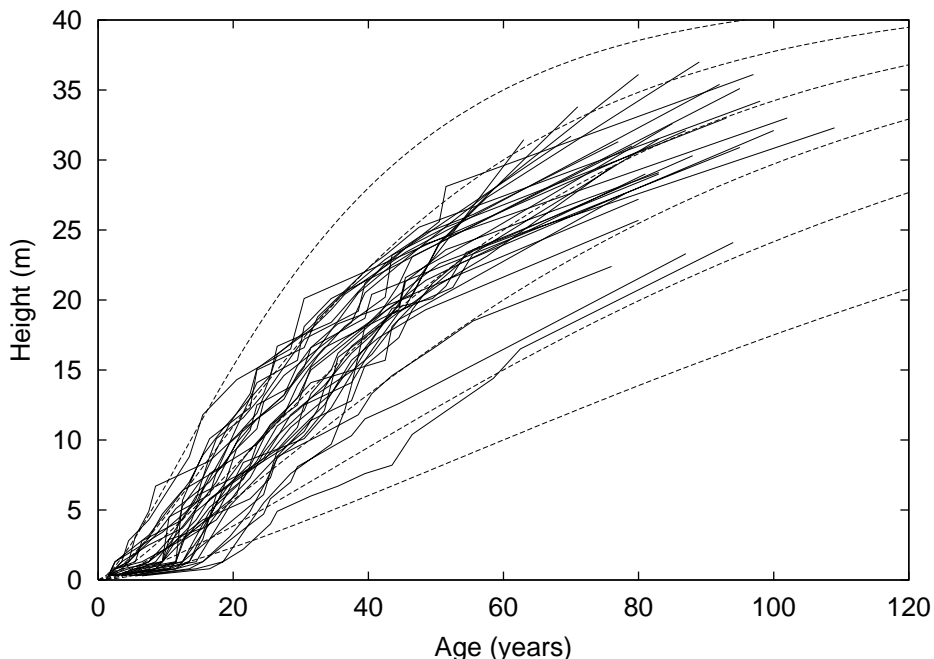
8

Fig. 3. Stem-analysis data, using total age. Successive observations on the same tree are joined by lines. Dashed: growth curves from model (3).

4.2 Breast-height age

With breast-height age the effect of early suppression is much reduced (Figure 4). Only data above breast-height is used to estimate the parameters. We also used a model with a local, and other with b local.

Again, the variant with b local produced a better fit than the one with a local, with a log-likelihood difference of 13.9 units. The model is:

$$H = 39.56[1 - 0.9587\,\mathrm{e}^{-b(t-0.5)}]^{1/0.9329}\,, \tag{4}$$

where t is now breast-height age (number of rings at breast-height).

To compare with total age, the b-local total-age model was re-estimated using only the measurements above breast height. The log-likelihood obtained for the total-age model was 50.2 units lower than that for the breast-height age model, confirming the better fit of this last one.

The differences between total and breast-height age ranged from 2 to 18, with a mean of 9.2 years.
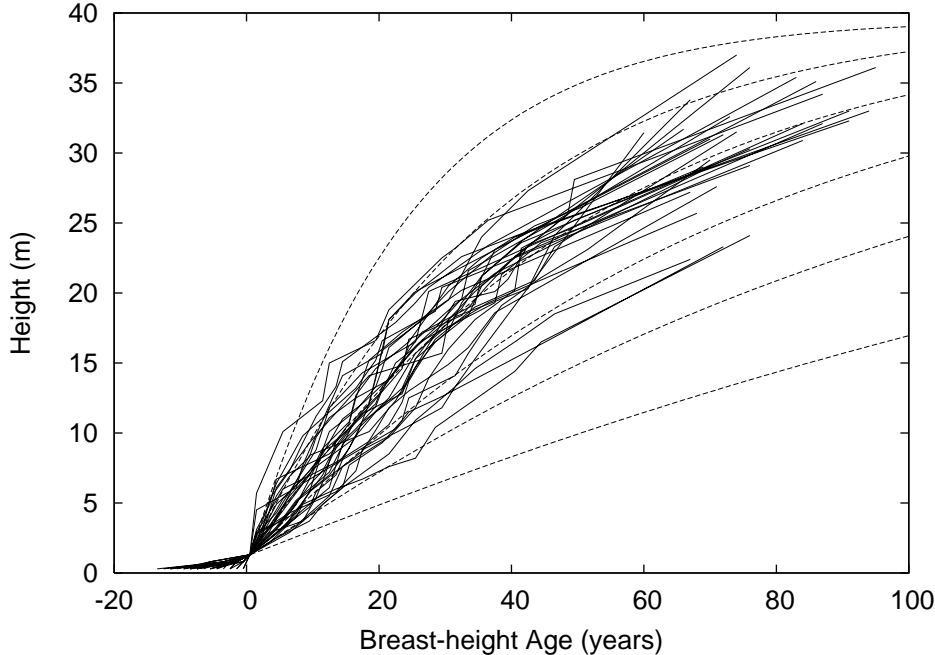
9

Fig. 4. Stem-analysis data, using breast-height age. Successive observations on the same tree are joined by lines. Dashed: growth curves from model (4).

4.3 Constraining the asymptote

The 40 m height asymptote in (4) seems inconsistent with our old-growth data, which suggests higher values. Uncertainty in this parameter estimated from the stem-analysis data is to be expected, given that the second-growth forests are relatively young. We therefore tried a model where the asymptote a is fixed at a specified level. Similar constraints have been advocated by Shifley and Brand (1984).

We used the code generation facility in EasySDE to produce a breast-height-age model variant with b local, as in (4), but with a fixed. An asymptote of 50 m gave a visually satisfactory fit (Figure 5):

$$H = 50[1 - 0.9801 \, \mathrm{e}^{-b(t-0.5)}]^{1/1.0746} \, . \tag{5}$$

The local parameter is related to a conventional site index S at a breast-height base age of t_b years by

$$b = -\ln\{[1 - (S/50)^{1.0746}]/0.9801\}/(t_b - 0.5) \, . \tag{6}$$

This can be substituted in (5) to produce an expression directly in terms of S:
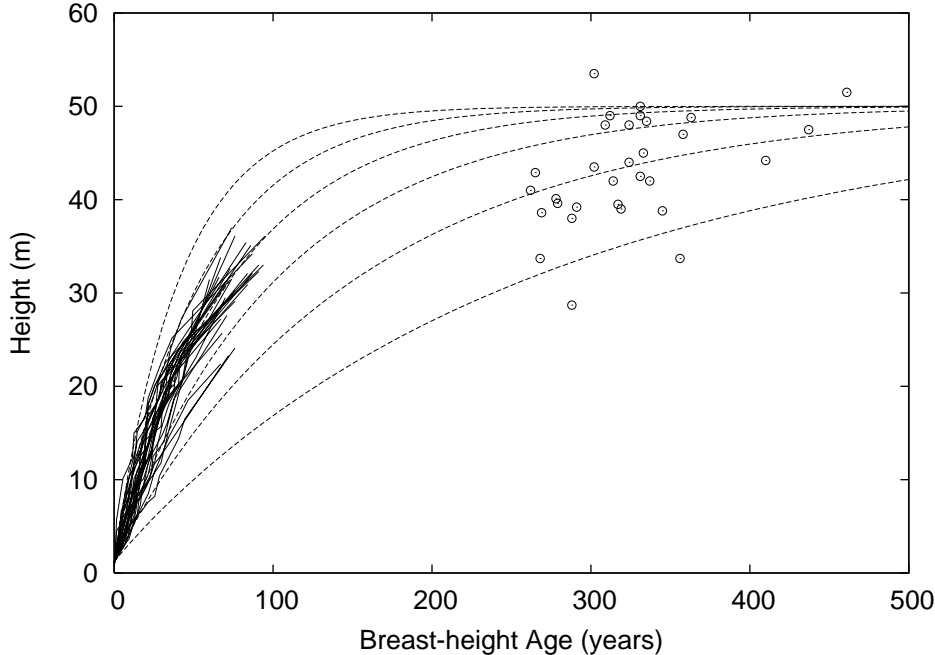
10

Fig. 5. Stem-analysis and old-growth data. Growth curves from model (5).

$$H = 50\{1 - 0.9801[(1 - (S/50)^{1.0746})/0.9801]^{\frac{t-0.5}{t_b-0.5}}\}^{1/1.0746} , \qquad (7)$$

Clearly, with the constrained asymptote the fit to the stem-analysis data can not be as good as that of the unconstrained model (4): the calculated log-likelihood is 10.5 units lower. A likelihood-ratio test for the null hypothesis $a = 50$ gives a p-value of 5×10^{-6}, although the relevance of such a test can be questioned (Shifley and Brand, 1984). From a practical point of view, the site index curves (base age 50) with or without the asymptote constraint differ by a maximum of about 6% over the range of the stem-analysis data.

5 Discussion and conclusions

Our stem-analysis data, with ages of up to about 100 years, extends considerably the age coverage previously available for the height development potential in *Nothofagus obliqua*. In addition, our old-growth height-age measurements help in defining upper height limits. The mean is somewhat higher than the 40 to 42 m in 400 years-old emergent individuals in roble remnants at Lake Villarrica reported by Veblen et al. (1979) and Donoso (1995, p. 337). We are not aware of any other similar roble observations in the literature. The heights are similar to those of other old-growth trees in Rucamanque for which we do not have age data.

11

Some trees suffered severe early height growth suppression for extended periods of time, before recovering and reaching dominant positions in the canopy. Because only one tree per plot was sampled, we cannot ascertain if this involved a single individual, or if the whole plot top height was affected. Similar trends in top heights, however, can be seen in the graphs of Trincado et al. (2002). Early height suppression is common in other species, and ring width measurements are sometimes used in screening of stem-analysis trees for site index modelling (Stage, 1963; Monserud, 1984).

The influence of early suppression on growth projections can be much reduced by using breast-height age. Breast-height age, not previously used in these forests, is also easier to obtain in the field. We recommend its use for site quality assessment, and for growth projections in existing stands.

For some planning purposes, an estimate of years-to-breast-height would be needed. This time seems to vary widely, and (site-dependent) estimates appropriate to various establishment and early development conditions should be used. From our data, and the plot b-estimates produced by EasySDE, we get $0.1127/b$, where b is as in (6). The average of 9.2 years is more than the 4 to 6 years commonly assumed in Chile for roble second-growth forests. Planted stands would also differ, and more research is needed on this subject.

It should be noted that, even excluding "abnormal" development during the establishment phase, stem analysis can introduce bias in height-age predictions due to the fact that trees that are dominant at sampling time may not have been dominant in previous years (Dahms, 1963; Magnussen and Penner, 1996; García, 2005). This would increase the steepness of estimated curves, although the magnitude of such bias is uncertain.

There is some variation and ambiguity in the literature about precise age definitions and adjustments for bias in stem analysis. We defined breast-height age as the number of growth rings at breast height (1.3 m). Goudie's correction seems a good alternative to the commonly used Carmean's method, being simple and logical. Differences between these methods or definitions generally amount to fractions of one year, and may be of little consequence in this instance. They can be more important with fast growing species.

The common practice of interpolating cross-section information to every year seems undesirable; we used only one height-age pair associated to each cross-cut. The stochastic differential equation approach proved to be flexible and easy to use for consistently applying different model variants and hypotheses, and produced satisfactory fits to the data.

Although it is not as close to the younger stem-analysis data as (4), we prefer model (5), which reflects more realistically the heights observed in old-growth trees. If using it as a site index model, however, it should be remembered that

it is based on a relatively small sample, from a restricted geographical area. Ideally, new models should be obtained in the future combining this with the data of Donoso et al. (1993) and other data sources.

Acknowledgments

References

Carmean, W., 1972. Site index curves for upland oaks in the central states. Forest Science 18 (2), 109–120.

Christie, M., Miller, A. C., Brumm, L. E., 1974. Nothofagus yield tables. paper 106, Forestry Commission Research and Development, 5 p.

Dahms, W. G., 1963. Correction for possible bias in developing site index curves from sectioned tree data. Journal of Forestry 61, 25–27.

Danby, N. P., 1991. Nothofagus in Wales. Quarterly Journal of Forestry LXXXV (2), 103–116.

Donoso, C., 1995. Bosques Templados de Chile y Argentina. Variación, Estructura y Dinámica, 3rd Edition. Editorial Universitaria, Santiago, Chile, 484 p.

Donoso, P., Donoso, C., Sandoval, V., 1993. Proposición de zonas de crecimiento de renovales de roble (*Nothofagus obliqua*) y raulí (*Nothofagus alpina*) en su rango de distribución natural. Bosque 14 (2), 37–55.

Dyer, M. E., Bailey, R. L., 1987. A test of six methods for estimating true heights from stem analysis data. Forest Science 33 (1), 3–13.

Frank, D., 1998. Umweltauswirkungen des Landnutzungswandels in der IX. Region Chiles : Untersuchung von Waldökosystemen und forstlichen Monokulturen am Beispiel der Umgebung Temucos. Mesnsch und Buch Verlag, Berlin, Germany, 222 p.

García, O., 1983. A stochastic differential equation model for the height growth of forest stands. Biometrics 39, 1059–1072.

García, O., 2005. Comparing and combining stem analysis and permanent sample plot data in site index models. Forest Science 51 (4), 277–283.

Grosse, H., 1989. Renovales de raulí, roble, coigüe y tepa, expectativas de rendimiento. Ciencia e Investigación Forestal 3 (6), 37–72.

Grosse, H., Cubillos, V., 1991. Antecedentes generales para el manejo de renovales de raulí, roble, coigüe y tepa. Informe Ténico 127, Instituto Forestal, Concepción, Chile.

Grosse, H., Quiroz, I., 1999. Silvicultura de los bosques de segundo crecimiento de roble, raulí y coigüe en la región centro-sur de Chile. In: Donoso, C., Lara, A. (Eds.), Silvicultura de los Bosques Nativos de Chile. Editorial Universitaria, Santiago, Chile, pp. 95–125.

Haddon, B. D. (Ed.), 1988. Forest Inventory Terms in Canada. Canadian Forest Inventory Committee, Forestry Canada, Chalk River, Ontario, `http://nfi.cfs.nrcan.gc.ca/terms/terminology_e.html`.

Hann, D. W., 1995. A key to the literature presenting site-index and dominant-height-growth curves and equations for species in the Pacific Northwest and California. Research Contribution No. 7, Forest Research Laboratory, Oregon State University, Corvallis, USA, 26 p.

Husch, B., 1956. Use of age at D. B. H. as a variable in the site index concept. Journal of Forestry 54 (5), 340.

Husch, B., Miller, C. I., Beers, T. W., 1972. Forest Mensuration, 2nd Edition. Wiley.

Lorimer, C. G., 1980. Age structure and disturbance history of a Southern Appalachian virgin forest. Ecology 61 (5), 1169–1184.

Magnussen, S., Penner, M., 1996. Recovering time trends in dominant height from stem analysis. Canadian Journal of Forest Research 29, 9–22.

Monserud, R. A., 1984. Height growth and site index curves for Inland Douglas-fir based on stem analysis data and forest habitat type. Forest Science 30 (4), 943–965.

Moreno, P., 2001. Proposición preliminar de curvas de índice de sitio para renovales de raulí. Tesis Ingeniero Forestal, Universidad de Chile, Santiago, Chile, 72 p.

Nigh, G. D., 1995. Compatibility improvements and bias reduction in height-age models. Research Report 03, British Columbia Ministry of Forests, Research Branch, 10 p.

Nigh, G. D., 2000. Western redcedar site index models for the interior of British Columbia. Research Report 18, Ministry of Forests Research Program, Victoria, B.C., Canada, 24 p.

Pearce, M. L., 1977. The Nothofagus in Britain. Forestry and British Timber 6 (1), 20–22.

Puente, M., Donoso, C., Peñaloza, R., Morales, E., 1979. Estudio de raleo y otras técnicas para el manejo de renovales de raulí (*Nothofagus alpina*) y roble (*Nothofagus obliqua*). Etapa I: Identificación y caracterización de renovales de raulí y roble. Informe de convenio No. 5, Proyecto CONAF/PNUD/FAO-CHI/76/003, Santiago, Chile, 88 p.

Richards, F. J., 1959. A flexible growth function for empirical use. Journal of Experimental Botany 10 (29), 290–300.

Rothermel, H., 2002. Economía del Manejo Sustentable — Una Alternativa para el Bosque Nativo. Ediciones Universidad Mayor — Serie Libros, Santiago, Chile, 173 p.

Salas, C., 2001. Caracterización básica del relicto de biodiversidad Rucamanque. Bosque Nativo 29, 3–9.

Salas, C., 2002. Ajuste y validación de ecuaciones de volumen para un relicto del bosque de roble-laurel-lingue. Bosque 23 (2), 81–92.

Salas, C., Marinao, J., 2005. Modelo estimador de la EDAP en función del diámetro a la altura del pecho para *Nothofagus obliqua* (roble). Informe Interno No. 1 Proyecto DIUFRO 110201, Departamento de Ciencias Forestales, Universidad de La Frontera, Temuco, Chile, 10 p.

Shifley, S. R., Brand, G. J., 1984. Chapman-Richards growth function constrained for maximum tree size. Forest Science 30 (4), 1066–1070.

Smith, D. M., Larson, B. C., Kelty, M. J., 1997. The Practice of Silviculture: Applied Forest Ecology, 9th Edition. Wiley, New York, USA, 537 p.

Stage, A. R., 1963. A mathematical approach to polymorphic site index curves for grand fir. Forest Science 9 (2), 167–180.

Trincado, G., Kiviste, A., von Gadow, K., 2002. Preliminary site index models for native roble (*Nothofagus obliqua*) and raulí (*N. alpina*) in Chile. New Zealand Journal of Forestry Science 32 (3), 322–333.

Tuley, G., 1980. Nothofagus in Britain. Forest Record 122, Forestry Commission.

Vanclay, J. K., 1994. Modelling Forest Growth and Yield: Applications to Mixed Tropical Forests. CABI International.

Veblen, T. T., Ashton, D. H., Schlegel, F. M., 1979. Tree regeneration strategies in a lowland Nothofagus-dominated forest in south-central Chile. Journal of Biogeography 6 (4), 329–340.

Veblen, T. T., Hill, R. S., Read, J. (Eds.), 1996. The Ecology and Biogeography of *Nothofagus* Forests. Yale University Press.