Site quality

Site:
- place + environmental conditions

Quality:
- potential productivity (for a species or forest type)

Main growth factors:
- site
- density

Read Chapter 2 of Clutter *et al* “Timber Management: A Quantitative Approach”

Methods

A. Direct
   1. Historical
   2. Volume
   3. Height

B. Indirect
   1. Inter-species relationships
   2. Indicator species
   3. Environmental variables (climate, soil, topography)

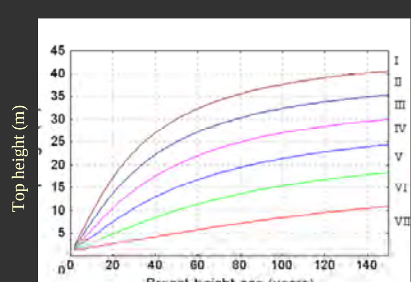
A1 is generally impractical in forestry.
A2 can fail with variable densities.
A3 usually OK with even-aged stands.
B is only choice with bare land or uneven-aged (but see Vanclay for research on uneven-aged direct methods).

Site Index (height-age)

- For even-aged stands
- Stratification
- Age:
  total, planting, breast-height
- Height:
  dominant / co-dominant, n tallest, n fatest
  (dominant, predominant, top height, site height)

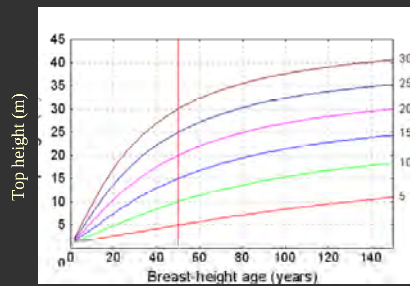
Should be used **within** bio-climatic regions.
BC Forest Productivity Council reserves the term “top height” for heights according to a particular standard method, recommends “site height” otherwise.

Site index curves



Interior lodgepole pine curves (Goudie 1984). In: Thrower, J.S. *et al*. 1994 *“Site index curves and tables for British Columbia interior species”* (2nd edition). BC MOF Res.Br., Land Manage.Handb. Field Guide Insert 6.
Labelling of curves is arbitrary (quality classes here).

Site index curves



Conventional site index labeling: height at *index* or *base age*.
Index age should not matter.

Growth intercept methods

- E.g., 5-year intercept: 5 whorls above breast height
- Use directly ("growth intercept index"). Or relate to SI
- Sometimes easier than SI. Good for young stands
- Two points. "Abnormal" establishment conditions.

For details see publications in www.for.gov.bc.ca/hre/sitetool/publicat.htm

- Indicator plants: Finland (Cajander 1926)
- Regressions on soil, climate, topography
- SIBEC

  Site Index from BEC (BC Biogeoclimatic Ecosystem Classification)

SIBEC combines vegetation, soil properties, etc. See www.for.gov.bc.ca/hre/sibec

Data for site index

- Temporary plots
- PSPs
- Stem analysis

Stem analysis. From *Münch and Thompson*, "The Structure and Life of Forest Trees", Chapman & Hall, 1929.

Reventlow (Denmark, ca. 1800)

Table B. Diameter Increment 1763–1792.

Forest	Numbers	1792	1791	1790	1789	1788	1787	1786	1785	1784	1783	1782	1781	1780	1779	1778
		The diameter increment in lines was in														
Pederstrup and Skjelstofte	8	10.0	10.2	9.7	12.0	10.8	8.7	8.1	8.8	8.2	9.1	9.6	7.6	8.2	9.4	10.7
Average		1.25	1.27	1.21	1.50	1.35	1.09	1.01	1.10	1.02	1.14	1.20	0.95	1.02	1.17	1.34
Christiansæde	10	10.0	9.9	11.2	12.7	12.2	11.3	12.0	11.5	13.5	13.7	14.5	12.2	13.7	13.6	13.2
Average		1.00	0.99	1.12	1.27	1.22	1.13	1.20	1.15	1.35	1.37	1.45	1.22	1.37	1.36	1.32

Forest	1777	1776	1775	1774	1773	1772	1771	1770	1769	1768	1767	1766	1765	1764	1763
	The diameter increment in lines was in														
Pederstrup and Skjelstofte	8.0	7.5	7.5	7.0	9.0	7.2	8.0	8.6	8.3	7.5	6.6	7.5	6.6	6.2	7.4
Average	1.00	0.94	0.94	0.87	1.12	0.90	1.00	1.07	1.04	0.94	0.82	0.94	0.82	0.77	0.92
Christiansæde	12.0	12.5	12.5	15.0	12.0	12.7	11.7	10.5	12.7	12.2	12.2	11.7	12.0	12.5	12.0
Average	1.20	1.25	1.25	1.50	1.20	1.27	1.17	1.05	1.27	1.22	1.22	1.17	1.20	1.25	1.20

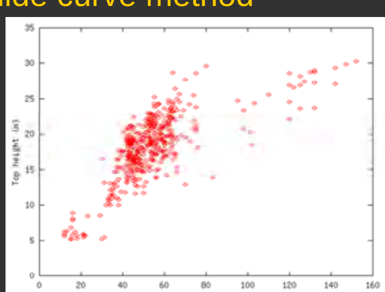
Diameter increments from rings of trees felled in 1792.

Table 3. The Height Increment of 22 Sorø Oaks.

Tree No.	Number of annual rings at butt	Number of annual rings at upper end	Trunk has grown up in years	Height of stump feet	Height of trunk with stump feet	Height of the tree feet	Average annual increment of trunk feet	Average annual increment of crown feet
1	139	74	65	0.50	37.00	58.00	0.56	0.28
2	126	67	59	0.58	36.08	43.58	0.60	0.11
3	136	75	61	0.50	31.50	40.00	0.51	0.11
4	139	80	59	0.17	32.17	50.17	0.54	0.23
5	136	106	30	0.75	28.75	51.75	0.93	0.22
6	138	107	31	0.50	22.50	48.50	0.71	0.24
7	138	98	40	0.75	21.75	47.75	0.52	0.27
8	138	109	29	1.00	18.00	46.00	0.59	0.26
9	139	100	39	0.66	27.66	53.66	0.69	0.26
10	135	106	29	0.75	31.75	43.75	1.07	0.11
11	130	100	30	0.66	15.66	38.66	0.50	0.23
12	129	109	20	1.00	20.00	52.00	0.95	0.29
13	143	100	43	0.83	28.83	60.83	0.65	0.32
14	130	91	39	0.83	22.83	48.83	0.56	0.29
15	135	88	47	0.50	42.50	62.50	0.89	0.23
16	127	92	35	0.66	29.66	50.66	0.83	0.23
17	139	80	59	0.58	31.58	57.58	0.53	0.32
18	136	94	42	0.66	24.66	62.66	0.57	0.40
19	128	95	33	0.66	26.66	58.66	0.79	0.34
20	187	96	91	0.85	22.75	56.75	0.24	0.35
21	143	93	50	0.58	32.58	66.58	0.64	0.37
22	183	95	88	0.66	34.66	64.66	0.39	0.32
Total	3074	2055	1019	14.53	619.53	1163.53	14.26	5.78
Average ...	139.73	93.41	46.32	0.66	28.16	52.89	0.65	0.26

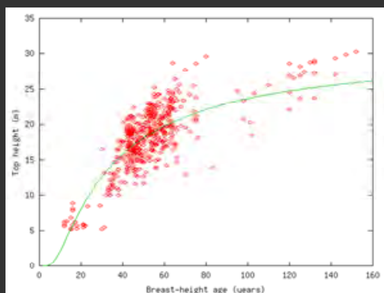
Reventlow: height calculations from stem analysis.

Guide curve method



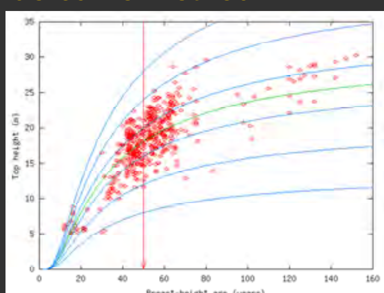
Temporary sample plots (actually, PSP lodgepole pine data from VDYP overlay files, but we ignore re-measuring info for now).

Guide curve method



Guide curve.

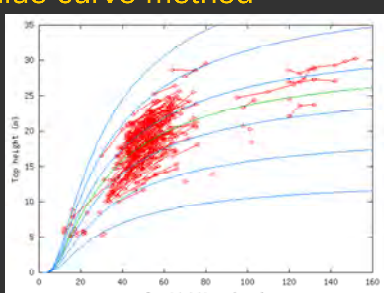
Guide curve method



Other curves proportional (along the vertical) to the guide curve. “Anamorphic”.

Non-proportional curves (“polymorphic”, i.e., not anamorphic) are sometimes used.

Guide curve method



PSPs give more information. The info could be used with hand-drawn curves, but it is not used with equation-based guide curve methods.

Guide curve method

Guide curve: $H = f(t)$

Site index curves: $H = k\, f(t)$

$= [S / f(50)]\, f(t)$ (anamorphic)

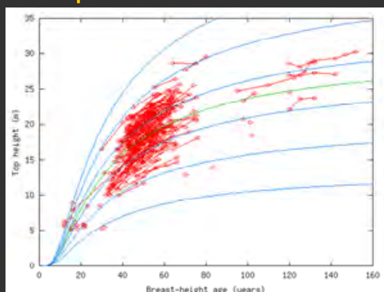
E.g., Schumacher: $H = a \exp(- b / t)$

fit $\ln H = \ln a - b / t$, or $\ln H = \alpha - b\, (1 / t)$

Schumacher’s model is often used.

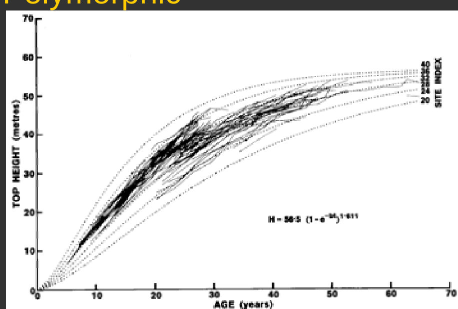
Usually fitted after logarithmic transformation. Allows linear regression to be used, and makes the variance more homogeneous (more uniform over time, improving estimation).

Anamorphic



Anamorphic = “same shape”: vertically proportional.
Difficult to know for sure with data like this (far from asymptote).

“Polymorphic”



Terminology not very appropriate, these happen to be horizontally proportional (same shape).
Richards equation (aka von Bertalanffy, Chapman-Richards). Note that site-dependent parameter *b* affects the time scale.
New Zealand radiata pine PSPs, index age 20.

Exponents, exponentials, logs

$9 \times 9 \times 9 \times 9 = 9^4$ $\quad aa \ldots a = a^n$

$a^3 a^2 = (aaa)(aa) = a^5$ $\quad a^m a^n = a^{m+n}$ $\quad (a^m)^n = a^{mn}$

$a^1 = a$ $\quad a^0\ ?$ $\quad a^0 a = a$ $\quad a^0 = 1$

$a^{-1}\ ?$ $\quad a^{-1} a^1 = a^0$ $\quad a^{-1} a = 1$ $\quad a^{-1} = \frac{1}{a}$

$a^{2.5} = aa a^{1/2}$ $\quad a^{1/2} a^{1/2} = a$ $\quad a^{1/2} = \sqrt{a}$ $\quad a^{1/n} = \sqrt[n]{a}$

$a^x a^y = a^{x+y}$ $\quad (a^x)^y = a^{xy}$

$10^4 = 10000$ $\quad 2^n$ $\quad e^x = \exp(x)$ $(e = 2.718\ldots)$

Mathematical interlude.

Exponents, exponentials, logs

$a^x = y \quad \Leftrightarrow \quad x = \log_a y$

$(a^u)(a^v) = a^{u+v}$

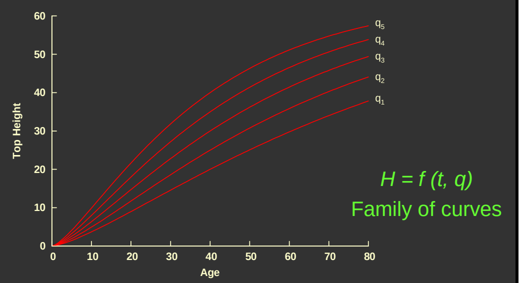
$\log_a(xy) = \log_a x + \log_a y$ $\quad \log_a(x^z) = z \log_a x$

$\log_{10} x$, $\log x$ common logarithm

$\log_e x$, $\log x$, $\ln x$ natural logarithm

$\frac{d \log_a x}{dt} = (\log_e a) \frac{1}{x} \frac{dx}{dt}$ $\quad \dot{\ln x} = \dot{x}/x$

Site Index - Deterministic

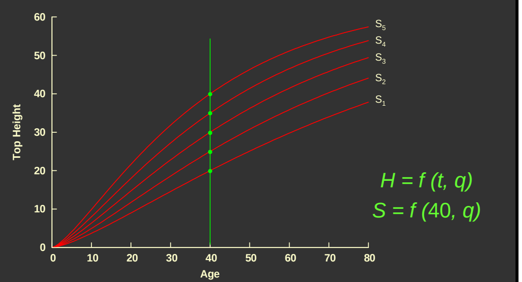


Basic idea: different curves for different site qualities, better quality at the top.

Infinite number of curves, labeled by parameter *q*.

Any one-to-one transformation of *q* would serve as well.

Site Index - Deterministic



Most common labeling scheme: *Site index* = top height at an *index* or *base age*.

Obtainable from any other *q*, and vice-versa.

Nothing magic about the base age.

Site Index - Deterministic

- In specific models $H = f(t, q)$ there are adjustable parameters $\mathbf{p} = (p_1, p_2, \ldots)$:
  $H = f(t, \mathbf{p}, q)$
- The p_i are common to all stands or plots
  "Global"
- q is site-dependent, specific to each stand or plot
  "Local"

Site Index - Deterministic

- Examples:
  - Anamorphic Schumacher
    $H = a \exp(-b / t)$, a (=q) local, b global
    $S = a \exp(-b / 40) \rightarrow a = S / \exp(-b / 40)$
    $\rightarrow H = S \exp[-b (1 / t - 1/40)]$
  - "Polymorphic" Richards
    $H = a [1 - \exp(-b t)]^c$, b local, a and c globals
    $S = a [1 - \exp(-40 b)]^c \rightarrow b = -\ln [1 - (S/a)^{1/c}] / 40$
    $\rightarrow H = a \{1 - [1 - (S/a)^{1/c}]^{t/40}\}^c$

Equivalent forms with *q* or *S*.

In a site index model only the globals need to be given specific numeric values. The *S* or other local parameter chooses a particular curve from the family.

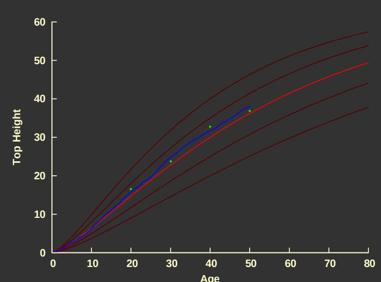
Recap

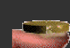
- Site quality. Site index, SI curves, SI equations
  - Schumacher: $H = a \exp(-b / t)$, one of a or b is "local" (site-dependent, q), the other is "global".
    - E.g., $a = q$ local $\rightarrow$ anamorphic
  - Richards: $H = a\,[1 - \exp(-b\,t)]^c$, one of a, b or c is local.
  - General: $H = f(t, \mathbf{p}, q)$, q is local, $\mathbf{p} = (p_1, p_2, \ldots)$ is global.

Recap

- Temporary plots $\rightarrow$ guide curve methods
  - Anamorphic
  - Polymorphic (using SD by age classes)
- PSP's or stem analysis $\rightarrow$ more info, many methods

Variability - Stochastics





Site index?

In reality, growth rate will vary because of weather, etc. (blue).
In addition, there may be measurement and/or sampling errors (green).
Confusion and controversy.
Question 1: which/what is the site index now?

Variability - Stochastics

1. Stand height at age 50
   "Stand site-index"
2. "Expected" height at age 50
   "Site site-index"

- Site index: Most likely top height at a base age among all the hypothetical stands that could grow on the site.

Two views:
1. Literal. Point on blue curve. Index is a property of the stand!
2. More convoluted, trying to stick to the original concept (property of the site): point on the red curve.

Definitions cannot be wrong; they may be more or less natural, more or less useful.
I choose definition 2.

Modelling approaches (PSP / SA)

- Parameter prediction
- Mixed effects
- "Difference equation" (Bailey-Clutter)
- Stochastic differential equation (SDE)

Parameter prediction method

1. Fit $H = f(t, a, b, \ldots)$ to each plot
2. Calculate S-estimate (H at $t=50$) for each plot
3. Regress $a = g_1(S)$, $b = g_2(S)$, ... $\rightarrow H = f'(t, S)$
4. Optionally, tweak to pass through S at $t=50$

Not "base-age invariant". Unrealistic statistical assumptions (independence, ...)

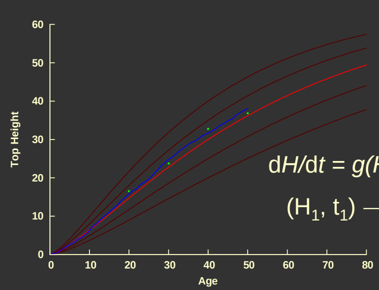
Most common. Details vary.
Two stages; fit each plot separately first.
Often a previously published equation is used, omitting steps 1 and 3, and estimating *S* in step 2 by interpolation.
Needs long time series: plots should have measurements near the index age for good results. Isolated data far from it cannot be used.
Results depend of chosen index age (method not "base-age invariant").

Mixed effect models

- $H = f(t, \mathbf{p}, q)$
- Assume q "random", with given distribution (usually Normal)
- Assume the stands in the data are a random sample
- Usually assume some covariance structure for successive measurements
- Use linear or non-linear mixed effects models from standard statistical packages

But growth data is rarely, if ever, a simple random sample from the population.

Bailey-Clutter, SDE



$dH/dt = g(H, t, \mathbf{p}, q)$

$(H_1, t_1) \rightarrow (H_2, t_2)$

Rational estimation requires a reasonable (stochastic) model for the error structure.
To understand the blue curve, think in terms of increments.

Bailey-Clutter, SDE

- $H = f(t, \mathbf{p}, q)$ ↔ $dH/dt = g(H, t, \mathbf{p}, q)$
  - B-C: $dH/dt = g(H, t, \mathbf{p})$ (site-independent)
  - SDE: $dH/dt = g(H, \mathbf{p}, q)$ + *noise*
- $H_2 = F(H_1, t_1, t_2, \mathbf{p}, q)$
  - B-C: $H_2 = F(H_1, t_1, t_2, \mathbf{p})$
  - SDE: $H_2 = F(H_1, t_2 - t_1, \mathbf{p}, q)$ + *accum. noise*

Differential equation obtained from the height curve model, or vice-versa

Integration gives transition function

SDE

- $dH/dt = g(H, \mathbf{p}, q, u(t))$

  $u(t)$ is "environmental noise" (a stochastic process)

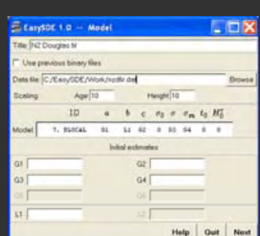
  E.g., $dH^c/dt = b\,(a^c - H^c) + u(t)$ → Richards
- $h_i = H(t_i) + \varepsilon_i$

  (measurement / sampling error)
- Integrate, estimate "most likely" $\mathbf{p}$

Scary math

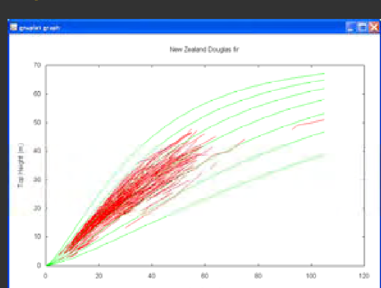
General formulation.

SDE software



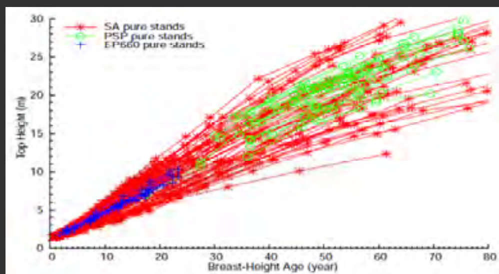
forestgrowth.unbc.ca/sde

Example



With good data most approaches give reasonable results.

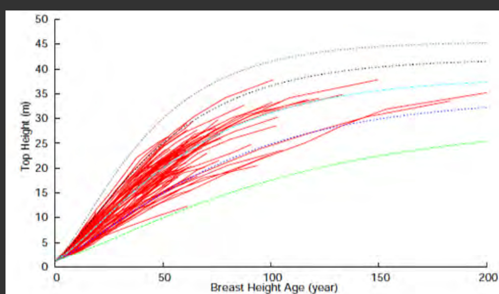
Example: Spruce in the SBS



Zhengjun Hu, UNBC MSc thesis

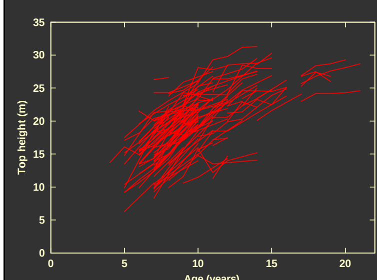
Combining stem analysis and PSSP data. Dominant trees selected for stem análisis may not have been dominant when young: potencial for bias.

Example: Spruce in the SBS



SDE model.

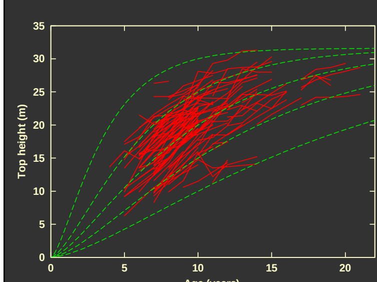
Example



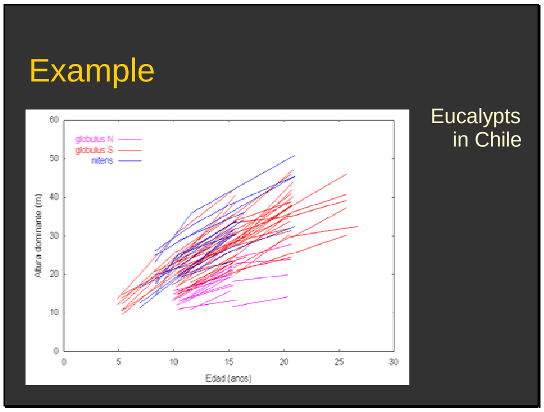
Eucalypt in Spain

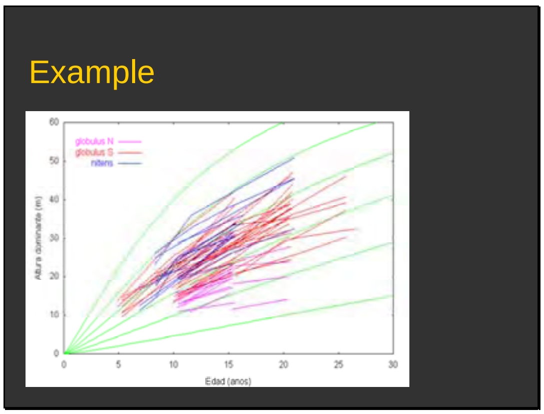
Poor data is more challenging.

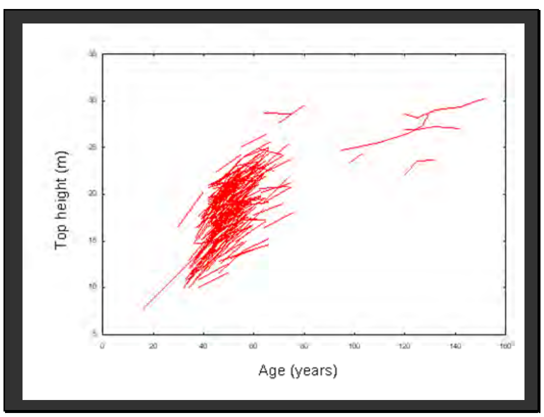
Example



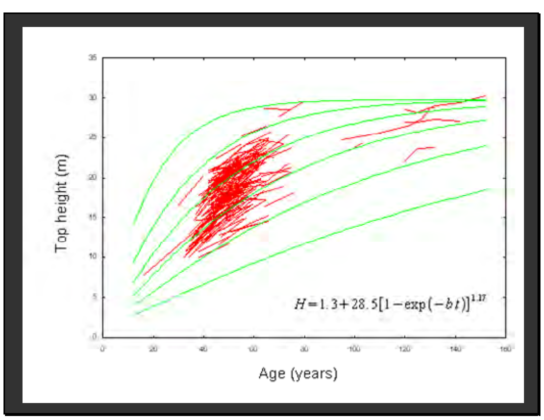
For.Ecol.Man. 173: 49-62, 2003







Our lodgepole pine data set, only plots with two or more measurements.



Polymorphic Richards (slightly better fit than polymorphic), obtained with method from *Biometrics 39*:1059-1072, 1983.

Note site-dependent parameter *b* which gives rise to the family of site index curves.

In summary...

- Guide curve:
  Curve through the middle. Others relative to it, usually proportional (anamorphic).
- Parameter prediction:
  Curve through each plot. "Harmonize": parameters as functions of estimated *S*.
- State-space (Bailey-Clutter, SDE):
  Growth rate from current *H* (and *t* in B-C) . Site-dependent parameter in SDE. Integrate rate.

See http://web.unbc.ca/~garcia/publ/SiteSDEj2.pdf

Site Index in BC …

- SI models for most species. SiteTools. Through parameter-prediction methods, stem-analysis data.
- At UNBC (to appear):
  - Adrian Batho: S.I. for lodgepole pine. SDE approach. Stem-analysis + PSPs
  - Zhengjun Hu: Same for spruce, + stand growth model

www.for.gov.bc.ca/hre/sitetool

… Site Index in BC

- Site index species conversion
  $S_1 = a + b\ S_2$
- Growth intercept models
- Site iIndex Estimates by Site Series (SIBEC)
- Several SI / site variables research studies

www.for.gov.bc.ca/hre/sibec